7-7

SF - alheimur með N_γ = fasti

$$N = 10^{20} \frac{1}{cm^3}$$

Getum ekki notað $\tau_E \equiv \frac{2\pi\hbar^2}{M}\left(\frac{n}{2.612}\right)^{2/3}$

þ.s. $M = 0$

Í dæmi (4-1) fékkst $N_e = \frac{2.404\, V\tau^3}{\pi^2\hbar^3 C^3}$

Finna τ_c þ.a. fyrir $\tau < \tau_c$ $N_e < N$

$$n_e = \frac{2.404\,\tau^3}{\pi^2\hbar^3 C^3} \rightarrow n = \frac{2.404\,\tau_c^3}{\pi^2\hbar^3 C^3}$$

$$\rightarrow \tau_c^3 = \frac{n \cdot \pi^2\hbar^3 C^3}{2.404} \rightarrow \tau_c = \hbar C \sqrt[3]{\frac{n\pi^2}{2.404}}$$

$$\tau_c = 1.05 \cdot 10^{-27} \,\text{erg s} \; 3 \cdot 10^{10} \,\frac{\text{cm}}{\text{s}} \sqrt[3]{\frac{10^{20} \frac{1}{\text{cm}^3} \pi^2}{2.404}}$$

$$= 2.34 \cdot 10^{-10} \,\text{erg} \qquad = 146 \text{ eV}$$

$$\rightarrow \quad T_c = \tau_c / k_B = 1.7 \cdot 10^{6} \,\text{K}$$

7-9 Bōse-eindir í einni vídd

Rekna $N_e(\tau)$

Ein vídd

Eins og sést í 7-1 er

$$\mathcal{D}_1(\varepsilon) = \frac{L}{2\pi}\sqrt{\frac{2m}{\hbar^2\varepsilon}}$$

$$N \simeq N_0 + \int_0^\infty d\varepsilon\, \mathcal{D}_1(\varepsilon) f(\varepsilon,\tau)$$

$$= N_0 + N_\varepsilon$$

$$N_\varepsilon = \frac{L}{2\pi}\sqrt{\frac{2m}{\hbar^2}}\int_0^\infty d\varepsilon\, \frac{f(\varepsilon,\tau)}{\sqrt{\varepsilon}}$$

$$= \frac{L}{2\pi}\sqrt{\frac{2m}{\hbar^2}}\int_0^\infty \frac{d\varepsilon}{\sqrt{\varepsilon}\left\{\lambda^{-1}e^{\frac{\varepsilon}{\tau}}-1\right\}}$$

ef $\lambda \sim 1$

$$N_\varepsilon = \frac{L\tau}{2\pi}\sqrt{\frac{2m}{\hbar^2\tau}}\int_0^\infty \frac{dx}{\sqrt{x}\,(e^x-1)}$$

$$= \frac{L}{2\pi}\sqrt{\frac{2m\tau}{\hbar^2}}\int_0^\infty \frac{dx}{\sqrt{x}\left\{e^x-1\right\}}$$

fallið $\dfrac{1}{\sqrt{x}(e^{x}-1)} \xrightarrow[x\to 0]{} \dfrac{1}{\sqrt{x}(1-x+\cdots-1)} \sim x^{-3/2}$

er ósamleitið og ekki heildanlegt

$\to$ Lagt er til að heildið að framan sé ekki notað.

7-11 fyrir fermi-eindir (eitt svigrúm)

$$\langle (\Delta N)^2 \rangle = \langle N \rangle \{1 - \langle N \rangle\}$$

Eitt svigrúm með setnu 0 eða 1

$$\langle (\Delta N)^2 \rangle = \langle (N - \langle N \rangle)^2 \rangle = \langle N^2 \rangle - \langle N \rangle^2$$

fyrir eittsvigrúm gildir $N^2 = N \rightarrow \langle N^2 \rangle = \langle N \rangle$

$$\rightarrow \langle (\Delta N)^2 \rangle = \langle N \rangle - \langle N \rangle^2 = \langle N \rangle \{1 - \langle N \rangle\}$$

7-12 fyrir Bose-eindir með eitt svigrúm

$$\langle (\Delta N)^2 \rangle = \langle N \rangle \left(1 + \langle N \rangle \right)$$

Höfum (5-59)

$$\langle N \rangle = \frac{\tau}{\mathfrak{z}} \frac{\partial \mathfrak{z}}{\partial \mu} \,, \qquad \mathfrak{z} = \sum_{N=0}^{\infty} \exp\left\{ \frac{(N\mu - \varepsilon)}{\tau} \right\}$$

$$\langle N^2 \rangle = \frac{1}{\mathfrak{z}} \sum_{N=0}^{\infty} N^2 \exp\left\{ \frac{(N\mu - \varepsilon)}{\tau} \right\} = \frac{\tau^2}{\mathfrak{z}} \frac{\partial^2 \mathfrak{z}}{\partial \mu^2}$$

$$\rightarrow \langle (\Delta N^2) \rangle = \langle N^2 \rangle - \langle N \rangle^2 = \tau^2 \left\{ \frac{1}{\mathfrak{z}} \frac{\partial^2 \mathfrak{z}}{\partial \mu^2} - \frac{1}{\mathfrak{z}^2} \left(\frac{\partial \mathfrak{z}}{\partial \mu} \right)^2 \right\}$$

$$= \tau \frac{\partial \langle N \rangle}{\partial \mu}$$

$$\rightarrow \langle (\Delta N^2) \rangle = \tau \frac{\partial \langle N \rangle}{\partial \mu} = \tau \frac{\partial}{\partial \mu} \left\{ \frac{1}{e^{\frac{\varepsilon-\mu}{\tau}} - 1} \right\}$$

$$= \frac{e^{\frac{\varepsilon-\mu}{\tau}}}{\left[e^{\frac{\varepsilon-\mu}{\tau}} - 1 \right]^2} = \left\{ \frac{1}{e^{\frac{\varepsilon-\mu}{\tau}} - 1} \right\} \left\{ \frac{e^{\frac{\varepsilon-\mu}{\tau}}}{e^{\frac{\varepsilon-\mu}{\tau}} - 1} \right\}$$

$$= \langle N \rangle \left\{ \langle N \rangle + 1 \right\}$$