7-2

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Afstæðilegt fermi gas

$\varepsilon \gg mc^2$

$\rightarrow \varepsilon \simeq pc$

$V = L^3$: teningur

Gerum ráð fyrir að

a)

$$p = \frac{\pi\hbar}{L}\underbrace{\sqrt{n_x^2+n_y^2+n_z^2}}$$

$$\varepsilon_F = c\,p_F = c\,\frac{\pi\hbar}{L}\,n_F$$

$$N = 2\times\frac{1}{8}\times\frac{4\pi}{3}n_F^3$$

$$n_F = \underbrace{\left(\frac{3N}{\pi}\right)^{1/3}}$$

$$\rightarrow \varepsilon_F = \frac{c\pi\hbar}{L}\left(\frac{3N}{\pi}\right)^{1/3}$$

$$= \hbar\pi c\left(\frac{3N}{\pi L^3}\right)^{1/3}$$

$$= \hbar\pi c\left(\frac{3n}{\pi}\right)^{1/3}$$

b) finna U_0

$$U_0 = 2\sum_{n\le n_F}\varepsilon_n$$

$$= 2\times\frac{1}{8}\times 4\pi\int_0^{n_F} dn\, n^2\varepsilon_n$$

$$U_0 = 2 \times \frac{1}{8} \times 4\pi \int_0^{n_F} dn\, n^3 \, \frac{c\pi\hbar}{L}$$

$$N = \frac{4\pi}{3} n_F^3 \cdot 4^{-1}$$

$$= \frac{\pi^2 c\hbar}{L} \int_0^{n_F} dn\, n^3 = \frac{\pi^2 c\hbar}{L\,4} n_F^4 = \frac{\pi^2 c\hbar}{L4} \cdot \frac{3\cdot 4}{4\pi} N \cdot n_F$$

En áður var komið

$$\Sigma_F = c p_F = c\,\frac{\pi\hbar}{L} n_F \;\rightarrow\; n_F = \frac{\Sigma_F L}{c\pi\hbar}$$

$$\rightarrow U_0 = \frac{\pi^2 c\hbar}{4L} \frac{3\cdot 4}{4\pi} N \frac{\Sigma_F L}{c\hbar\pi} = \frac{3}{4} N \Sigma_F$$

7-3

a) Sýna að fyrir fermigas í grunnástandi gildi

$$P = \frac{(3\pi^2)^{2/3}}{5} \cdot \frac{\hbar^2}{m} \left(\frac{N}{V}\right)^{5/3}$$

$$U_0 = \frac{3}{5} N \varepsilon_F$$ (þýðir að $\tau = 0$), $\varepsilon_F = \frac{\hbar^2}{2m}(3\pi^2 n)^{2/3}$

$n = \frac{N}{V}$

Vitum að $p = -\left(\frac{\partial U}{\partial V}\right)_{\tau N} = -\left(\frac{\partial U_0}{\partial V}\right)_{\tau N}$

Áður verið kynnt fyrir okkur að breyting á V þ.a. kerfið haldist í vissu ástandi er jafn óreiðubreyting

$$\rightarrow p = -\left(\frac{\partial U_0}{\partial V}\right)_N = -\frac{3}{5} N \left(\frac{\partial \varepsilon_F}{\partial V}\right)_N = \frac{3}{5} \cdot \frac{2}{3} \frac{N}{V} \varepsilon_F$$

$$= \frac{2}{5} \frac{N}{V} \varepsilon_F = \frac{2}{5} n \varepsilon_F$$

b) finna σ þegar $\tau \ll \varepsilon_F$

$$C_V = \tau \left(\frac{\partial \sigma}{\partial \tau}\right)_V$$

Vitum að $C_V \xrightarrow[\tau \to 0]{} \alpha \tau$

$$\Longrightarrow \left(\frac{\partial \sigma}{\partial \tau}\right)_V = \alpha$$

Þá $\sigma = \alpha \tau = C_V$

Ef $\sigma \xrightarrow[\tau \to 0]{} 0$

þá kemur engin fasti til greina í $\sigma = \alpha \tau + \sigma_0$

$$\sigma = C_V = \frac{\pi^2 N \tau}{2 \varepsilon_F}$$

samkvæmt Eq. (37)

í bók á bls. 193

7-5 $\quad {}^3He$ vökvi, $I = \frac{1}{2}$

finna v_F, ε_F, og T_F fyrir ${}^3He$ við $\tau = 0$

gefið $\quad \rho = 0.081 \frac{g}{cm^3} \qquad n = \frac{\rho}{M}, \quad M = 3\,amu$

$amu = 1.66 \cdot 10^{-24}\,g$

$$\varepsilon_F = \frac{\hbar^2}{2M}\left(3\pi^2 n\right)^{2/3}$$

$$= \frac{\hbar^2}{2M}\left(3\pi^2 \frac{\rho}{M}\right)^{2/3} = \frac{\hbar^2}{2M^{5/3}}\left(3\pi^2 \rho\right)^{2/3}$$

$$= \frac{\left(1{,}05 \cdot 10^{-27}\,erg\,s\right)^2}{2 \cdot \left(3 \cdot 1{,}66 \cdot 10^{-24}\,g\right)^{5/3}}\left(3\pi^2 \cdot 0.081 \frac{g}{cm^3}\right)^{2/3} = 6.8 \cdot 10^{-16}\,erg$$

$$= 0.43\,meV$$

$$T_F = \frac{\varepsilon_F}{k_B} = 4.9\,K$$

$$\varepsilon_F = \frac{1}{2} M v_F^2 \quad \rightarrow \quad v_F = \sqrt{\frac{2\varepsilon_F}{M}}$$

$$v_F = \sqrt{\frac{2\cdot 6.8\cdot 10^{-16}\,\text{erg}}{(3\cdot 1.66\cdot 10^{-24})}} = 1.65\cdot 10^{4}\ \frac{\text{cm}}{\text{s}}$$

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b)

C_V fyrir $\tau \ll \tau_F$ Mælt gildi $C_V = 2.89\, N k_B T$

$C_V = \frac{\pi^2}{2} N \frac{\tau}{\tau_F}$ τ venjul. ein. $\frac{\pi^2}{2} N \frac{k_B T}{T_F}$

$$\frac{C_V}{N} = \frac{\pi^2}{2}\,\frac{k_B T}{T_F} = \left(\frac{\pi^2}{2 T_F}\right)\cdot k_B T = 1.0\ k_B T$$

Í þessum fermi vökva eru hreyfingar atómanna tengdar fylgni böndum, því er raun varmarýmdin hærri eins og atómin væru þyngri $\Leftrightarrow$ vökva eiginleikar
víxlverkan

7-6 Hvíturdvergur : M, R

Rafeindir kulgas
Róteindir klyggas

a) sjálforka (þyngdar)

Ef þéttleikinn er fastur

$$\frac{M}{V} = \rho$$

$$\rightarrow \rho = \frac{3M}{4\pi R^3}$$

því $V = \frac{4\pi}{3} R^3$

Sjálforkan er stöðuorkan
fyrir allan massann
(þyngdarstöðuorkan)

$$\Phi(r) = \frac{G\, m(r)}{r}$$

$$m(r) = \rho \cdot \frac{4\pi}{3} r^3 = M \frac{r^3}{R^3}$$

$$U_G = \int_0^R 4\pi r^2 dr\, \rho\, \Phi(r)$$

$$= \frac{GM}{R^3} 4\pi \left(\frac{3M}{4\pi R^3}\right) \int_0^R r^4 dr$$

$$= \frac{3GM^2}{R^6} \frac{R^5}{5} = \frac{3}{5} G M^2 \frac{1}{R}$$

Setjum stöðuorkuna 0 fyrir tvo massa þegar fjarlægð þeirra $\to \infty$

$$\to U_G = -\frac{3}{5}\frac{GM^2}{R}$$

ρ er í raun ekki fasti setjum sem stærðargráðu-nálgun

$$U_G \approx -\frac{GM^2}{R}$$

b) Meta hreyfiorku

Rafeindamassi: m

Róteindarmassi: M_H

$$N = N_e = N_H \approx \frac{M}{M_H}$$

$$U_0 = \frac{3}{5} N \varepsilon_F$$

$$\varepsilon_F = \frac{\hbar^2}{2m}\left(3\pi^2 n\right)^{2/3}$$

$$\to U_0 = \frac{3\hbar^2}{2m}\left(\frac{3\pi^2}{V}\right)^{2/3}\frac{3}{5} N^{5/3}$$

$$= \frac{3\hbar^2}{2m}\left(\frac{3\pi^2 \cdot 3}{4\pi R^3}\right)^{2/3}\frac{3}{5} N^{5/3}$$

$$= \frac{\hbar^2 N^{5/3}}{mR^2}\cdot \underbrace{\frac{9}{10}\left(\frac{3\pi}{4}\right)^{2/3}}_{\sim 1{,}6} \sim \frac{\hbar^2 N^{5/3}}{mR^2}$$

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$$U_0 \sim \frac{\hbar^2 N^{5/3}}{mR^2} = \frac{\hbar^2}{mR^2}\left(\frac{M}{M_H}\right)^{5/3}$$

$$RM^{1/3} \sim 8\cdot 10^{19}\ \frac{g^{1/3}}{cm} \sim 10^{20}\frac{g^{1/3}}{cm}$$

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c) Ef $U_0 \sim |U_G|$

$$\rightarrow \frac{\hbar^2}{mR^2}\left(\frac{M}{M_H}\right)^{5/3} \sim \frac{GM^2}{R}$$

$$\rightarrow \frac{\hbar^2}{mR}\left(\frac{M}{M_H}\right)^{5/3} M^{-2} \sim G$$

$$\frac{\hbar^2}{m}\left(\frac{1}{M_H}\right)^{5/3}\frac{1}{RM^{1/3}} \sim G$$

$$RM^{1/3} \sim \frac{\hbar^2}{m}\left(\frac{1}{M_H}\right)^{5/3}\frac{1}{G}$$

d) Ef $M = M_\odot = 2\cdot 10^{33}\,g$

hver er på pettlitidvergsins

$$RM^{1/3} \sim 10^{20}$$

$$\rightarrow R = \frac{10^{20}\ g^{1/3}/cm}{M^{1/3}}$$

$$\approx 8\cdot 10^{8}\ cm$$

$$\approx 8\cdot 10^{3}\ km$$

e) Nifteindastjarna

$$U_0 = \frac{\hbar^2}{m \cdot 1837 R^2} \left(\frac{M}{M_H}\right)^{5/3}$$

$$R M^{1/3} \sim \frac{\hbar^2}{1837 \cdot m} \left(\frac{1}{M_H}\right)^{5/3} \frac{1}{G}$$

$$\sim 4 \cdot 10^{16}$$

$$\rightarrow R \sim 4 \text{ Km}$$