16-02

(i) Sýnið að $\left(\frac{\partial T}{\partial V}\right)_U = -\frac{1}{C_V}\left\{T\left(\frac{\partial p}{\partial T}\right)_V - p\right\}$

notum (16.64)

$$\left(\frac{\partial T}{\partial V}\right)_U = -\left(\frac{\partial T}{\partial U}\right)_V \left(\frac{\partial U}{\partial V}\right)_T$$

og

$$\left(\frac{\partial U}{\partial T}\right)_V = C_V$$

enn fremur

$$dU = TdS - pdV \rightarrow \left(\frac{\partial U}{\partial V}\right)_T = T\left(\frac{\partial S}{\partial V}\right)_T - p$$

$$\left(\frac{\partial T}{\partial V}\right)_U = -\frac{1}{C_V}\left[T\left(\frac{\partial S}{\partial V}\right)_T - p\right]$$

og Maxwell gefur $\left(\frac{\partial S}{\partial V}\right)_T = \left(\frac{\partial p}{\partial T}\right)_V$

þannig fæst að lokum

$$\left(\frac{\partial T}{\partial V}\right)_U = -\frac{1}{C_V}\left\{T\left(\frac{\partial p}{\partial T}\right)_V - p\right\} \qquad (*)$$

Sem minnir á að lausn mín á 14-07 er röng
Ég gerði ráð fyrir að $dS = 0$ þar (í stað að $U =$ fasti)
$dQ_{rev} = 0$ → jafngengt óvermið ferli er líka
jafn óvermið ferli, en Joul-þenslan er ekki jafngeng
því þarf að nota í dæminu (*)

$$\Delta T = -\frac{1}{C_V}\int_{V}^{xV} dV\left\{T\left(\frac{\partial p}{\partial T}\right)_V - p\right\}$$

og þegar ástandsjafnan

$$p = \frac{nRT}{V-nb} - \frac{n^2a}{V^2}$$

er notuð fæst

$$\left\{T\left(\frac{\partial p}{\partial T}\right)_V - p\right\} = \frac{n^2a}{V^2}$$

og því

$$\Delta T = -\frac{n^2a}{C_V}\int_V^{\alpha V}\frac{dV'}{V'^2} = +\frac{n^2a}{C_V}\frac{1}{V'}\Bigg|_V^{\alpha V}$$

$$= \frac{n^2a}{C_V}\left\{\frac{1}{\alpha V} - \frac{1}{V}\right\} = \frac{n^2a}{C_V V}\left\{\frac{1-\alpha}{\alpha}\right\}$$

$$= -\frac{n^2a}{C_V \cdot V}\left\{\frac{\alpha-1}{\alpha}\right\}$$

(ii) Sýna að

$$\left(\frac{\partial T}{\partial V}\right)_S = -\frac{1}{C_V} T \left(\frac{\partial p}{\partial T}\right)_V$$

sem var einmitt það sem ég reiknaði í 14-07

$$dS = \left(\frac{\partial S}{\partial T}\right)_V dT + \left(\frac{\partial S}{\partial V}\right)_T dV$$

$$dS = 0 \;\rightarrow\; \left(\frac{\partial S}{\partial T}\right)_V dT + \left(\frac{\partial S}{\partial V}\right)_T dV = 0$$

$$\rightarrow \left(\frac{\partial T}{\partial V}\right)_S = -\left(\frac{\partial T}{\partial S}\right)_V \left(\frac{\partial S}{\partial V}\right)_T$$

notum $\frac{C_V}{T} = \left(\frac{\partial S}{\partial T}\right)_V$

$$= -\frac{T}{C_V}\left(\frac{\partial S}{\partial V}\right)_T$$

og síðan Maxwell

$$\left(\frac{\partial S}{\partial V}\right)_T = \left(\frac{\partial p}{\partial T}\right)_V$$

$$= -\frac{T}{C_V}\left(\frac{\partial p}{\partial T}\right)_V$$

(iii) Sýna að

$$\left(\frac{\partial T}{\partial p}\right)_H = \frac{1}{C_p}\left\{T\left(\frac{\partial V}{\partial T}\right)_p - V\right\}$$

Byrjum með

$$\left(\frac{\partial T}{\partial p}\right)_H = -\left(\frac{\partial T}{\partial H}\right)_p\left(\frac{\partial H}{\partial p}\right)_T$$

notum

$$dH = TdS + Vdp \quad \longrightarrow \left(\frac{\partial H}{\partial T}\right)_p = T\left(\frac{\partial S}{\partial T}\right)_p = C_p$$

$$\longrightarrow \left(\frac{\partial H}{\partial p}\right)_T = T\left(\frac{\partial S}{\partial p}\right)_T + V$$

þá fæst

$$\left(\frac{\partial T}{\partial p}\right)_H = -\frac{1}{C_p}\left\{T\left(\frac{\partial S}{\partial p}\right)_T + V\right\}.$$

Maxwell gefur $\left(\frac{\partial S}{\partial p}\right)_T = -\left(\frac{\partial V}{\partial T}\right)_p$

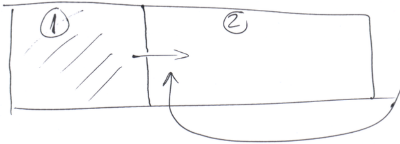
$$\rightarrow \left(\frac{\partial T}{\partial p}\right)_H = +\frac{1}{C_p}\left\{T\left(\frac{\partial V}{\partial T}\right)_p - V\right\}$$

(i) $\leftarrow$ Joule þensla $\left(\frac{\partial T}{\partial V}\right)_U$ ekki jafngeng

(ii) $\leftarrow$ óvermin jafngeng þensla $\left(\frac{\partial T}{\partial V}\right)_S$

(iii) $\leftarrow$ Joule-Kelvin þensla (Joule-Thomson)

óvermin, ekki jafngeng



í gegnum ventil
tvær bullur eins og í
dæmi 12-05

b) Kjörgas

$$p = \frac{nRT}{V}$$

$$\left(\frac{\partial T}{\partial V}\right)_U = -\frac{1}{C_V}\left\{T\left(\frac{\partial p}{\partial T}\right)_V - p\right\}$$

$$= 0$$

$$\left(\frac{\partial T}{\partial p}\right)_H = +\frac{1}{C_p}\left\{T\left(\frac{\partial V}{\partial T}\right)_p - V\right\} = 0$$

$$\left(\frac{\partial T}{\partial V}\right)_S = -\frac{1}{C_V}T\left(\frac{\partial p}{\partial T}\right)_V = -\frac{T}{C_V}\frac{nR}{V}$$

$$\rightarrow dT = -\frac{nRT}{C_V}\frac{dV}{V} = -\frac{2}{3}T\frac{dV}{V}$$

$$\rightarrow \frac{dT}{T} = -(\gamma - 1)\frac{dV}{V}$$

$$\frac{C_V}{Rn} = \frac{3}{2}$$

$$\gamma = \frac{5}{3}$$

16-03 Σήμα δέ

$$\left(\frac{\partial U}{\partial V}\right)_T = \frac{C_p - C_V}{V\beta_p} - p$$

p.s. $\beta_p = \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_p$

Notam dé $U(V,T)$

$$\left.\begin{aligned} dU &= đQ + đW = đQ - p\,dV \\ dU &= \left(\frac{\partial U}{\partial T}\right)_V dT + \left(\frac{\partial U}{\partial V}\right)_T dV \end{aligned}\right\} \rightarrow \boxed{đQ = \left(\frac{\partial U}{\partial T}\right)_V dT + \left\{\left(\frac{\partial U}{\partial V}\right)_T + p\right\} dV}$$

$$C_V = \left(\frac{\partial Q}{\partial T}\right)_V = \left(\frac{\partial U}{\partial T}\right)_V$$

$$C_p = \left(\frac{\partial Q}{\partial T}\right)_p = \left(\frac{\partial U}{\partial T}\right)_V + \left[\left(\frac{\partial U}{\partial V}\right)_T + p\right]\left(\frac{\partial V}{\partial T}\right)_p$$

$$\rightarrow \left(\frac{\partial U}{\partial V}\right)_T + p = C_p \underbrace{\left(\frac{\partial T}{\partial V}\right)_p}_{\frac{1}{\beta_p V}} - \underbrace{\left(\frac{\partial U}{\partial T}\right)_V}_{C_V} \underbrace{\left(\frac{\partial T}{\partial V}\right)_p}_{\frac{1}{\beta_p V}}$$

$$\rightarrow \boxed{\left(\frac{\partial U}{\partial V}\right)_T = \frac{C_p - C_V}{\beta_p V} - p}$$

16-04 $\quad U = U(S,V)$ — náttúrulegar breytur

finnið þá jöfnur fyrir T og p

1. Lögmálið $\quad dU = TdS - pdV$

$$dU = \left(\frac{\partial U}{\partial S}\right)_V dS + \left(\frac{\partial U}{\partial V}\right)_S dV$$

$$\rightarrow \quad T = \left(\frac{\partial U}{\partial S}\right)_V \quad \text{og} \quad p = -\left(\frac{\partial U}{\partial V}\right)_S$$

b) Setjum sem svo að við þekkjum V, T og $U(V,T)$

Hver er þá jafnan fyrir p?

notum Maxwell

$$dU = T\,dS - p\,dV$$

$$\rightarrow \left(\frac{\partial U}{\partial V}\right)_T = T\left(\frac{\partial S}{\partial V}\right)_T - p = T\left(\frac{\partial p}{\partial T}\right)_V - p$$

Komum yfir í eina afleiðu með

$$\left(\frac{\partial U}{\partial V}\right)_T = T^2\left(\frac{\partial\left(\frac{p}{T}\right)}{\partial T}\right)_V \rightarrow \left(\frac{\partial\left(\frac{p}{T}\right)}{\partial T}\right)_V = \frac{1}{T^2}\left(\frac{\partial U}{\partial V}\right)_T$$

heildum

$$\rightarrow \boxed{\frac{p}{T} = \int \frac{dT}{T^2}\left(\frac{\partial U}{\partial V}\right)_T + f(V)}$$

(16-07) Höfum (16.79) Kjörgas

$$S = C_v \ln T + R \ln V + \text{fasti}$$

notum $pV = RT$ (fyrir eitt mól)

$$\rightarrow \quad S = C_v \ln(pV) + R\ln V + C_1 \qquad \Big| \ \frac{R}{\gamma-1} = C_v$$

$$= C_v \ln(pV) + C_v(\gamma-1)\ln V + C_1$$

$$= C_v \ln(pV) + C_v \ln\left(V^{\gamma-1}\right) + C_1$$

$$= C_v \ln\left(pV^{\gamma}\right) + C_1 \quad , \qquad \rho = \frac{M}{V}$$

$$= C_v \ln\left(\frac{p}{\rho^{\gamma}}\right) + C_2$$

18-02

$$H = G - T\left(\frac{\partial G}{\partial T}\right)_p$$

$$\hookrightarrow G - H = T\left(\frac{\partial G}{\partial T}\right)_p$$

$$\rightarrow \Delta G - \Delta H = T\left(\frac{\partial \Delta G}{\partial T}\right)_p$$

$$dG = Vdp - SdT \rightarrow \Delta G - \Delta H = -T\Delta S$$

Ef $T \rightarrow 0 \rightarrow \Delta S \rightarrow 0$

$$\rightarrow \Delta G - \Delta H \rightarrow 0$$