Dreifing Maxwells og Boltzmanns

Gerum ráð fyrir mjög smáum atómum (← kerfi), sem rekast ekki á en eru í varmajafnvægi við öll hin atómin (← geymir) í ílátinu

Engin víxlverkun atóma
⟹ aðeins hreyfiorka

$$\frac{1}{2}mv^2 = \frac{1}{2}m\left\{v_x^2 + v_y^2 + v_z^2\right\}$$

Hraðadreifing — Hlutfall atóma eða sameinda með hraða milli v_x og $v_x + dv_x$ er $g(v_x)dv_x$

Boltzmann →

$$g(v_x) \sim \exp\left\{-\frac{mv_x^2}{2k_BT}\right\}$$

stöðlum dreifinguna

$$\int_{-\infty}^{\infty} dv_x \exp\left\{-\frac{m v_x^2}{2k_B T}\right\} = \sqrt{\frac{\pi}{m/(2k_B T)}} = \sqrt{\frac{2\pi k_B T}{m}}$$

báðar áttir

$$\longrightarrow \boxed{g(v_x) = \sqrt{\frac{m}{2\pi k_B T}} \exp\left\{-\frac{m v_x^2}{2k_B T}\right\}}$$

Reynum meðaltöl

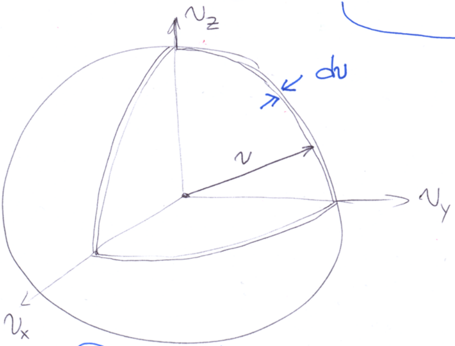
$$\langle v_x \rangle = \int_{-\infty}^{\infty} dv_x \, v_x \, g(v_x) = 0 \qquad \text{(oddstætt fall)}$$

$$\langle |v_x| \rangle = 2\int_{0}^{\infty} dv_x \, v_x \, g(v_x) = \sqrt{\frac{2k_B T}{\pi m}}$$

$$\langle v_x^2 \rangle = \int_{-\infty}^{\infty} dv_x \; v_x^2 \, g(v_x) = \frac{k_B T}{m}$$

Meðal ferð sameindanna (speed)

Á bilinu $v \to v + dv$ — rúmmál $4\pi v^2 \cdot dv$



Hraðarúmið

$\to$ Ferðardreifing

$$f(v)dv \sim v^2 dv \, e^{-\frac{mv^2}{2k_B T}}$$

stöðlun

$$\int_0^\infty dv\, v^2 \exp\left\{-\frac{mv^2}{2k_BT}\right\} = \frac{1}{4}\sqrt{\frac{\pi}{m/(2k_BT)^3}}$$

$$\rightarrow \quad \boxed{f(v)\,dv = \frac{4}{\sqrt{\pi}}\left(\frac{m}{2k_BT}\right)^{3/2} v^2\,dv\,\exp\left\{-\frac{mv^2}{2k_BT}\right\}}$$

Maxwell-Boltzmann dreifing feðar

Meðaltöl

$$\langle v \rangle = \int_0^\infty dv\, v f(v) = \sqrt{\frac{8k_BT}{\pi m}}$$

$$\langle v^2 \rangle = \int_0^\infty dv\, v^2 f(v) = \frac{3k_BT}{m}$$

Samantekt

Áður sást $\langle v_x^2 \rangle + \langle v_y^2 \rangle + \langle v_z^2 \rangle = \frac{3k_B T}{m}$

sem fellur saman við $\langle v^2 \rangle = \frac{3k_B T}{m}$

Tökum líka eftir að

$$v_{rms} = \sqrt{\langle v^2 \rangle} = \sqrt{\frac{3k_B T}{m}} \sim \frac{1}{\sqrt{m}}$$

Meðalorkan

$$\langle E_{KE} \rangle = \frac{1}{2} m \langle v^2 \rangle = \frac{3}{2} k_B T$$

↑

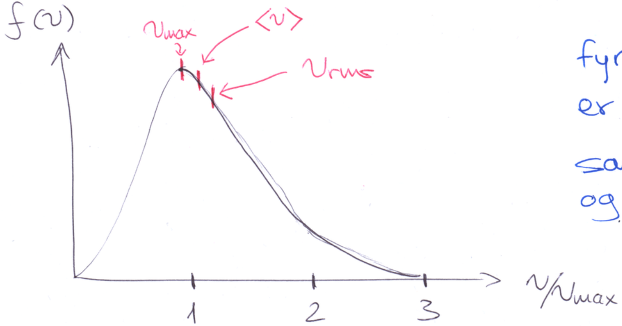
finnum síðar á annan hátt

Hágildi $f(v)$

Finnum með $\frac{df}{dv} = 0$ $\longrightarrow$ $v_{max} = \sqrt{\frac{2k_BT}{m}}$

Því fæst $v_{max} < \langle v \rangle < v_{rms}$

$$\left\{ \sqrt{2} < \sqrt{\frac{8}{\pi}} < \sqrt{3} \right\}$$



fyrir N_2 við $T = 300K$
er $v_{rms} \approx 500$ m/s

sama stærðargráða
og hljóðhraðinn