4-1

Finna meðal heildarfjölda ljóseinda

$\sum \langle s_n \rangle$ í jafnvægi við τ í holrúmi

með V

$$\sum_{n=1}^{\infty} \langle s_n \rangle = \sum_{n=1}^{\infty} \frac{1}{e^{\hbar\omega_n/\tau} - 1} = N_{ph}$$

Gerum ráð fyrir stóru holi til þess að umrita í heildi

$$\omega_n = n\pi \frac{c}{L}$$

2 skautanir

$$N_{ph} \sim \frac{2}{8} \int_0^{\infty} 4\pi n^2 dn \frac{1}{e^{\frac{\hbar c n \pi}{L\tau}} - 1}$$

jákvæð n_x, n_y, n_z

notum $x = \frac{\hbar c n \pi}{L\tau}$

$$dx = \frac{\hbar c \pi}{L\tau} dn$$

$$n = x \frac{L\tau}{\hbar c \pi}$$

$$N_{ph} \sim \frac{2\cdot 4\pi}{8}\left(\frac{L\tau}{\hbar c\pi}\right)^3 \int_0^\infty x^2 dx \,\frac{1}{e^x-1} = 2\cdot\frac{4\pi}{8}\left(\frac{L\tau}{\hbar c\pi}\right)^3 \Gamma(3)\,\zeta(3)$$

þ.s. (GR. 3.411.1) er notað

$$N_{ph} \sim 2\cdot\frac{V}{2\pi^2}\left(\frac{\tau}{\hbar c}\right)^3 \cdot 2 \cdot 1.20205 \cdot \simeq 2.404\,\frac{V}{\pi^2}\left(\frac{\tau}{\hbar c}\right)^3$$

$$\zeta(z) \equiv \sum_{n=1}^{\infty} \frac{1}{n^z} \qquad \text{(GR 9.522.1)}$$

Óreiða alheimsins gæti að mestu leyti verið vegna ljóseinda

4-6

Ljóseindagas

a) Sýna að

$$p = -\left(\frac{\partial U}{\partial V}\right)_\tau = -\sum_j s_j \hbar \left(\frac{d\omega_j}{dV}\right)$$

s_j er fjöldi ljóseinda í hætti j.

Höfum

$$U = \sum_j \langle \varepsilon_j \rangle = \sum_j \langle s_j \rangle \hbar\omega_j \,, \quad \omega_j = \frac{j\pi c}{V^{1/3}}$$

$$p = -\left(\frac{\partial}{\partial V}\sum_j \langle s_j \rangle \hbar\omega_j\right)_\tau = -\sum_j \left(\frac{\partial \langle s_j \rangle}{\partial V}\right)_\tau \hbar\omega_j - \sum_j \langle s_j \rangle \left(\frac{\partial \hbar\omega_j}{\partial V}\right.$$

fyrir ljóseindir gaf (23)

$$\sigma = \frac{4\pi^2 V}{45}\left(\frac{\tau}{\hbar c}\right)^3$$

og dæmi 4-1

$$N_{ph} = \text{fasti} \cdot V \left(\frac{\tau}{\hbar c}\right)^3$$

$$\rightarrow \left(\frac{\partial \langle s_j \rangle}{\partial V}\right)_\sigma = 0$$

og því

$$p = -\sum_j \langle s_j \rangle \frac{\partial \hbar\omega_j}{\partial V}$$

b)

$$\frac{d\omega_j}{dV} = \frac{d}{dV}\left(\frac{j\pi c}{V^{1/3}}\right) = -\frac{j\pi c}{V^{4/3}} \cdot \frac{1}{3}$$

$$= -\frac{j\pi c}{V^{1/3}} \cdot \frac{1}{3V}$$

$$= -\frac{\omega_j}{3V}$$

c)

$$p = -\sum_j \langle s_j \rangle \frac{\partial \hbar\omega_j}{\partial V}$$

$$= \sum_j \langle s_j \rangle \frac{\hbar\omega_j}{3V}$$

$$= \frac{U}{3V}$$

d) H-atóm 1 mole/cm^3

Við hvaða hitastig er þrýstingur ljóseinda-gasins jafn H-gasins

H-gas: $pV = N\tau = Nk_BT$

$= nRT$

ljóseindir: $pV = \frac{U}{3} = \frac{\pi^2 k_B^4 T^4}{45\hbar^3 c^3} V$

$\rightarrow \frac{nRT}{V} = \frac{\pi^2 k_B^4 T^4}{45\hbar^3 c^3}$

$$T = \left(\frac{45\hbar^3 c^3 nR}{\pi^2 k_B^4 V} \right)^{1/3}$$

$= 3.2 \cdot 10^7$ K

$\hbar \approx 1.05 \cdot 10^{-27}$ erg$\cdot$s

$c \approx 3 \cdot 10^{11}$ cm/s

$R \approx 8.31 \cdot 10^7 \frac{\text{erg}}{\text{mol}\cdot\text{K}}$

$k_B = 1{,}38 \cdot 10^{-16} \frac{\text{erg}}{\text{K}}$

$n = 1$ mole/cm^3

$V = 1$ cm^3

4-9

Ljóseindagas í 1D

Finna $C_V(\tau)$

Jaðarstilyrði $E(0), E(L) = 0$

$$v^2 \frac{\partial^2 E}{\partial x^2} = \frac{\partial^2 E}{\partial t^2} \rightarrow$$

Lausn $E(x) = \sin(\omega t) \cdot \sin\left(\frac{n\pi x}{L}\right) \cdot E_0$

$n = \{1, \ldots \infty\}$

Innsetning gefur

$$\frac{v^2 \pi^2 n^2}{L^2} = \omega^2 \rightarrow \omega_n = \pi n \frac{v}{L}$$

$$U = \sum_{n=1}^{\infty} \langle \varepsilon_n \rangle = \sum_{n=1}^{\infty} \frac{\hbar \omega_n}{e^{\frac{\hbar \omega_n}{\tau}} - 1}$$

gennem vææ fyris langrī linea

$$\frac{\hbar \pi n v}{\tau L} = \frac{\hbar \omega_n}{\tau} = x$$

$$\rightarrow dn = dx \cdot \frac{\tau L}{\hbar \pi v}$$

$$U \simeq \int_0^\infty dn \cdot \frac{\hbar \omega_n}{e^{\frac{\hbar \omega_n}{\tau}} - 1}$$

$$= \frac{\tau^2 L}{\hbar \pi v} \int_0^\infty dx \frac{x}{e^x - 1}$$

$$= \frac{\tau^2 L}{\hbar \pi v} \Gamma(2) \zeta(2)$$

$$U = \frac{\tau^2 L}{\hbar \pi v} \frac{\pi^2}{6} = \frac{\tau^2 L \pi}{\hbar v 6}$$

$$\rightarrow \frac{U}{L} = \frac{\tau^2 \pi}{\hbar v 6}$$

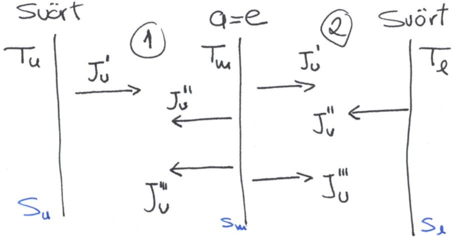
$$C_v(\tau) = \left(\frac{\partial U}{\partial \tau} \right)_v = \frac{\tau L \pi}{3 \hbar v}$$

En i 3D fost

$$C_v(\tau) = \left(\frac{\partial}{\partial \tau} \frac{\pi \tau^4 V}{15 \hbar^3 c^3} \right)_v$$

$$= \frac{4 \pi \tau^3 V}{15 \hbar^3 c^2}$$

4-19



$r = 1 - a$

Á svæði 1

$J_u' = \sigma_B T_u^4$ Geislun Su svarthluts

$J_u'' = \sigma_B T_m^4 \cdot e$ Geislun Sm, ekki svarthluts

$J_u''' = \sigma_B T_u^4 \cdot r$ endurkast geislunar frá Su af fleti Sm

$\rightarrow \sigma_B \cdot a \cdot (T_u^4 - T_m^4)$

$$J_u^{(1)} = \sigma_B \left(T_u^4 - e T_m^4 - r T_u^4\right) = \sigma_B \left(T_u^4 (1-r) - a T_m^4\right)$$

Á svæði 2

$$J_v' = \sigma_B T_m^4 \cdot a$$ Geislun sléttu S_m, ekki svört

$$J_v'' = \sigma_B T_e^4$$ Geislun sléttu S_e, svört

$$J_v''' = \sigma_B T_e^4 \cdot r$$ endurkast geislunar frá S_e af S_m

$$J_v^{(2)} = \sigma_B \left(T_m^4 \cdot a + T_e^4 \cdot r - T_e^4 \right)$$

$$= \sigma_B \left(T_m^4 \cdot a - T_e^4 \cdot a \right)$$

$$= \sigma_B \, a \left(T_m^4 - T_e^4 \right)$$

$$J_u^{(1)} = J_u^{(2)} \to \sigma_B a\left(T_u^4 - T_m^4\right) = \sigma_B a\left(T_m^4 - T_e^4\right)$$

$$\to 2T_m^4 = T_u^4 + T_e^4$$ sama svar og í dæmi 4.8

notum í $J_u^{(1)}$ til að finna orku flæði þéttnina

$$J_u^{(1)} = \sigma_B a\left(T_u^4 - T_m^4\right) = \sigma_B a\left(T_u^4 - \frac{T_u^4 + T_e^4}{2}\right)$$

$$= \sigma_B a \frac{1}{2}\left(T_u^4 - T_e^4\right) = \sigma_B (1-r)\frac{1}{2}\left(T_u^4 - T_e^4\right)$$

$(1-r)$ bætist við svarið úr 4-8. $r=1 \to$ ekkert flæði