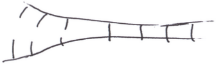
3-7 Rennilás N-tengi

Lokuð tengi orka: 0

Opið tengi orka: ε

opnast aðeins frá vinstri



s getur opnast ef $(1, 2, \ldots, s-1)$ eru opin

Ástand p: p hlekkir opnir

a)

$$Z = \sum_{p=0}^{N} e^{-p\epsilon/\tau} = \sum_{p=0}^{N} \left(e^{-\epsilon/\tau}\right)^p$$

$$= 1 + \sum_{p=1}^{N} \left(e^{-\epsilon/\tau}\right)^p = 1 + \frac{\left\{\left(e^{-\epsilon/\tau}\right)^N - 1\right\}\left(e^{-\epsilon/\tau}\right)}{\left(e^{-\epsilon/\tau}\right) - 1}$$

(GR: 0.112)

$$= \frac{e^{-(N+1)\varepsilon/\tau} - 1}{e^{-\epsilon/\tau} - 1}$$

b) Fyrir $\epsilon \gg \tau$ finna meðal fjölda opinna klefa

$$\langle p \rangle = \frac{\sum_{p=0}^{N} p\, e^{-p\epsilon/\tau}}{Z} = \frac{\sum_{p=0}^{N} p\, x^{p}}{\sum_{p=0}^{N} x^{p}} \quad \text{ef } x = e^{-\epsilon/\tau}$$

$$= x \frac{d}{dx} \ln Z = x \frac{d}{dx} \left\{ \ln \left[\frac{1 - x^{N+1}}{1 - x} \right] \right\}$$

$$= x \frac{d}{dx} \left\{ \ln(1 - x^{N+1}) - \ln(1 - x) \right\}$$

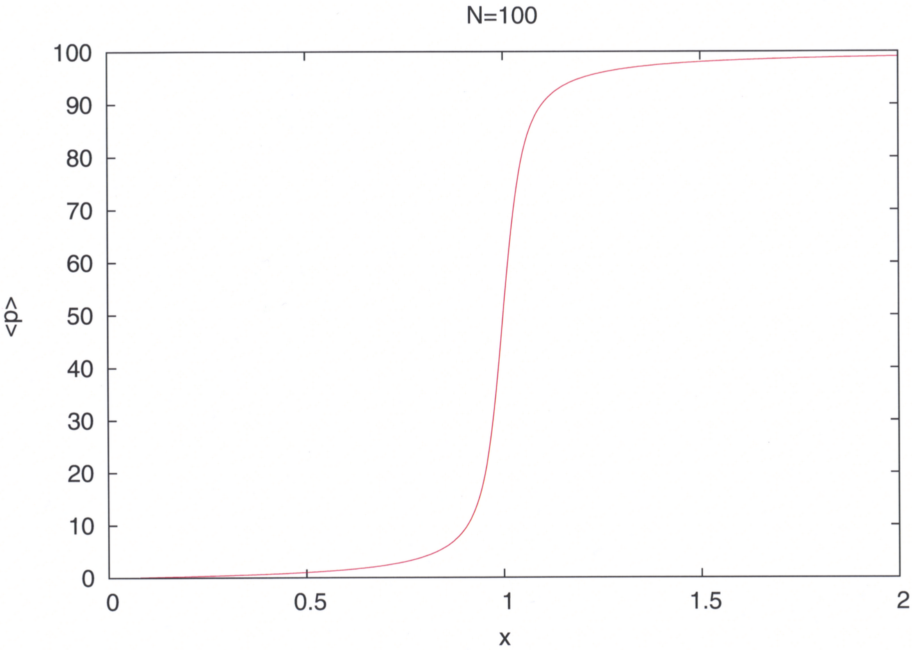
$$= x \left\{ \frac{-(N+1) x^{N}}{1 - x^{N+1}} - \frac{(-1)}{1 - x} \right\} = \left\{ \frac{x}{1 - x} - \frac{(N+1) x^{N+1}}{1 - x^{N+1}} \right\}$$

$$= \left\{ \frac{e^{-\epsilon/\tau}}{1 - e^{-\epsilon/\tau}} - \frac{(N+1) e^{-(N+1)\epsilon/\tau}}{1 - e^{-(N+1)\epsilon/\tau}} \right\}$$

pegar $\epsilon \gg \tau$, $\epsilon > 0$ $\rightarrow$ $x \ll 1$

$$\langle p \rangle = \left\{ x + x^2 + x^3 + \cdots - (N+1)x^{N+1} \cdot \left(1 + x^{N+1} + x^{2(N+1)} + \cdots\right) \right\}$$

$$\simeq x = e^{-\epsilon/\tau}$$


N=100
100
90
80
70
60
50
40
30
20
10
0
<p>
0
0.5
1
1.5
2
x

3-8 Teningur með hlið L

Orka eins atóms

$$\varepsilon_n = \frac{\hbar^2}{2M}\left(\frac{\pi}{L}\right)^2 (n_x^2 + n_y^2 + n_z^2)$$

$$n_i \in \{1, 2, 3, \dots \infty\}$$

→ Grunnástand

$$\varepsilon_1 = \frac{\hbar^2}{2M}\left(\frac{\pi}{L}\right)^2 3$$

Finna L (veðan n) þ.a.

$$\varepsilon_1 = \tau$$

$$\frac{\hbar^2 \pi^2 3}{2M\, V^{2/3}} = \tau$$

$$\to V^{2/3} = \frac{3\hbar^2\pi^2}{2M\tau}$$

$$\to V = \left(\frac{3\hbar^2\pi^2}{2M\tau}\right)^{3/2}$$

$$\to n = \frac{1}{V} = \left(\frac{2M\tau}{3\hbar^2\pi^2}\right)^{3/2}$$

$$= \left(\frac{M\tau}{2\pi\hbar^2}\right)^{3/2} \left(\frac{2\cdot 2}{3\pi}\right)^{3/2}$$

$$= n_Q \left(\frac{4}{3\pi}\right)^{3/2}$$

$$\sim n_Q \cdot 0{,}42$$

3-9

Sýna að kórsumma tveggja óháðra kerfa í varmatengslum við sama hitastig sé

$$Z(1+2) = Z(1)Z(2)$$

Í hreinu ástandi (eiginástöndum)

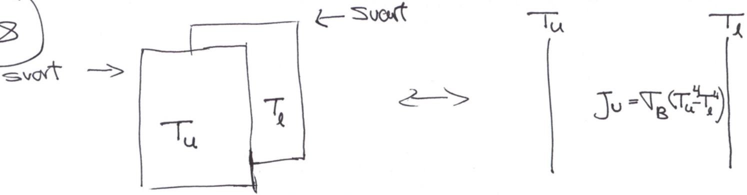
$$Z_1(\tau) = \sum_s e^{-\epsilon^1_s/\tau}, \qquad Z_2(\tau) = \sum_s e^{-\epsilon^2_s/\tau}$$

Orka ~~þessara~~ tveggja kerfa er $\epsilon^1_s + \epsilon^2_r$ fyrir öll gildi á s og r

$$\rightarrow Z_{1+2} = \sum_{sr} e^{-(\epsilon^1_s+\epsilon^2_r)/\tau} = \sum_{sr} e^{-\epsilon^1_s/\tau} e^{-\epsilon^2_r/\tau}$$

$$= \sum_s e^{-\epsilon^1_s/\tau} \sum_r e^{-\epsilon^2_r/\tau} = Z_1 \cdot Z_2$$

4-8



Þriðja sléttan er sett á milli (líkasvört)
fær hitastigið T_m eftir smátíma
Finna T_m og sýna að varmaflæðið sé
helmingað

Upphaflega

$$J_U = \sigma_B \left(T_u^4 - T_l^4 \right)$$

Síðan

$$J_U' = \sigma_B \left(T_u^4 - T_m^4 \right) \quad \text{og} \quad J_U'' = \sigma_B \left(T_m^4 - T_l \right)$$

Kerfið er í stöðugu ástandi með T_m fast

$$\to \quad J_u' = J_u'' \quad \to \quad T_u^4 - T_m^4 = T_m^4 - T_e^4$$

$$\to 2T_m^4 = T_u^4 + T_e^4$$

$$\to \boxed{T_m^4 = \frac{1}{2}\left(T_u^4 + T_e^4\right)}$$

og

$$J_u' = \sigma_B\Big(T_u^4 - \overbrace{\tfrac{1}{2}\left(T_u^4 + T_e^4\right)}^{T_m^4}\Big) = \sigma_B\left(\tfrac{1}{2}T_u^4 - \tfrac{1}{2}T_e^4\right)$$

$$= \frac{1}{2}\sigma_B\left(T_u^4 - T_e^4\right) = \frac{1}{2}J_u$$

orkuflæðið helmingast —> varmaskjöldur