# (3-1) Two state system $0, \varepsilon$

a) finna F

$$Z = 1 + e^{-\frac{\varepsilon}{\tau}}, \quad F = -\tau \ln Z = -\tau \ln\left\{1 + e^{-\frac{\varepsilon}{\tau}}\right\}$$

b) finnum $\sigma$ frá

$$\left(\frac{\partial F}{\partial \tau}\right)_V = -\sigma \quad \rightarrow \boxed{\sigma = \tau \frac{e^{-\frac{\varepsilon}{\tau}}\left(+\frac{\varepsilon}{\tau^2}\right)}{1 + e^{-\varepsilon/\tau}} + \ln\left\{1 + e^{-\varepsilon/\tau}\right\}}$$

og U frá

$$U = F + \tau\sigma$$

$$= -\tau \ln\left\{1 + e^{-\frac{\varepsilon}{\tau}}\right\} + \tau \ln\left\{1 + e^{-\frac{\varepsilon}{\tau}}\right\} + \frac{\varepsilon e^{-\frac{\varepsilon}{\tau}}}{1 + e^{-\varepsilon/\tau}}$$

$$= \boxed{\frac{\varepsilon e^{-\frac{\varepsilon}{\tau}}}{1 + e^{-\varepsilon/\tau}}}$$

Eðlilegra að nota

$$U = \frac{\sum_s \varepsilon_s e^{-\frac{\varepsilon}{\tau}}}{Z} = \tau^2 \left( \frac{\partial \ln Z}{\partial \tau} \right)$$

$$= \frac{\varepsilon e^{-\frac{\varepsilon}{\tau}}}{1 + e^{-\varepsilon/\tau}}$$

eins og við sáum áður

3-3 Einn HO $\qquad \varepsilon_s = s\cdot\hbar\omega \ , \quad s = \{0, 1, 2, \ldots\ldots\}$

$$Z(\tau) = \sum_s \exp\left(-\frac{\varepsilon_s}{\tau}\right) = \sum_s \exp\left(-s\cdot\frac{\hbar\omega}{\tau}\right)$$

$$= \sum_s \left\{\exp\left(-\frac{\hbar\omega}{\tau}\right)\right\}^s = \frac{1}{1-\exp\left(-\frac{\hbar\omega}{\tau}\right)}$$

ef $e^{-\frac{\hbar\omega}{\tau}} < 1$

sem er alltaf uppfyllt fyrir endanlegt hitastig

$$F = -\tau \ln Z$$

$$= \tau \ln\left\{1 - e^{-\frac{\hbar\omega}{\tau}}\right\}$$

Ef $\tau \gg \hbar\omega$

$$F \approx \tau \ln\left\{1 - 1 + \frac{\hbar\omega}{\tau} + \cdots\right\} = \tau \ln\left(\frac{\hbar\omega}{\tau}\right)$$

b) finn $\sigma$

$$\sigma = -\left(\frac{\partial F}{\partial \tau}\right)_V = -\left(\frac{\partial}{\partial \tau}\tau \ln\left\{1-e^{-\frac{\hbar\omega}{\tau}}\right\}\right)_V$$

$$= -\ln\left\{1-e^{-\frac{\hbar\omega}{\tau}}\right\} - \tau \frac{e^{-\frac{\hbar\omega}{\tau}}}{1-e^{-\frac{\hbar\omega}{\tau}}}\left(-\frac{\hbar\omega}{\tau^2}\right)$$

$$= -\ln\left\{1-e^{-\frac{\hbar\omega}{\tau}}\right\} + \left(\frac{\hbar\omega}{\tau}\right)\frac{e^{-\frac{\hbar\omega}{\tau}}}{1-e^{-\frac{\hbar\omega}{\tau}}}$$

$$= -\ln\left\{1-e^{-\frac{\hbar\omega}{\tau}}\right\} + \frac{\left(\frac{\hbar\omega}{\tau}\right)}{e^{\frac{\hbar\omega}{\tau}}-1}$$

og $\lim\limits_{\tau \to 0} \sigma = 0$

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tvíatóma

Snúningur sameinda

$$\varepsilon(j) = j(j+1)\varepsilon_0 \,, \quad j = 0, 1, \ldots$$

Finna Kórsummuna $Z_R(\tau)$ fyrir eina sameind

margfeldni hvers orkustigs er $g(j) = 2j+1$

$$Z_R(\tau) = \sum_j g(j)\, e^{-\varepsilon(j)\varepsilon_0/\tau}$$

$$= \sum_{j=0}^{\infty} (2j+1)\, e^{-\frac{j(j+1)\varepsilon_0}{\tau}}$$

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b) Reikna $Z_R(\tau)$ þ. $\tau \gg \varepsilon_0$, þétt orkuróf miðað við $\tau$

$\rightarrow$ breyta í heildi

$$Z_R(\tau) \xrightarrow[\frac{\varepsilon_0}{\tau} \ll 1]{} \int_0^\infty (2x+1)\, e^{-x(x+1)\frac{\varepsilon_0}{\tau}}\, dx$$

← gróf nálgun sem ég leyfi mér hér

sjá Thermodynamics and statistical Mechanics, eftir Greiner, Neise og Stöcker á bls. 230 ~~með~~ betri umformun yfir í heildi

breyta $x(x+1) = y \rightarrow (2x+1)dx = dy$

$$\rightarrow Z_R(\tau) \xrightarrow[\frac{\varepsilon_0}{\tau} \ll 1]{} \int_0^\infty dy\, e^{-y\frac{\varepsilon_0}{\tau}} = \frac{\tau}{\varepsilon_0}$$

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c) Reikna $Z_R(\tau)$ þ. $\tau \ll \varepsilon_0$

$$Z_R(\tau) \xrightarrow[\frac{\varepsilon_0}{\tau} \gg 1]{} \left\{ 1 + 3e^{-\frac{2\varepsilon_0}{\tau}} + \cdots \right\}$$

d) finna $U$ og $C$ sem föll af $\tau$ í báðum markgildum

$$U = +\tau^2 \frac{\partial \ln Z}{\partial \tau}$$

$\underline{\tau \ll \varepsilon_0:}$ $\longrightarrow$

$$U \simeq +\tau^2 \frac{\partial}{\partial \tau} \ln\left\{1 + 3e^{-\frac{2\varepsilon_0}{\tau}}\right\}$$

$$= +\tau^2 \frac{3e^{-\frac{2\varepsilon_0}{\tau}} \left(+\frac{2\varepsilon_0}{\tau^2}\right)}{1 + 3e^{-2\varepsilon_0/\tau}}$$

$$= \frac{6\varepsilon_0 e^{-\frac{2\varepsilon_0}{\tau}}}{1 + 3e^{-2\varepsilon_0/\tau}} \simeq 6\varepsilon_0 e^{-\frac{2\varepsilon_0}{\tau}}$$

þegar $\tau \ll \varepsilon_0$

$$C_V = \left(\frac{\partial U}{\partial \tau}\right)_V = 12\left(\frac{\varepsilon_0}{\tau}\right)^2 e^{-\frac{2\varepsilon_0}{\tau}}$$

skammtakerfi fyrir lágt hitastig, lítil örvun fárra stiga

$\tau \gg \varepsilon_0$ :

$$U \simeq +\tau^2 \frac{\partial}{\partial \tau}\ln\left(\frac{\tau}{\varepsilon_0}\right) = +\tau^2 \frac{1/\varepsilon_0}{\tau/\varepsilon_0} = +\tau$$

$$C_V = \left(\frac{\partial U}{\partial \tau}\right)_V = 1$$

← Klassíska markgildið fyrir hátt hitastig mörg ástönd örvuð