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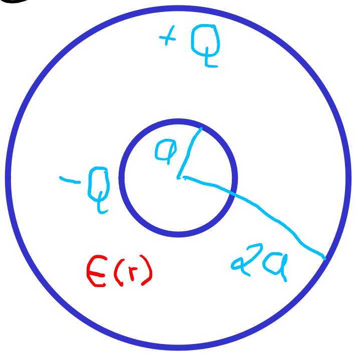

1

$$\varepsilon(r) = \frac{2a\varepsilon_0}{3a-r}$$ only free charges, no electred

$$\oint \bar{D} \cdot d\bar{s} = Q$$ ε(r) does not break the spherical symmetry

Gauss law and the symmetry can used to find that the field outside the spherical capacitor, and for $r < a$, is 0. we start thus between the plates, the only region with nonzero D-field.

a) $$\oint_S \bar{D} \cdot d\bar{s} = -Q \longrightarrow 4\pi r^2 D_r = -Q$$

$$\longrightarrow \boxed{\bar{D} = -\frac{Q\hat{a}_r}{4\pi r^2}}$$

$$\bar{D} = \varepsilon(r)\bar{E}$$

$$\longrightarrow \boxed{\bar{E} = \frac{\bar{D}}{\varepsilon(\bar{r})} = -\frac{Q\hat{a}_r}{4\pi r^2}\frac{(3a-r)}{2a\varepsilon}}$$ for later use

b) Find ρ_p and ρ_{ps}

$$\bar{D} = \epsilon_o \bar{E} + \bar{P} \quad \text{and} \quad \rho_{ps} = \bar{P} \cdot \hat{a}_n$$

$$\rho_p = -\bar{\nabla} \cdot \bar{P}$$

$$\bar{P} = \bar{D} - \epsilon_o \bar{E}$$

$$= -\frac{Q \hat{a}_r}{4\pi r^2} + \frac{Q \hat{a}_r}{4\pi r^2} \frac{(3a - r)}{2a} = -\frac{Q \hat{a}_r}{8\pi r^2} \left[\frac{r}{a} - 1 \right]$$

$$\rightarrow \boxed{\begin{aligned} &\rho_{ps}(a^+) = 0 \\ &\rho_{ps}(2a^-) = \frac{Q}{8\pi (2a)^2} \left\{ \frac{2a}{a} - 1 \right\} = \frac{Q}{32\pi a^2} \end{aligned}}$$

$$\boxed{\rho_p = -\bar{\nabla} \cdot \bar{P} = -\frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 P_r \right) = \frac{Q}{8\pi r^2 \cdot a}}$$

c) $\boxed{W_E} = \frac{1}{2}\int_V dv\, \bar{D}\cdot\bar{E}$

$$= \frac{1}{2}\frac{Q^2}{(4\pi)^2}\frac{4\pi}{\epsilon_0}\int_a^{2a} r^2 dr\, \frac{1}{r^4}\left(\frac{3a-r}{2a}\right)$$

$$= \frac{Q^2}{16\pi\epsilon_0 a}\left[\frac{3}{2}+\ln\left(\frac{1}{2}\right)\right]$$

$$\boxed{= \frac{Q^2}{16\pi\epsilon_0 a}\left[\frac{3}{2}-\ln(2)\right]} > 0$$

d) Capacity C

$$V(2a) - V(a) = -\int_a^{2a} \overline{E} \cdot d\overline{l} = -\int_a^{2a} E_r \, dr$$

$$= \int_a^{2a} dr \left[\frac{Q}{4\pi\epsilon_0 \cdot 2} \left(\frac{3}{r^2} - \frac{1}{ra} \right) \right]$$

$$= \frac{Q}{8\pi\epsilon_0 a} \left[\frac{3}{2} - \ln(2) \right]$$

$$\boxed{C = \frac{Q}{\Delta V} = \frac{8\pi\epsilon_0 a}{\left[\frac{3}{2} - \ln(2) \right]}} > 0$$

2 $\bar{A}(\bar{r},t) = \Phi(r,t)\,\hat{a}_z$, $\Phi = C\dfrac{e^{i(kr-\omega t)}}{r}$

a) Find $\bar{B}$ and $\bar{E}$

$$\bar{A}(\bar{r},t) = \Phi(r,t)\left[\hat{a}_r \cos\theta - \hat{a}_\theta \sin\theta\right]$$

$\rightarrow$ $A_r = \Phi\cos\theta$

$A_\theta = -\Phi\sin\theta$

$A_\phi = 0$

$k = \dfrac{\omega}{c}$

$= \omega\sqrt{\epsilon\mu}$

$\eta = \dfrac{\omega\mu}{k}$

$= \sqrt{\dfrac{\mu}{\epsilon}}$

$$\bar{B} = \bar{\nabla}\times\bar{A} = \hat{a}_\phi \frac{1}{r}\left[\frac{\partial}{\partial r}(rA_\theta) - \frac{\partial A_r}{\partial\theta}\right]$$

$$= C\,\hat{a}_\phi\, k^2 \sin\theta\left\{\frac{1}{ikr} - \frac{1}{(ikr)^2}\right\} e^{i(kr-\omega t)}$$

$$\bar{E} = \frac{1}{i\omega\epsilon_0\mu_0}\,\bar{\nabla}\times\bar{B} \qquad (\text{as } \bar{M} = 0)$$

$$\rightarrow \quad E_\phi = 0$$

$$E_r = -\frac{Ck^2}{i\omega\epsilon_0\mu_0}\left\{\frac{1}{ikr} - \frac{1}{(ikr)^2}\right\} e^{i(kr-\omega t)}\,\frac{1}{r\sin\theta}\,\frac{\partial}{\partial\theta}\sin^2\theta$$

$$= 2Ck^2\cos\theta\,\underbrace{\left\{\frac{1}{(ikr)^2} - \frac{1}{(ikr)^3}\right\}}\, e^{ikr - i\omega t}$$

all terms of a near field, no far-field in the radial direction

$$E_\theta = - \frac{C k^2 \sin\theta}{i\omega \epsilon_0 \mu_0} \frac{1}{r} \frac{\partial}{\partial r} \left\{ r \left(\frac{1}{ikr} - \frac{1}{(ikr)^2} \right) e^{i(kr - \omega t)} \right\}$$

$$= - C k^2 \sin\theta \left\{ \frac{1}{ikr} + \frac{1}{(ikr)^2} + \frac{1}{(ikr)^3} \right\} e^{i(kr - \omega t)}$$

far-field perpendicular to the direction of propagation and the far-field component of B

The far fields ~ $1/(kr)$ that lead to energy flow to infinity all come from the phase factor $\exp(ikr)$ in A, which comes from the retarted radiation of the source with respect to the observer.

3

$$V(r) = \frac{q\, e^{-\mu r}}{4\pi\epsilon_0 r}$$ spherical

$$\bar{E} = -\bar{\nabla} V = -\frac{q}{4\pi\epsilon_0} \frac{\partial}{\partial r}\left[\frac{1}{r} e^{-\mu r}\right] \hat{a}_r$$

$$= +\hat{a}_r \frac{q e^{-\mu r}}{4\pi\epsilon_0 r^2}\left\{1 + \mu r\right]$$

a) $\bar{E} \sim \hat{a}_r \longrightarrow \bar{\nabla} \times \bar{E} = 0$

V and E have translational symmetry, and for an individua point charge we can always shift the coordinate system. More complex charge configuration can be modeled by summation of point charges and superposition

b) $$\oint \bar{E} \cdot d\bar{a} = 4\pi r^2 \frac{q e^{-\mu r}}{4\pi \epsilon_0 r^2} \{1 + \mu r\} \Big|_{r = R_0}$$

$$= \frac{q}{\epsilon_0} e^{-\mu R_0} (1 + \mu R_0)$$

$$\to 0 \quad \text{as} \quad R_0 \to \infty$$

but check

$$\int_0^{R_0} dr\, V(r) = \frac{q\, 4\pi}{4\pi \epsilon_0} \int_0^{R_0} dr \left(\frac{r^2}{r}\right) e^{-\mu r}$$

$$= \frac{q}{\epsilon_0 \mu^2} \left\{ 1 - (\mu R_0 + 1) e^{-\mu r} \right\}$$

$$\boxed{\oint \bar{E}\cdot d\bar{a} + \mu^2 \int dv\, V} = \frac{q}{\epsilon_0} e^{-\mu r}(1 + \mu R_0)$$

$$+ \frac{q}{\epsilon_0}\left[1 - (\mu R_0 + 1)\, e^{-\mu r}\right]$$

$$\boxed{= \frac{q}{\epsilon_0}}$$

<u>independent of R_0</u>