

Problem 1

a) Relation between

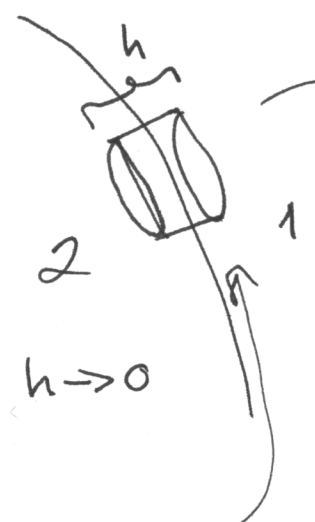
$$E_{1t} = E_{2t} \quad \text{and} \quad B_{1u} = B_{2u}$$

b) and between

$$\hat{a}_{n2} \cdot (\bar{D}_1 - \bar{D}_2) = f_s \quad \text{and} \quad \hat{a}_{n2} \times (\bar{H}_1 - \bar{H}_2) = \bar{J}_s$$

a) We can rewrite these two conditions as

$$\hat{a}_{n2} \cdot (\bar{B}_1 - \bar{B}_2) = 0 \quad \text{and} \quad \hat{a}_{n2} \times (\bar{E}_1 - \bar{E}_2) = 0$$



Consider a small volume. If

$$\hat{a}_{n2} \cdot (\bar{B}_1 - \bar{B}_2) = 0$$

at the surface then also

$$\hat{a}_{n2} \cdot (\partial_t \bar{B}_1 - \partial_t \bar{B}_2) = 0$$

Now we use $\nabla \cdot (\hat{a}_{n2} \times \bar{E}) = \bar{E} \cdot (\nabla \times \hat{a}_{n2}) - \hat{a}_{n2} \cdot (\nabla \times \bar{E})$
 to get

$$\begin{aligned} \hat{a}_{n2} \cdot (\partial_t \bar{B}_1 - \partial_t \bar{B}_2) &= \overset{\text{Maxwell - Faraday}}{-} \hat{a}_{n2} \cdot (\nabla \times \bar{E}_1 - \nabla \times \bar{E}_2) \\ &= \nabla \cdot \left\{ \hat{a}_{n2} \times (\bar{E}_1 - \bar{E}_2) \right\} = 0 \end{aligned}$$

integrate over the volume
 use Gauss theorem to see that on the end surfaces

$$\underline{\hat{a}_{n2} \times (\bar{E}_1 - \bar{E}_2) = 0}$$

$$\hat{a}_{u2} \cdot (\bar{D}_1 - \bar{D}_2) = \rho_s$$

$$\hat{a}_{u2} \cdot (\partial_t \bar{D}_1 - \partial_t \bar{D}_2) = \partial_t \rho_s$$

$$\hat{a}_{u2} \cdot (\nabla \times \bar{H}_1 - \bar{J}_1 - (\nabla \times \bar{H}_2 - \bar{J}_2)) = -\nabla \cdot \bar{J}_s$$

$$\hat{a}_{u2} \cdot (\nabla \times \bar{H}_1 - \nabla \times \bar{H}_2) = -\nabla \cdot \bar{J}_s + \hat{a}_{u2} \cdot (\bar{J}_1 - \bar{J}_2)$$

$$-\nabla \cdot \left\{ \hat{a}_{u2} \times (\bar{H}_1 - \bar{H}_2) \right\} = -\nabla \cdot \bar{J}_s + \underbrace{\hat{a}_{u2} \cdot (\bar{J}_1 - \bar{J}_2)}_{=0 \text{ orthogonal}}$$

integral as before

$$\rightarrow \underline{\hat{a}_{u2} \times (\bar{H}_1 - \bar{H}_2) = \bar{J}_s}$$