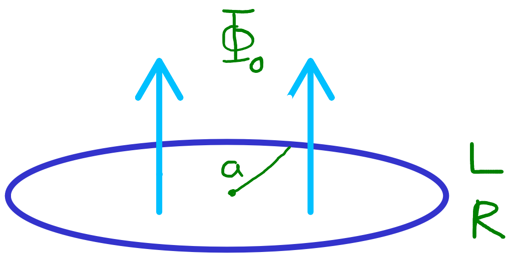
Problem 1



$$\Phi(t) = \Phi_0 \, e^{-\Gamma t} (\Gamma t)^2 \quad \text{external}$$

Faraday $$\underbrace{\oint_c \bar{E} \cdot d\bar{\ell} = -\frac{d\Phi}{dt}}$$

Total flux = external flux + self-induced flux

$$\Phi_{total}(t) = \Phi(t) + L \cdot i(t)$$

$$R \cdot i(t) = -\frac{d\Phi(t)}{dt} - L \cdot \frac{di(t)}{dt}$$

$$\boxed{L \frac{di(t)}{dt} + R \cdot i(t) = -\frac{d\Phi(t)}{dt} = -\Phi_0 \Gamma e^{-\Gamma t} (\Gamma t)\left[2 - (\Gamma t)\right]}$$

Lausn

$$i(t) = \underbrace{i(0)}_{=0} - e^{-\frac{R}{L}t} \frac{\Phi_0}{L} \int_0^t ds \, e^{\frac{R}{L}s - \Gamma s} (\Gamma s)\left[2 - (\Gamma s)\right]$$

$$i(t) = -\frac{\Phi_0}{L}\frac{1}{A}\Big\{ e^{-\frac{R}{L}t}\, 2L^2R - \Big[(\Gamma L R^2 - 2\Gamma^2L^2R + \Gamma^3L^3)t^2 + (2\Gamma L^2R - 2LR^2)t + 2L^2R\Big]e^{-\Gamma t}\Big\}$$

with

$$A = R^3 - 3\Gamma L R^2 + 3\Gamma^2L^2R - \Gamma^3L^3$$

Two time scales

$$\left[\frac{L}{R}\right] = T \,, \quad \left[\frac{1}{\Gamma}\right] = T$$: dimension

Select

$$\boxed{\frac{R}{L} = \alpha\Gamma}$$

fix L and Γ
Keep R variable

$$\boxed{i(t) = -\frac{\Phi_0}{L\Gamma^2}\left\{ \frac{2e^{-\alpha(\Gamma t)}}{\alpha^2 - 3\alpha + 3 - \frac{1}{\alpha}} - \frac{(\alpha - 2 + \frac{1}{\alpha})(\Gamma t)^2 - 2(1-\alpha)(\Gamma t) + 2}{\alpha^2 - 3\alpha + 3 - \frac{1}{\alpha}}\, e^{-\Gamma t}\right\}}$$

There seems to be a resonance for the case when the time-scales are the same, i.e. $\alpha = 1$

Maxima shows that the limit for i(t) --> infinity as α --> 1 and similar behavior is seen in gnuplot

we see a resonance behavior as the time scales become equal

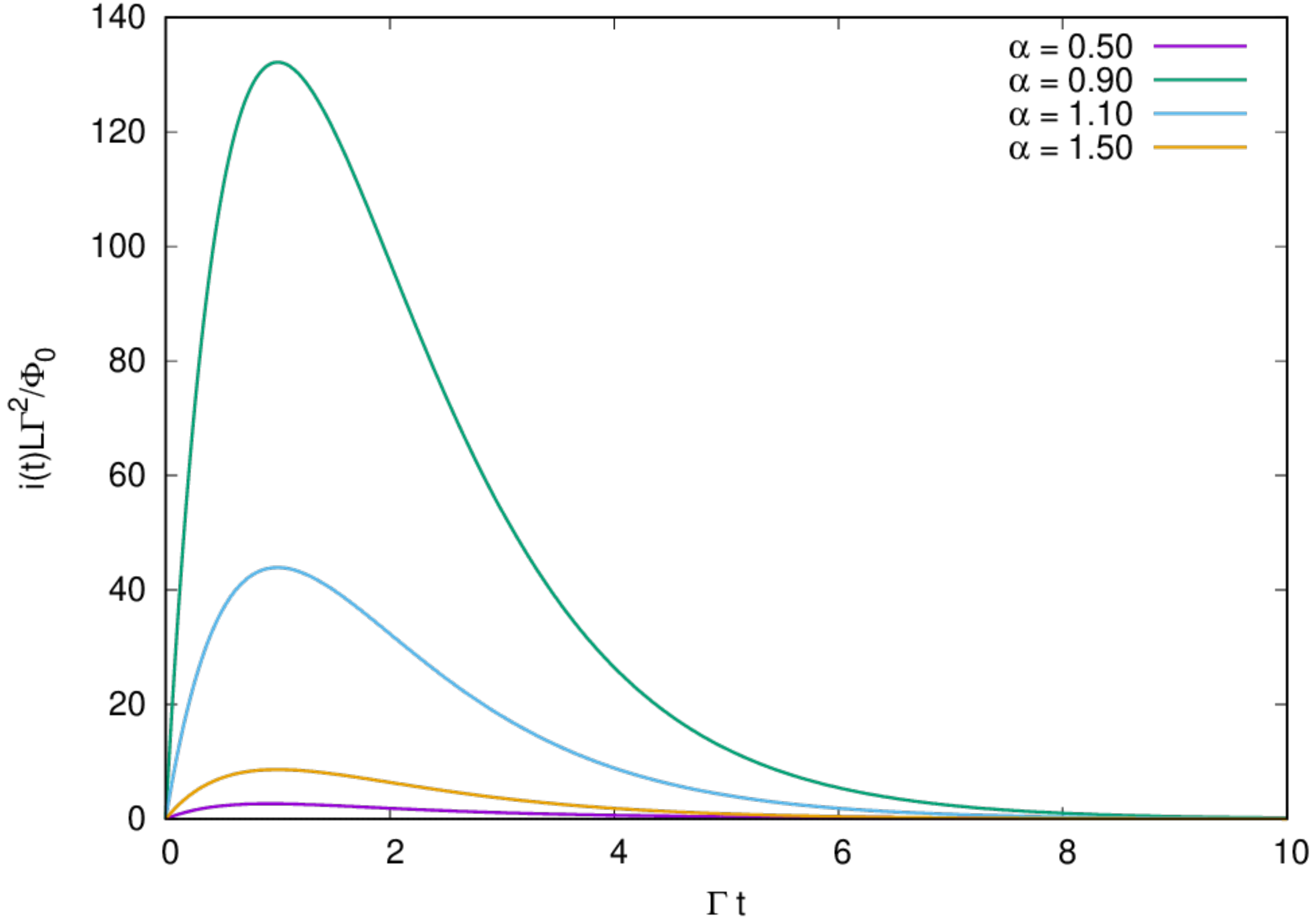
we also see a regime for low $\alpha = R/(L\Gamma)$, for low R compared to $L\Gamma$, when the reaction of the loop is changing sign for the regime of low Γt

I tend to think that the resonance was not expected....

I can rewrite the solution to make the resonance explicit

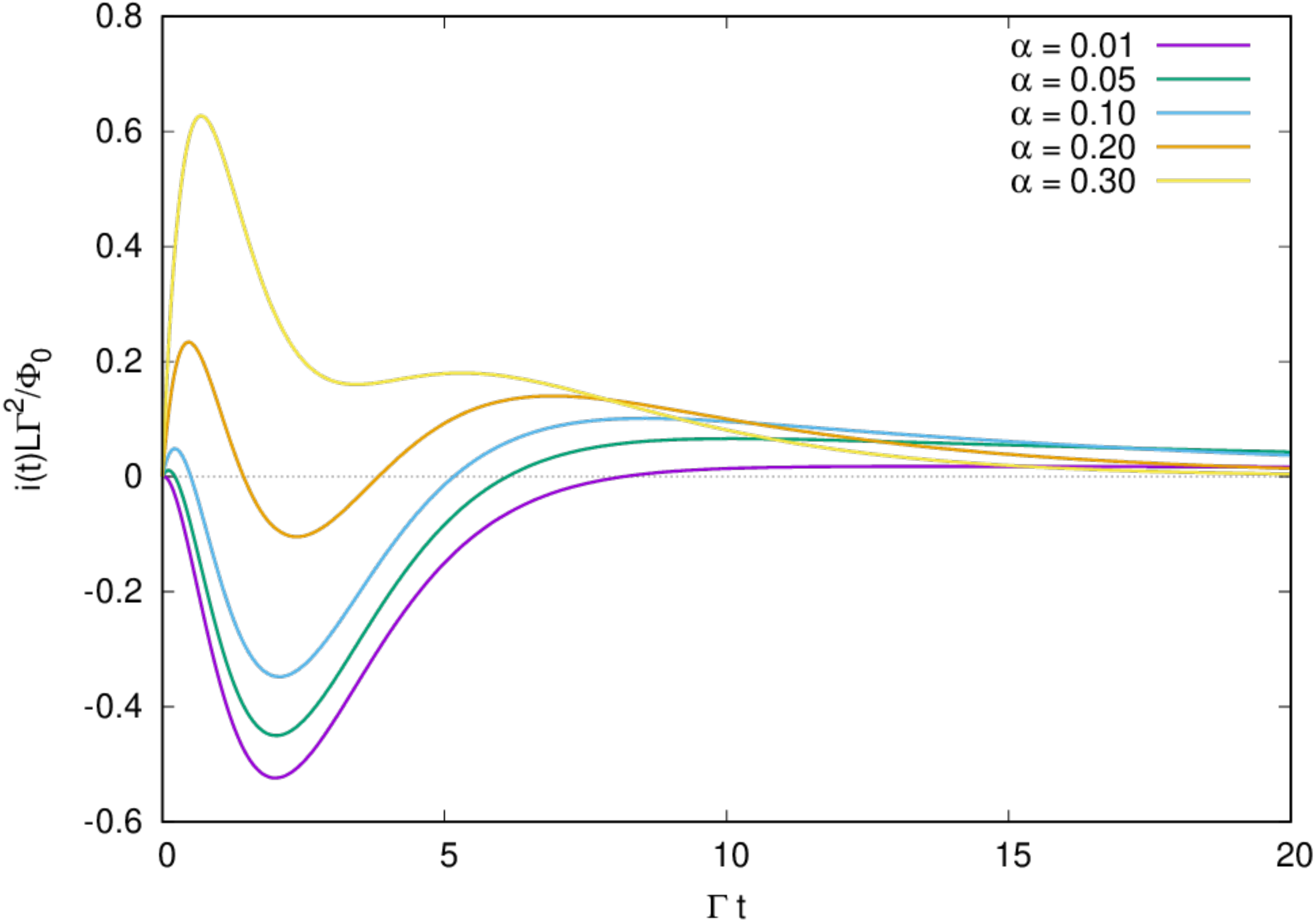
$$i(t) = \frac{\Phi_0 \alpha}{L\Gamma^2(\alpha-1)^3}\left[2e^{-\alpha(\Gamma t)} - \left(\frac{1}{\alpha}(\alpha-1)^2(\Gamma t^2) + 2(\alpha-1)(\Gamma t) + 2\right)e^{-\Gamma t}\right\}$$

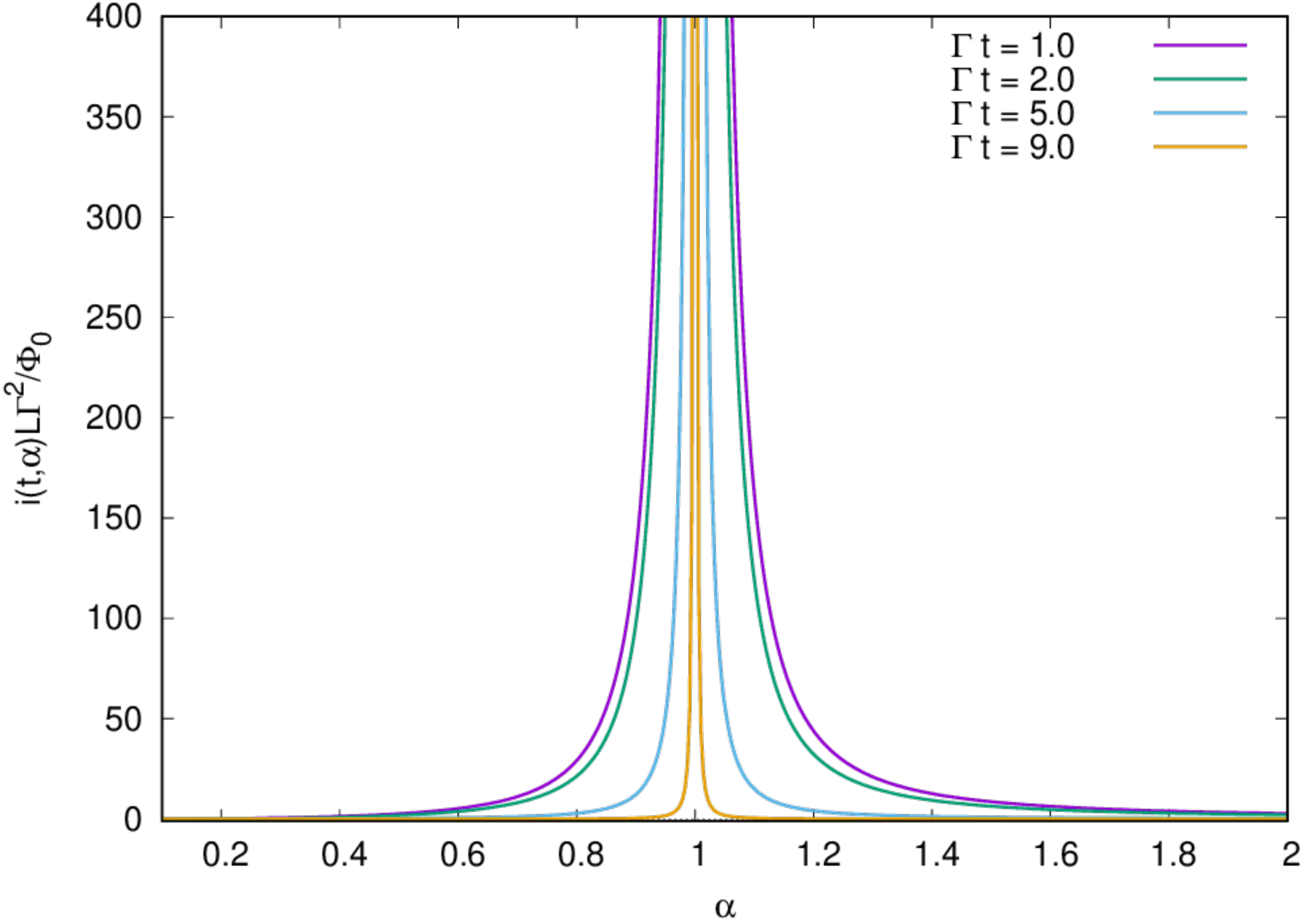
4



α = 0.50
α = 0.90
α = 1.10
α = 1.50
i(t)LΓ^2/Φ_0
Γ t
0
2
4
6
8
10
0
20
40
60
80
100
120
140

5


0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
0
5
10
15
20
$i(t)L\Gamma^2/\Phi_0$
Γt
$\alpha = 0.01$
$\alpha = 0.05$
$\alpha = 0.10$
$\alpha = 0.20$
$\alpha = 0.30$


Γ t = 1.0
Γ t = 2.0
Γ t = 5.0
Γ t = 9.0
i(t,α)LΓ^-2/Φ_0
α
0
50
100
150
200
250
300
350
400
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2

$$\left.\begin{array}{l}\tau_\Phi = \dfrac{1}{\Gamma} \\[2ex] \tau_L = \dfrac{L}{R}\end{array}\right\} \rightarrow \alpha = \frac{\tau_\Phi}{\tau_L}$$

but the time scales are a bit elusive

The loop has to react to the rise of the flux pulse and its decay, the graphs indicate that only for low enough R can it oppose to the rise of the flux

For higher R the graphs show that the reaction of the loop is mainly to retain the flux