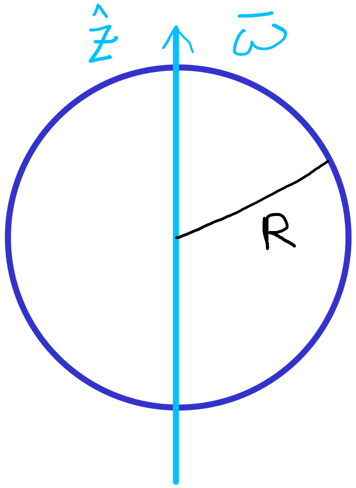
Problem 1



According to the Book the vector potential in- and outside an evenly charged thin shell with uniform charge is

$$\bar{A}(r,\theta,\phi) = \begin{cases} \dfrac{\mu_0 R\omega\sigma}{3} r\sin\theta\,\hat{\phi} & r \leq R \\ \dfrac{\mu_0 R^4\omega\sigma}{3}\dfrac{\sin\theta}{r^2}\,\hat{\phi} & r \geq R \end{cases}$$

Now we want to find the magnetic field inside a rotating sphere with a even charge distribution all over. we need to integrate "R" over thin shells from from o --> R and notice that the definition of the function changes at each shell (at "R") the derivative is discontinuous there.

$$\bar{A}(r,\theta,\phi) = \hat{\phi}\frac{\mu_0\omega\rho}{3}\left\{\frac{\sin\theta}{r^2}\int_0^r dr'(r')^4 + r\sin\theta\int_r^R dr'\,r'\right\}$$

as we take σ into $\rho dr'$

check the dimension

$$= \hat{\phi}\frac{\mu_0\omega\rho}{3}\left\{\frac{\sin\theta}{r^2}\frac{r^5}{5} + r\sin\theta\left(\frac{R^2}{2} - \frac{r^2}{2}\right)\right\}$$

2

$$= \hat{\phi}\, \frac{\mu_0 \omega \rho}{3} r\sin\theta \left[\frac{R^2}{2} - \frac{3r^2}{10}\right] = \hat{\phi}\, \frac{\mu_0 \omega \rho}{2} r\sin\theta \left[\frac{R^2}{3} - \frac{r^2}{5}\right]$$

and the magnetic field is

$$\bar{B} = \bar{\nabla} \times \bar{A} = \frac{\hat{r}}{r\sin\theta} \frac{\partial}{\partial\theta}\left[\sin\theta A_\phi\right] - \frac{\hat{\theta}}{r} \frac{\partial}{\partial r}\left[r A_\phi\right]$$

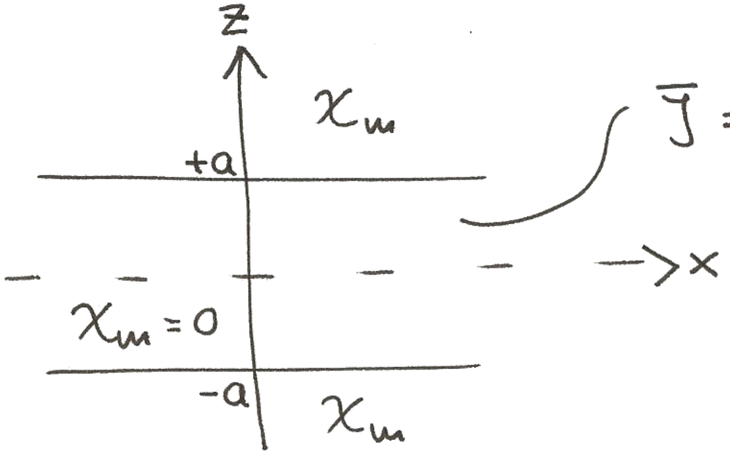
$$= \frac{\hat{r}}{r\sin\theta} \frac{\mu_0 \omega \rho}{2} r\, 2\sin\theta\cos\theta \left(\frac{R^2}{3} - \frac{r^2}{5}\right)$$

$$- \frac{\hat{\theta}}{r} \frac{\mu_0 \omega \rho}{2} \sin\theta \left(2\frac{rR^2}{3} - \frac{4r^3}{5}\right)$$

$$\boxed{= \mu_0 \omega \rho \left\{\hat{r}\left(\frac{R^2}{3} - \frac{r^2}{5}\right)\cos\theta - \hat{\theta}\left(\frac{R^2}{3} - \frac{2r^2}{5}\right)\sin\theta\right\}}$$

The magnetic field is definitely neither constant any more, in direction nor magnitude as it was in the rotating shell

(3)



Ég vil beita lögmáli Ampères

$$\oint_C \bar H \cdot d\bar l = I$$

en vil hafa stuðning af

$$\bar A(\bar r) = \frac{\mu_0}{4\pi} \int_{V'} d^3 r' \, \frac{\bar J(\bar r')}{|\bar r - \bar r'|}$$

til að sjá að $\bar A$ og $\bar J$ eru samsíða

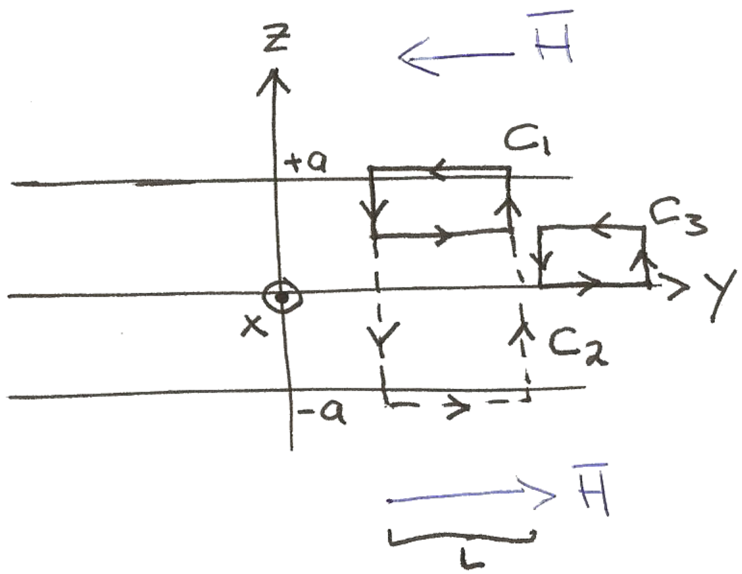
$\bar A$ er því með x-hnit A_x

$$\to \bar B = \bar\nabla \times \bar A = \hat a_y \frac{\partial A_x}{\partial z}$$

er með y-hnit. $\bar B$ liggur þvert á strauminn

Fyrir utan leiðarann legg ég lykkju í y-z-sléttu. Í gegnum hana er aldrei neinn straumur óháð lögun og staðsetningu

$\hookrightarrow$ $\boxed{\bar H \text{ er fasti utan leiðara}}$



Ef við notum leið C_2 af mynd sést að $\overline{H}$ verður að hafa <u>andstæðar</u> áttir <u>ofan</u> og <u>neðan</u> leiðara

$$\oint_{C_2} \overline{H} \cdot d\overline{l} = I$$

$$2H \cdot L = J_0 \cdot 2a \cdot L$$

$$\rightarrow \boxed{H = J_0 a \quad \text{utan leiðara}}$$

<u>Ofan leiðara</u>: $\overline{H} = -\hat{a}_y J_0 a$

<u>Neðan leiðara</u>: $\overline{H} = +\hat{a}_y J_0 a$

$\overline{H}$ verður að vera <u>andsamhverft</u> um $z = 0$. Notum þá C_3 til að reikna $\overline{H}$ innan leiðara

$$\oint_{C_3} \overline{H} \cdot d\overline{l} = I_{enc}$$

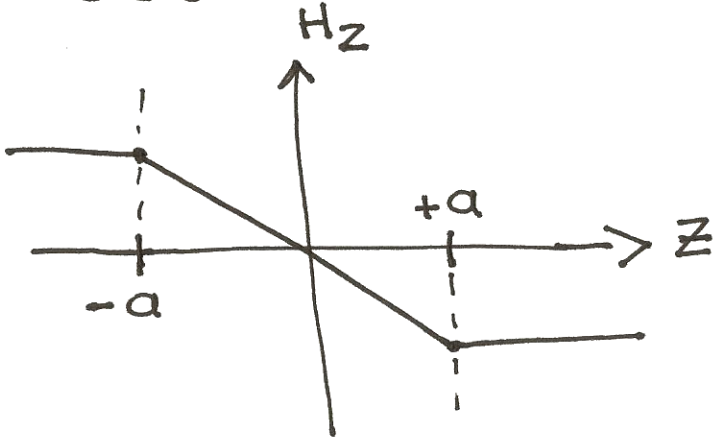
$$H(z) \cdot L = J_0 \cdot L \cdot z \quad , \quad z > 0$$

$$\hookrightarrow H(z) = J_0 z$$

Því fæst innan leiðara

$$\bar{H} = -\hat{a}_y(J_0 z)$$

eða



<u>Innan leiðara</u> er $\chi_m = 0$

$$\rightarrow \bar{B} = \mu_0 \bar{H}$$

$$\bar{M} = 0$$

$$\bar{B} = -\hat{a}_y(\mu_0 J_0 z)$$

<u>Utan leiðara</u> er $\chi_m \neq 0$

Því er $\bar{B} = \mu_0(1+\chi_m)\bar{H}$

Einnig gildir $\bar{H} = \dfrac{\bar{B}}{\mu_0} - \bar{M}$

eða $\bar{M} = \dfrac{\bar{B}}{\mu_0} - \bar{H}$

$$= \left\{(1+\chi_m) - 1\right\}\bar{H}$$

$$= \chi_m \bar{H}$$

<u>Ofan leiðara</u>

$$\bar{M} = -\hat{a}_y \chi_m J_0 a$$

$$\bar{B} = -\hat{a}_y \mu_0(1+\chi_m) J_0 a$$

<u>Neðan leiðara</u>

$$\bar{M} = +\hat{a}_y \chi_m J_0 a$$

$$\bar{B} = +\hat{a}_y \mu_0 (1+\chi_m) J_0 a$$

<u>Jafngildisstraumar</u>

Hvergi eru bol jafngildis-straumar því

$$\bar{J}_m = \bar{\nabla} \times \bar{M} = 0$$

en

$$\bar{J}_{ms} = \bar{M} \times \hat{a}_n$$

<u>Ofan á leiðara</u>

$$\hat{a}_n = \hat{a}_z$$

$$\rightarrow \bar{J}_{ms} = -\hat{a}_y \times \hat{a}_z \chi_m J_0 a$$
$$= -\hat{a}_x \chi_m J_0 a$$

<u>Neðan á leiðara</u>

$$\hat{a}_n = -\hat{a}_z$$

$$\rightarrow \bar{J}_{ms} = -\hat{a}_y \times \hat{a}_z \chi_m J_0 a$$
$$= -\hat{a}_x \chi_m J_0 a$$

Jafngildisstraumarnir eru á báðum yfirborðum andstæðir við bol frjálsa strauminn.