Problem set 08

Problem 1

The magnetic flux in a circular loop with radius a, self-inductance L, and resistance R changes in time

$$\Phi(t) = \Phi_0 \exp(-\Gamma t)(\Gamma t)^2$$

Find the time-dependent current in the loop, $i(t)$ if initially there is no current in the loop. Analyze the solution graphically.

P.S. A related problem was solved 2016. You can consult the solution method there, but here some curious effects can be found, that are not obvious there. Clearly two time scales are competing in the solution, they are determined by the paramaters R/L for the loop, and Γ for the magnetic flux. The best strategy is to write the solution using these time scales to make the terms in it dimensionless. We can assume Γ and L to be constant but vary the resistance R of the loop and introduce a connection between the length scales

$$\frac{R}{L} = \alpha\Gamma.$$

Writing the solution in terms of α and Γ, plot the current in the loop as a function of the dimensionless quantity Γt for several values of α, and on another graph plot the current as a function of α for a fixed value of Γt. What do you find?

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The solution for the current in the loop was found from the differential equation of the system in 2016, but notice the appendix to the solution showing how it can be found from an integral equation, and what further information can be extracted from the integral equation.

The problem is due Monday March 10 2025 at 20:00