4.27

$$\chi = A\begin{pmatrix} 3i \\ 4 \end{pmatrix}$$

a) Finna normunarfastann A

$$\chi^*\chi = |A|^2(-3i, 4)\begin{pmatrix} 3i \\ 4 \end{pmatrix} = |A|^2\{9+16\} = |A|^2 25$$

$$\rightarrow A = \frac{1}{5}$$

b) finna væntigildi S_x, S_y, og S_z í χ

$$\langle S_x \rangle_\chi = \frac{\hbar A^2}{2}(-3i, 4)\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 3i \\ 4 \end{pmatrix} = \frac{A^2\hbar}{2}(4, -3i)\begin{pmatrix} 3i \\ 4 \end{pmatrix} = \frac{A^2\hbar}{2}(12i - 12i)$$

$$= 0$$

$$\langle S_y\rangle_x = \frac{\hbar A^2}{2}(-3i,4)\begin{pmatrix}0 & -i\\ i & 0\end{pmatrix}\begin{pmatrix}3i\\ 4\end{pmatrix} = \frac{\hbar A^2}{2}(4i\,,-3)\begin{pmatrix}3i\\ 4\end{pmatrix} = \frac{\hbar A^2}{2}\{-12-12\}$$

$$= -\frac{6\hbar}{25}$$

$$\langle S_z\rangle = \frac{\hbar^2 A^2}{2}(-3i,4)\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}\begin{pmatrix}3i\\ 4\end{pmatrix} = \frac{\hbar A^2}{2}(-3i,-4)\begin{pmatrix}3i\\ 4\end{pmatrix} = \frac{\hbar A^2}{2}\{9-16\}$$

$$= -\frac{\hbar A^2}{2}7 = -\frac{\hbar 7}{50}$$

c)

$$\left.\begin{aligned} \sigma_z^2 &= \begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}\\ \sigma_x^2 &= \begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix} = \begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}\\ \sigma_y^2 &= \begin{pmatrix}0 & -i\\ i & 0\end{pmatrix}\begin{pmatrix}0 & -i\\ i & 0\end{pmatrix} = \begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}\end{aligned}\right\} \longrightarrow \langle \sigma_i^2\rangle_x = \frac{\hbar^2 A^2}{4}(-3i,4)\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}\begin{pmatrix}3i\\ 4\end{pmatrix}$$

$$= \frac{\hbar^2 A^2}{4}(-3i,4)\begin{pmatrix}3i\\ 4\end{pmatrix}$$

$$= \frac{\hbar^2 A^2}{4}\{9+16\} = \frac{\hbar^2}{4}$$

$$\sigma_{S_x} = \sqrt{\langle S_x^2\rangle_x - \langle S_x\rangle_x^2} = \sqrt{\frac{\hbar^2}{4}} = \frac{\hbar}{2}$$

$$\sigma_{S_y} = \sqrt{\langle S_y^2\rangle_x - \langle S_y\rangle_x^2} = \sqrt{\frac{\hbar^2}{4} - \frac{\hbar^2 6^2}{25^2}} = \hbar\sqrt{\frac{1}{4} - \frac{6^2}{25^2}}$$

$$\simeq \hbar \cdot 0{,}43863$$

$$\sigma_{S_z} = \sqrt{\langle S_z^2\rangle_x - \langle S_z\rangle_x^2} = \sqrt{\frac{\hbar^2}{4} - \frac{\hbar^2 49}{(50)^2}} = \hbar \cdot 0.48$$

d) Nú verður að gilda að

$$\sigma_{S_x} \cdot \sigma_{S_y} \geq \frac{\hbar}{2} |\langle S_z\rangle|$$

$$\sigma_{S_x} \cdot \sigma_{S_y} = \frac{\hbar^2}{2} \cdot 0{,}43863$$

$$= \hbar^2 \cdot 0{,}2193$$

$$\frac{\hbar}{2}|\langle L_z\rangle| = \frac{\hbar^2}{2}\left|\frac{7}{50}\right| = \hbar^2 0{,}07$$

og áfram .....

4.29

a) Finna eigingildi og vigra $S_y = \frac{\hbar}{2}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$

Eigingildin eru $\pm \frac{\hbar}{2}$

með eiginvigra $\frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ i \end{pmatrix}$ og $\frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -i \end{pmatrix}$

b) S_y mælt fyrir almennt ástand $\chi = \begin{pmatrix} a \\ b \end{pmatrix}$

Til ~~þess~~ að sjá þarf að liða χ í eiginástöndunum

$$\chi = \frac{1}{\sqrt{2}}\left\{ \frac{a+ib}{\sqrt{2}}\begin{pmatrix} 1 \\ -i \end{pmatrix} + \frac{a-ib}{\sqrt{2}}\begin{pmatrix} 1 \\ +i \end{pmatrix} \right]$$

(5)

Meligildid $+\frac{\hbar}{2}$ fost mee litumum $\left|\frac{a-ib}{\sqrt{2}}\right|^2$

$$= \frac{1}{2}|a-ib|^2 =$$

Gildid $-\frac{\hbar}{2}$ fost mee litumum $\left|\frac{a+ib}{\sqrt{2}}\right|^2 = \frac{1}{2}|a+ib|^2$

Heildelitumorem $\frac{1}{2}\left[|a-ib|^2 + |a+ib|^2\right]$

$$= \frac{1}{2}\left\{(a^*+ib^*)(a-ib) + (a^*-ib^*)(a+ib)\right\}$$

$$= \frac{1}{2}\left\{|a|^2+|b|^2+|a|^2+|b|^2 \cancel{-iba^*} \cancel{+ib^*a} \cancel{-ib^*a} \cancel{+iba^*}\right\}$$

$$= \frac{2}{2}\left\{|a|^2+|b|^2\right\} = |a|^2+|b|^2 = 1$$

c) Ef S_y^2 er mælt, hvaða gildi fæst með hvaða líkum?

Á tvennan hátt má sjá svarið

T.d. $S_y^2 = \frac{\hbar^2}{4} I$ einingar fylkið

$\frac{\hbar^2}{4}$ með líkum 1

Eða undir tekin

$$\left(+\frac{\hbar}{2}\right)\left(+\frac{\hbar}{2}\right)$$

$$\left(-\frac{\hbar}{2}\right)\left(-\frac{\hbar}{2}\right)$$

með öllum líkum 1