(1.9) (1)

$$\Psi(x,t) = A e^{-a\left[\frac{mx^2}{\hbar} + it\right]} \quad , A, a > 0 \text{ Rauntölur}$$

a) finna A

$$\int_{-\infty}^{\infty} dx\, |\Psi|^2 = A^2 \int_{-\infty}^{\infty} dx\, e^{-\frac{2amx^2}{\hbar}} = A^2 \sqrt{\frac{\hbar}{2am}} \int_{-\infty}^{\infty} d\left(\sqrt{\frac{2am}{\hbar}}\, x\right) e^{-\frac{2amx^2}{\hbar}}$$

$$= A^2 \sqrt{\frac{\hbar}{2am}} \int_{-\infty}^{\infty} du\, e^{-u^2} = A^2 \sqrt{\frac{\hbar}{2am}} \sqrt{\pi} = 1$$

$$\rightarrow A^2 = \sqrt{\frac{2am}{\hbar \pi}}$$

(2)

b) fyrir hvaða mætti V er Ψ lausn á jöfnu Schrödingers

Athugum $T\psi = \frac{p^2}{2m}\psi = -\frac{\hbar^2 d_x^2}{2m}\psi$

$$T\psi = -\frac{\hbar^2 A}{2m} d_x^2\, e^{-amx^2/\hbar} = -\frac{\hbar^2 A}{2m}\left\{4\left(\frac{am}{\hbar}\right)^2 x^2 - 2\left(\frac{am}{\hbar}\right)\right\} \cdot e^{-amx^2/\hbar}$$

$$= A e^{-\frac{amx^2}{\hbar}}\left\{-2a^2 m x^2 + \hbar a\right\}$$

Schrödinger jafnan er

$$H\psi = E\psi \quad \text{eða} \quad (T+V)\psi = E\psi$$

því lítur út hér að mættið sé

$$V(x) = +2ma^2x^2$$

(3)

og orkan sé $E = \hbar a$, sem einnig passar við tíma þátt bylgjufallsins

$$\varphi(t) = e^{-iat} = e^{-i\frac{E}{\hbar}t}$$

Passar við H.O. fyrir $a = \frac{\omega}{2}$

c) Reikna x, x^2, p, p^2

Bylgjufallið er jafnstætt $\rightarrow \langle x \rangle = 0$, $\langle p \rangle = 0$

$$\langle p^2 \rangle = A^2 \int_{-\infty}^{\infty} dx\, e^{-\frac{2mx^2a}{\hbar}} \left\{4\left(\frac{am}{\hbar}\right)^2 x^2 - 2\left(\frac{am}{\hbar}\right)\right\}(-\hbar^2)$$

$$= A^2 \int_{-\infty}^{\infty} dx\, \exp\left(-\frac{2max^2}{\hbar}\right)\left\{2\frac{am}{\hbar}\left(\frac{2max^2}{\hbar}\right) - 2\frac{am}{\hbar}\right\}(-\hbar^2)$$

(4)

$$\langle p^2 \rangle = A^2 \sqrt{\frac{2ma}{\hbar}} \int_{-\infty}^{\infty} \sqrt{\left(\frac{2ma}{\hbar}\right)}\, dx\, \exp\left(-\frac{2amx^2}{\hbar}\right)\left\{\left(\frac{2max^2}{\hbar}\right) - 1\right\}(-\hbar^2)$$

$$= A^2 \sqrt{\frac{2ma}{\hbar}}\, \hbar^2 \int_{-\infty}^{\infty} du\, e^{-u^2}(1-u^2)$$

$$= \sqrt{\frac{2am}{\hbar\pi}}\, \hbar^2 \sqrt{\frac{2ma}{\hbar}} \left\{\frac{\sqrt{\pi}}{2}\right\} = \hbar^2 \frac{am}{\hbar} = \hbar^2 \frac{m\omega}{2\hbar}$$

$$= \hbar^2 \frac{1}{2d}$$

þ.s. $d = \sqrt{\frac{\hbar}{m\omega}}$ náttúrulega lengdin fyrir H.O.

5

$$\langle x^2\rangle = A^2 \int_{-\infty}^{\infty} dx\; x^2\, e^{-\frac{2amx^2}{\hbar}} = A^2\sqrt{\frac{\hbar}{2ma}}\,\frac{\hbar}{2ma}\int_{-\infty}^{\infty} dx\sqrt{\frac{2am}{\hbar}} \cdot \frac{2amx^2}{\hbar}\, e^{-\frac{2amx^2}{\hbar}}$$

$$= A^2\sqrt{\frac{\hbar}{2ma}}\,\frac{\hbar}{2ma}\int_{-\infty}^{\infty} du\, u^2 e^{-u} = A^2\sqrt{\frac{\hbar}{2ma}}\,\frac{\hbar}{2ma}\,\frac{\sqrt{\pi}}{2}$$

$$= \frac{\hbar}{2ma}\,\frac{1}{2} = \frac{\hbar}{m\omega}\,\frac{1}{2} = \frac{d^2}{2}$$

---

d) $\sigma_x = \sqrt{\langle x^2\rangle - \langle x\rangle^2} = \frac{d}{\sqrt{2}}$, $\sigma_p = \sqrt{\langle p^2\rangle - \langle p\rangle^2} = \frac{\hbar}{\sqrt{2}\,d}$

$\to \sigma_x \cdot \sigma_p = \frac{\hbar}{2}$ minnsta mögulega

1.15 1

$$P(t) = \int_{-\infty}^{\infty} dx\, |\Psi(x,t)|^2 = e^{-t/\tau}$$

Gerum ráð fyrir $V = V_0 - i\Gamma$, $\Gamma > 0$, $\Gamma \in \mathbb{R}$

Sýna að í stað $d_t\int_{-\infty}^{\infty} dx\, |\Psi(x,t)|^2 = 0$

komi nú $d_t P = -\frac{2\Gamma}{\hbar} P$

Schrödinger jafnan er

$$i\hbar\partial_t\Psi = H\Psi = \left\{\frac{p^2}{2m} + V_0 - i\Gamma\right\}\Psi$$

og $-i\hbar\partial_t\Psi^* = \left\{\frac{p^2}{2m} + V_0 + i\Gamma\right\}\Psi^*$

2

$$d_t|\Psi|^2 = (\partial_t\Psi^*)\Psi + \Psi^*\partial_t\Psi$$

$$= \left\{\text{liðirnir vegna } \frac{p^2}{2m} + V_0\right\} - \frac{\Gamma}{\hbar}\Psi^*\Psi - \frac{\Gamma}{\hbar}\Psi^*\Psi$$

$\to d_t P = -\frac{2\Gamma}{\hbar} P$

---

b) $\frac{dP}{dt} = -\frac{2\Gamma}{\hbar} P \to \frac{dP}{P} = -\frac{2\Gamma\, dt}{\hbar}$

með lausn $P(t) = P_0 \exp\left\{-\frac{2\Gamma}{\hbar} t\right\}$

Náttúrulegur tímafasti: $\frac{\hbar}{2\Gamma}$

1.16 1

Sýna að $d_t\int_{-\infty}^{\infty} dx\, \Psi_1^*\Psi_2 = 0$

fyrir tvær normanlegar lausnir Schrödinger j.

$$d_t\int_{-\infty}^{\infty} dx\, \Psi_1^*\Psi_2 = \int_{-\infty}^{\infty} dx\left\{(\partial_t\Psi_1^*)\Psi_2 + \Psi_1^*(\partial_t\Psi_2)\right\}$$

$$= \int_{-\infty}^{\infty} dx\left\{\left(\frac{H}{-i\hbar}\Psi_1^*\right)\Psi_2 + \Psi_1^*\left(\frac{H}{i\hbar}\Psi_2\right)\right\}$$

H er Hermitískur virki

$$= \frac{i}{\hbar}\int_{-\infty}^{\infty} dx\left\{(H\Psi_1^*)\Psi_2 - \Psi_1^*(H\Psi_2)\right\} = 0$$

(2.6) (1)

$$\Psi(x,0) = A\left\{\psi_1(x) + e^{i\phi}\psi_2(x)\right\}$$

Reikna

$$\Psi(x,t) = A\left\{\psi_1(x)e^{-i\omega_1 t} + \psi_2(x)e^{-i\omega_2 t + i\phi}\right\}$$

Normunin er óbreytt $A = \frac{1}{\sqrt{2}}$

$$|\Psi(x,t)|^2 = \frac{1}{2}\left\{\psi_1^2(x) + \psi_2^2(x) + \psi_1(x)\psi_2(x)\left(e^{it(\omega_1-\omega_2)+i\phi} + e^{-it(\omega_1-\omega_2)-i\phi}\right)\right\}$$

$$= \frac{1}{2}\left\{\psi_1^2(x) + \psi_2^2(x) + 2\psi_1(x)\psi_2(x)\cos((\omega_1-\omega_2)t + \phi)\right\}$$

$$= \frac{1}{2}\left\{\psi_1^2(x) + \psi_2^2(x) + 2\psi_1(x)\psi_2(x)\cos(3\omega t - \phi)\right\}$$

(2)

$$\langle x\rangle = a\left\{\frac{1}{2} - \frac{16}{9\pi^2}\cos(3\omega t - \phi)\right\}$$

$\phi = \pi$ gefur $\cos(3\omega t - \pi) = -\cos(3\omega t)$ π úr fasa

$\phi = \frac{\pi}{2}$ — || — $\cos(3\omega t - \frac{\pi}{2}) = -\sin(3\omega t)$ $\frac{\pi}{2}$ úr fasa

(2.14) (1)

H.O. með tíðni ω, eind í grunnástandi

Allt í einu verður $\omega' = 2\omega$

Hver eru líkind fyrir því að mæla $E = \frac{\hbar\omega}{2}$?

— || — $E = \hbar\omega$?

---

fyrir óbreytt ω er

$$\psi_n(x) = \frac{1}{\sqrt{2^n n!\sqrt{\pi}a}} H_n\left(\frac{x}{a}\right) e^{-\frac{1}{2}\left(\frac{x}{a}\right)^2}, \quad a = \sqrt{\frac{\hbar}{m\omega}}$$

breytt $\psi_0'(x) = \frac{1}{\sqrt{\sqrt{\pi}a'}} e^{-\frac{1}{2}\left(\frac{x}{a'}\right)^2}$ $a' = \sqrt{\frac{\hbar}{m2\omega}} = \frac{a}{\sqrt{2}}$

$$= \sqrt{\frac{\sqrt{2}}{\sqrt{\pi}a}}\, e^{-\left(\frac{x}{a}\right)^2}$$

(2)

Nú þyrfti að liða $\psi_0'(x)$ í gamla grunninum $\{\psi_n\}$

$$\psi_0'(x) = \sum_{n=0}^{\infty} c_n \psi_n(x)$$

en við erum bara beðin um að finna c_0 og c_1

$$c_0 = \int dx\, \psi_0^*(x)\,\psi_0'(x) = \frac{\sqrt{\sqrt{2}}}{\sqrt{\pi}a}\int_{-\infty}^{\infty} dx\, e^{-\frac{3}{2}\left(\frac{x}{a}\right)^2}$$

$$= \frac{\sqrt{\sqrt{2}}}{\sqrt{\pi}}\int_{-\infty}^{\infty} du\, e^{-\frac{3}{2}u^2} = \frac{\sqrt{\sqrt{2}}}{\sqrt{\pi}}\,\frac{\sqrt{2}\sqrt{\pi}}{\sqrt{3}} = \sqrt{\frac{2\sqrt{2}}{3}}$$

$$c_1 = 0 = \sqrt{\frac{2\sqrt{2}}{\sqrt{\pi}}}\int_{-\infty}^{\infty} du\, u\, e^{-\frac{3}{2}u^2}$$ (oddstætt)

(3)

Ψ_0' er ekki eiginástand H, en Ψ_0' er liðað í eiginástöndum H, við getum aðeins mælt orku sem eigingildi H með líkindum $|C_n|^2$

$$|C_0|^2 = \frac{2\sqrt{2}}{3} \simeq 0{,}94281\ldots$$

$$|C_1|^2 = 0$$

(2.15) (1)

Hver eru líkindi þess að finna H.O. í grunnástandi utan sígildu markanna

$$V = \frac{1}{2}m\omega^2 x^2 \quad , \quad E_0 = \frac{\hbar\omega}{2}$$

$$\rightarrow \quad x_{cl} = \pm\sqrt{\frac{\hbar}{m\omega}} = \pm a$$

$|\Psi_0|^2$ er jafnstætt, því eru líkindin

$$2\int_a^\infty dx\, |\Psi_0|^2 = \frac{2}{\sqrt{\pi}}\int_a^\infty \frac{dx}{a}\, e^{-\left(\frac{x}{a}\right)^2} = \frac{2}{\sqrt{\pi}}\int_1^\infty du\, e^{-u^2}$$

$$= \frac{2}{\sqrt{\pi}\cdot 2}\,\Gamma\left(\tfrac{1}{2}, 1\right) = 0{,}07865$$

Incomplete gamma

$$\Gamma(a,x) = \int_x^\infty dt\, t^{a-1} e^{-t}$$

(2.19) (1)

líkindastraumsþéttleiki

$$J(x,t) = \frac{i\hbar}{2m}\left\{(\partial_x\Psi^*)\Psi - \Psi^*\partial_x\Psi\right\}$$

Reikna fyrir

$$\Psi_k(x,t) = A\exp\left\{i\left(kx - \omega_k t\right)\right\} \quad , \quad \omega_k = \frac{\hbar k^2}{2m}$$

$$J_k(x,t) = \frac{i\hbar}{2m}|A|^2\left\{-ik - ik\right\} = \frac{\hbar k}{m}|A|^2$$

Í hvaða átt?

fyrir þetta einvíða verkefni er straumurinn í sömu stefnu og k, einvídd býður upp á $\pm k$

(2.22) (1)

Frjáls eind, pakki

$$\Psi(x,0) = Ae^{-ax^2} \qquad a \in \mathbb{R},\ a > 0$$

a) ~~Staðla~~ $\Psi(x,0)$, skilgreini $\alpha^{-2} = a$, $\alpha = \frac{1}{\sqrt{a}}$

$$|A|^2\int_{-\infty}^{\infty} dx\, e^{-2ax^2} = |A|^2\alpha\int_{-\infty}^{\infty}\frac{dx}{\alpha}\, e^{-2\left(\frac{x}{\alpha}\right)^2} = |A|^2\alpha\int_{-\infty}^{\infty} du\, e^{-2u^2} = |A|^2\alpha\frac{\sqrt{\pi}}{\sqrt{2}}$$

$$\rightarrow |A|^2\alpha\frac{\sqrt{\pi}}{\sqrt{2}} = 1 \quad \text{eða} \quad A = \frac{\sqrt{\sqrt{2}}}{\sqrt{\alpha\sqrt{\pi}}} = \left(\frac{a2}{\pi}\right)^{1/4}$$

b) Finna $\Psi(x,t)$

$$\Psi(x,t) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} dk\, \phi(k)\, e^{i(kx - \omega_k t)}$$

(2)

$$\phi(k) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} dx\, \Psi(x,0)\, e^{-ikx} = \frac{\sqrt{2}}{\sqrt{2\pi\alpha\sqrt{\pi}}}\int_{-\infty}^{\infty} dx\, e^{-(\frac{x}{\alpha})^2 - ikx}$$

$$= \frac{\sqrt{2}}{\sqrt{2\pi\alpha\sqrt{\pi}}}\,\alpha\int_{-\infty}^{\infty}\frac{dx}{\alpha}\exp\left\{-\left(\frac{x}{\alpha}\right)^2 - ik\alpha\left(\frac{x}{\alpha}\right)\right\}$$

$$= \sqrt{\frac{\alpha}{\pi\sqrt{2\pi}}}\int_{-\infty}^{\infty} du\,\exp\left\{-u^2 - ik\alpha u\right\}$$

$$= \sqrt{\frac{\alpha}{\pi\sqrt{2\pi}}}\,e^{-(\frac{k\alpha}{2})^2}\int_{-\infty}^{\infty} du\, e^{-(u+\frac{ik\alpha}{2})^2} = \sqrt{\pi}\sqrt{\frac{\alpha}{\pi\sqrt{2\pi}}}\,e^{-(\frac{k\alpha}{2})^2}$$

$$= \sqrt{\frac{\alpha}{\sqrt{2\pi}}}\,e^{-(\frac{k\alpha}{2})^2} = \frac{1}{\sqrt{\sqrt{2\pi a}}}\,e^{-(\frac{k}{a})^2}$$

Engin hliðrun pakka, hann er jafnstæður (samhverfur) í k um $k=0$

(3)

$$\Psi(x,t) = \frac{1}{\sqrt{2\pi}}\sqrt{\frac{\alpha}{\sqrt{2\pi}}}\int_{-\infty}^{\infty} dk\, e^{i(kx-\omega_k t) - (\frac{k\alpha}{2})^2}$$

$$= \sqrt{\frac{\alpha}{2\pi\sqrt{2\pi}}}\int_{-\infty}^{\infty} dk\,\exp\left\{-\left(\frac{k\alpha}{2}\right)^2 - i\frac{\hbar k^2}{2m}t + i(k\alpha)\frac{x}{\alpha}\right\}$$

$$\left\{-\left(\frac{k\alpha}{2}\right)^2 - i\frac{\hbar t}{2m\alpha^2}(k\alpha)^2 + i(k\alpha)\frac{x}{\alpha}\right\} = \left\{-\left(\frac{k\alpha}{2}\right)^2\left[1 + i\frac{2\hbar t}{\alpha^2 m}\right] + i(k\alpha)\frac{x}{\alpha}\right\}$$

$$= \left\{-\left(\frac{k\alpha}{2}\right)^2\beta^2 + i(k\alpha)\frac{x}{\alpha}\right\} = -\left\{\frac{(k\alpha)\beta}{2} - \frac{i}{\beta}\left(\frac{x}{\alpha}\right)\right\}^2 - \left(\frac{x}{\beta\alpha}\right)^2$$

$$\to \Psi(x,t) = \frac{1}{\alpha}\sqrt{\frac{\alpha}{2\pi\sqrt{2\pi}}}\exp\left(-\left(\frac{x}{\beta\alpha}\right)^2\right)\int_{-\infty}^{\infty} d(k\alpha)\,\exp\left[-\left(\frac{k\alpha}{2}\beta - \frac{ix}{\beta\alpha}\right)^2\right]$$

(4)

$$\Psi(x,t) = \frac{1}{\sqrt{2\pi\alpha\sqrt{2\pi}}}\exp\left\{-\left(\frac{x}{\beta\alpha}\right)^2\right\}\int_{-\infty}^{\infty} du\,\exp\left[-\frac{\beta^2}{4}\left(u - \frac{ix2}{\beta^2\alpha}\right)^2\right]$$

$$= \frac{1}{\sqrt{2\pi\alpha\sqrt{2\pi}}}\exp\left\{-\left(\frac{x}{\beta\alpha}\right)^2\right\}\sqrt{\pi}\,\frac{2}{\beta}$$

$$= \frac{\sqrt{2}}{\sqrt{\alpha\sqrt{2\pi}}}\exp\left\{-\left(\frac{x}{\beta\alpha}\right)^2\right\}\frac{1}{\beta} = \left(\frac{2}{\alpha^2\pi}\right)^{1/4}\exp\left\{-\left(\frac{x}{\beta\alpha}\right)^2\right\}\frac{1}{\beta}$$

c) finna $|\Psi(x,t)|^2$ og tákna við $w^{-1} = \sqrt{\frac{a}{1+(\frac{2\hbar a t}{m})^2}}$

$$w^{-1} = \sqrt{\frac{1}{\alpha^2\left[1+\left(\frac{2\hbar t}{m\alpha^2}\right)^2\right]}} \qquad \beta^2 = \left\{1 + \frac{2\hbar i t}{\alpha^2 m}\right\}$$

ég breyti þ.a. $[W] \sim L$

(5)

$$|\Psi(x,t)|^2 = \sqrt{\frac{2}{\alpha^2\pi}}\exp\left\{-\left(\frac{x}{\alpha}\right)^2\left[\frac{2}{1+\left(\frac{2\hbar t}{\alpha^2 m}\right)^2}\right]\right\}\frac{1}{\sqrt{1+\left(\frac{2\hbar t}{\alpha^2 m}\right)^2}}$$

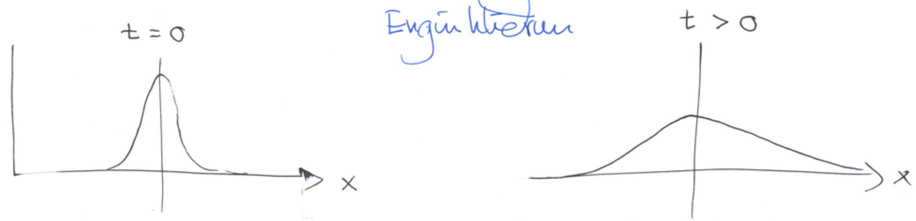
$$= \sqrt{\frac{2}{\pi}}\exp\left\{-2\left(\frac{x}{w}\right)^2\right\}\frac{1}{w}, \qquad [w] = L$$

Hvernig breytist $w(t)$ með tíma

$w(0) = \alpha$ upprunaleg stölun (náttúrulegur skali)

$w(t) \xrightarrow[t\to\infty]{} \infty$

Engin hliðrun



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d) samhverfur pakki í x og p

$$\rightarrow \langle x\rangle = 0 \quad , \quad \langle p\rangle = 0$$

$$\langle x^2\rangle = \sqrt{\frac{2}{\pi w^2}}\int_{-\infty}^{\infty} dx\; x^2\, e^{-2\left(\frac{x}{w}\right)^2}$$

$$= \sqrt{\frac{2}{\pi}}\, w^2 \int_{-\infty}^{\infty} du\, u^2 e^{-2u^2} = w^2\sqrt{\frac{2}{\pi}}\,\frac{\sqrt{\pi}}{2^{5/2}}$$

$$= w^2\, 2^{\frac{1}{2}-\frac{5}{2}} = \frac{w^2}{4}$$

$\langle x^2\rangle \sim w^2$

sem vex með tíma

7

$\langle p^2\rangle$: Frjálseind, engin ytri kraftur breytir skriðþunga

$$\Psi(x,t) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} dk\, \phi(k)\, e^{i(kx - i\omega_k t)}$$

fast fall, samhverft, sem við fundum

væntigildi $\langle p\rangle$ og $\langle p^2\rangle$ eru óháð tíma

$$\Psi(x,0) = \left(\frac{2}{\alpha^2\pi}\right)^{1/4} \exp\left\{-\left(\frac{x}{\alpha}\right)^2\right\} \qquad \text{því } \beta(0) = 1$$

$$-\hbar^2\partial_x^2\Psi(x,0) = -\hbar^2\left(\frac{2}{\alpha^2\pi}\right)^{1/4}\exp\left\{-\left(\frac{x}{\alpha}\right)^2\right\}\left\{\frac{4x^2}{\alpha^4} - \frac{2}{\alpha^2}\right\}$$

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$$\langle p^2\rangle = -\sqrt{\frac{2}{\pi}}\,\frac{\hbar^2}{\alpha}\int_{-\infty}^{\infty} dx\, \exp\left\{-2\left(\frac{x}{\alpha}\right)^2\right\}\left\{\frac{4x^2}{\alpha^4} - \frac{2}{\alpha^2}\right\}$$

$$= -\sqrt{\frac{2}{\pi}}\,\frac{\hbar^2}{\alpha^2}\int_{-\infty}^{\infty} \frac{dx}{\alpha}\, \exp\left\{-2\left(\frac{x}{\alpha}\right)^2\right\}\cdot\left\{4\left(\frac{x}{\alpha}\right)^2 - 2\right\}$$

$$= \frac{\hbar^2}{\alpha^2}\left\{\frac{4}{2}\sqrt{\frac{2}{\pi}}\left[\sqrt{\frac{\pi}{2}} - \frac{\sqrt{\pi}}{2^{3/2}}\right]\right\} = \frac{\hbar^2}{\alpha^2}\left\{2\left(1-\frac{1}{2}\right)\right\} = \frac{\hbar^2}{\alpha^2}$$

e) $$\Delta x\cdot\Delta p = \frac{w(t)}{2}\cdot\frac{\hbar}{\alpha} = \frac{\hbar}{2}\left(\frac{w(t)}{\alpha}\right)$$

$$= \frac{\hbar}{2}\sqrt{1 + \left(\frac{2\hbar t}{m\alpha^2}\right)^2} \geq \frac{\hbar}{2}$$

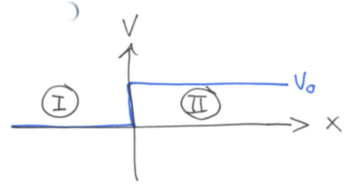
minnst þegar t = 0 í upphafi

og vex með tíma

1

2.34 Mættisþrep

$$V(x) = \begin{cases} 0 & \text{ef } x \leq 0 \\ V_0 & \text{ef } x > 0 \end{cases}$$



a) Reikna endurkast ef $E < V_0$

I $$k_1 = \frac{\sqrt{2mE}}{\hbar} \quad , \quad \kappa = \frac{\sqrt{2m(V_0 - E)}}{\hbar}$$

$$\psi(x) = e^{ik_1 x} + Be^{-ik_1 x}$$

eindin smýgur aðeins inn í vegginn

II $$\psi(x) = Ce^{-\kappa x}$$

vaxandi lausnin $e^{\kappa x}$ er ekki möguleg

Samfella ψ

$$1 + B = C \quad (i)$$

Samfella ψ'

$$\psi_I'(0) = \psi_{II}'(0)$$

$$ik_1 - ik_1 B = -\kappa C \quad (ii)$$

2 jöfnur, 2 óþekktar stærðir

$(i) \to (ii)$

$$ik_1(1-B) = -\kappa(1+B)$$

$$B(\kappa - ik_1) = -ik_1 - \kappa$$

$$B = \frac{-ik_1 - \kappa}{-ik_1 + \kappa}$$

$$|B|^2 = B^*B = \frac{\kappa^2 - k_1^2}{\kappa^2 - k_1^2} = 1$$

eindin kemur alltaf til baka þó hún smjúgi aðeins inn í vegginn

---

b) Speglun ef $E > V_0$

$$k_1 = \frac{\sqrt{2mE}}{\hbar}, \quad k_2 = \frac{\sqrt{2m(E-V_0)}}{\hbar}$$

I $\quad \psi(x) = e^{ik_1x} + Be^{-ik_1x}$

Aðeins bylgja frá vinstri

II $\quad \psi(x) = Ce^{ik_2x}$

Samfella í ψ

$$1 + B = C \quad (i)$$

Samfella í ψ'

$$ik_1 - ik_1 B = ik_2 C \quad (ii)$$

$(i) \to (ii)$

$$ik_1(1-B) = ik_2(1+B)$$

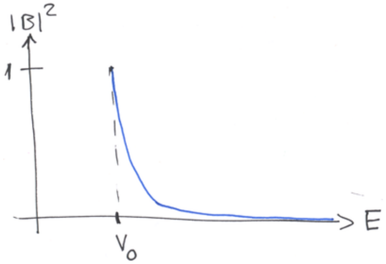
$$B(-ik_1 - ik_2) = -ik_1 + ik_2$$

$$B = -\frac{ik_2 - ik_1}{ik_2 + ik_1} = -\frac{k_2-k_1}{k_2+k_1}$$

$$|B|^2 = \frac{(k_2-k_1)^2}{(k_2+k_1)^2}$$

$$|B|^2 = \frac{\left(\sqrt{E-V_0} - \sqrt{E}\right)^2}{\left(\sqrt{E-V_0} + \sqrt{E}\right)^2}$$

Ef $V_0 = 0 \to |B|^2 = 0$



c) Rifjum upp dæmi (2.19) um líkinda straumþéttleika

þar sást að straumur ástands

$$\Psi_k(x,t) = A\exp\{i(kx - \omega_k t)\}$$

$$J(x,t) = \frac{i\hbar}{2m}\left\{(\partial_x\Psi)^*\Psi - \Psi^*\partial_x\Psi\right\}$$

$$= \frac{i\hbar}{2m}|A|^2\{-ik - ik\} = \frac{\hbar k}{m}|A|^2$$

$E > V_0$ Reiknum C

$$1 + B = C$$

$$ik_1 - ik_1 B = ik_2 C$$

eins og áður

$$B = C - 1$$

$$ik_1(1-(C-1)) = ik_2 C$$

$$ik_1(2-C) = ik_2 C$$

$$2ik_1 = iC(k_2+k_1)$$

$$C = \frac{2k_1}{k_2+k_1}$$

straumur einda inn

$$J_{inn} = \frac{\hbar k_1}{m}|A|^2 = \frac{\hbar k_1}{m}$$

endurkast straumur

$$J_R = -\frac{\hbar k_1}{m}|B|^2 = -\frac{\hbar k_1}{m}\frac{(k_2-k_1)^2}{(k_2+k_1)^2}$$

útstraumur

$$J_T = \frac{\hbar k_2}{m}|C|^2 = \frac{\hbar k_2}{m}\frac{4k_1^2}{(k_2+k_1)^2}$$

Ef líkindi R eru $|B|^2$ þá eru líkindi $T = |C|^2\frac{k_2}{k_1}$

$$T = \frac{4k_1^2}{(k_2+k_1)^2}\cdot\frac{k_2}{k_1} = \frac{4k_1k_2}{(k_2+k_1)^2}$$

$$T = |C|^2\sqrt{\frac{E-V_0}{E}}$$

vegna straumvarðveislu

d)

$$T = \frac{4k_1k_2}{(k_2+k_1)^2}$$

$$R = \frac{(k_2-k_1)^2}{(k_2+k_1)^2}$$

$$\rightarrow T+R=1$$

Varðveisla straums

$$J_{in} + J_R = J_T \quad \Big| \quad \frac{\hbar k_1}{m} - \frac{\hbar k_1}{m}|B|^2 = \frac{\hbar k_2}{m}|C|^2$$

$$\frac{\hbar k_1}{m} - \frac{\hbar k_1}{m}\frac{(k_2-k_1)^2}{(k_2+k_1)^2} = \frac{\hbar k_2}{m}\frac{4k_1^2}{(k_2+k_1)^2}$$

$$\frac{\hbar k_1}{m}\left[1 - \frac{(k_2-k_1)^2}{(k_2+k_1)^2} = \frac{4k_2k_1}{(k_2+k_1)^2}\right]$$

2.44

$$V(x) = \begin{cases} \alpha\delta(x) & \text{fyrir } -a<x<+a \\ \infty & -\|- \quad |x|\geq a \end{cases}$$

Mættið er samhverft → lausnir eru annaðhvort odd- eða jafnstæðar og verða að uppfylla

$$d_x\psi(0^+) - d_x\psi(0^-) = +\frac{2m\alpha}{\hbar^2}\psi(0)$$

vegna δ-veggsins í brunninum

Odd-stæðar lausnir

Fyrir lausn sem er oddstæð gildir að $\psi(0)=0$ → δ-veggurinn hefur engin áhrif og lausnin eru oddstæðu lausnir óendanlegs brunns með breidd $2a$

oddstæðar lausnir eru því með orkuna

$$E_n^{odd} = \frac{(2n)^2\pi^2\hbar^2}{2m(2a)^2}, \quad n=1,2,3,\ldots$$

$$\psi_n^{odd}(x) = \sqrt{\frac{2}{2a}}\sin\left(\frac{2n\pi x}{2a}\right) = \sqrt{\frac{1}{a}}\sin\left(\frac{n\pi x}{a}\right)$$

jafnstæðar lausnir

Ef δ-veggurinn væri ekki, þ.a $\alpha=0$

$$E_n^{even} = \frac{(2n-1)^2\pi^2\hbar^2}{2m(2a)^2}, \quad n=1,2,3,\ldots$$

$$\psi^{even}(x) = \sqrt{\frac{2}{2a}}\cos\left(\frac{(2n-1)\pi x}{2a}\right) = \sqrt{\frac{1}{a}}\cos\left(\frac{(2n-1)\pi x}{2a}\right)$$

en ef $\alpha\neq 0$

jafnstæð lausn

$$\psi_n(x) = \begin{cases} -A\sin(k_n(x+a)) & -a\leq x<0 \\ +A\sin(k_n(x-a)) & 0<x\leq +a \end{cases}$$

Samfella í ψ er sjálfkrafa

$$-A\sin(k_na) = A\sin(-k_na)$$

Brot í afleiðu

$$+Ak_n\cos(k_na) + Ak_n\cos(k_na) = +\frac{2m\alpha}{\hbar^2}A\sin(-k_na)$$

$$k_n\cos(k_na) = -\frac{2m\alpha}{\hbar^2}\sin(k_na)$$

$$k_n\cot(k_na) = -\frac{2m\alpha}{\hbar^2}$$

jafna (óbein) sem ákvarðar k_n

(4)

umskrifum sem

$$\cot(k_n a) + \frac{2m\alpha}{\hbar^2 k_n} = 0$$

$$\cot(k_n a) + \frac{2m\alpha a}{\hbar^2 (k_n a)} = 0$$

$$\cot(k_n a) + \frac{2ma^2}{\hbar^2}\frac{\left(\frac{\alpha}{a}\right)}{(k_n a)} = 0$$

$$\cot(k_n a) + \left(\frac{\alpha}{E_1 a}\right)\frac{1}{(k_n a)} = 0$$

Allt víddarlausar stærðir í sviganum

$$\cot(k_n a) + \beta\frac{1}{k_n a} = 0$$

þ. $\beta \to 0$

$\alpha \ll E_1 a$

$$\cot(k_n a) \sim 0$$

$$\to k_n a = \frac{\pi}{2} n, \quad n = 1,2,3$$

$$E \simeq \frac{\hbar^2 (k_n a)^2}{2ma^2} = \frac{\hbar^2 \pi^2 n^2}{2m(2a)^2}$$

Lausnin fyrir engan vegg

þ. $\beta \to \infty$

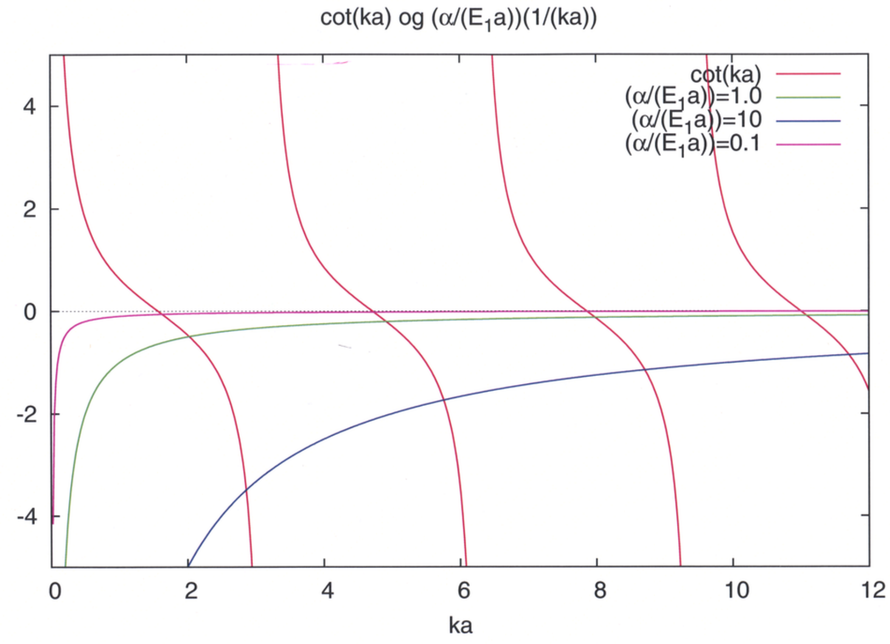
þá eru ræturnar

$$k_n a = n\pi, \quad n = 1,2,3$$

$$E \simeq \frac{\hbar^2 (k_n a)^2}{2ma^2} = \frac{\hbar^2\pi^2 n^2}{2ma^2} = \frac{\hbar^2\pi^2 (2n)^2}{2m(2a)^2} \quad n = 1,2,3,\dots$$

Sama lausn og fyrir oddstöðu

(5)



(6)

Sjá S. Flügge bls. 38

Fig. 4

38 One-Body Problems without Spin. One-Dimensional Problems

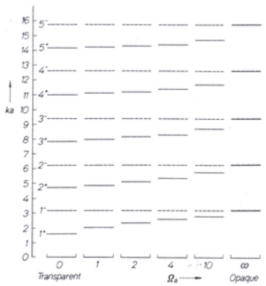


Fig. 3. Level positions for different values of wall opacity. Full lines even, broken lines odd parity

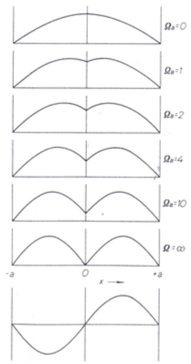


Fig. 4. Lowest eigenfunction for different wall opacities. Above 1^+, below 1^-, as limiting cases

$$\Omega = \frac{2m\alpha}{\hbar^2}$$

$$\Omega a = \frac{2m\left(\frac{\alpha}{a}\right)a^2}{\hbar^2}$$

$$= \left(\frac{\alpha}{E_1 a}\right)$$

(1)

2.46

Eind með massa m á hring með geisla L.
Lotubundið bylgjufall $\Phi(x+L) = \Phi(x)$

Notum ekki pólhnitaoktar á jöfnu Schrödinger í 2 ~~eða~~ 3 víddum

$$-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}\Phi = E\Phi$$

Notum að $x = R\phi = \frac{L\phi}{2\pi}$, því $L = 2\pi R$, $\phi \in [0, 2\pi]$

$$-\frac{\hbar^2}{2mR^2} d_\phi^2 \Phi(\phi) = E\Phi(\phi) \to d_\phi^2 \Phi = -K^2\Phi$$

með

$$K = \frac{\sqrt{+2mR^2E}}{\hbar}$$

Lausnir jöfnunnar eru

$$\psi = A e^{\pm i M \phi}, \quad M = 0, \pm 1, \pm 2$$

og þess vegna $K^2 = M^2$ og orku gildin

$$M^2 = \frac{2mR^2E}{\hbar^2} \rightarrow E_M = \frac{\hbar^2 M^2}{2mR^2} = \frac{\hbar^2 4\pi^2 M^2}{2mL^2}$$

Orkulægsta ástandið, grunnástandið með $M=0$, $E_0 = 0$
Einfalt, en öll hin eru tvöföld

Stöðlun

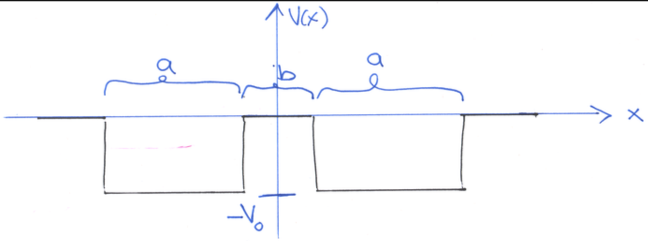
$$\int_0^L dx|\psi|^2 = \frac{L}{2\pi}\int_0^L d\phi\,|\psi|^2 = \frac{L}{2\pi}\int_0^{2\pi} d\phi A^2 = |A|^2\frac{L}{2\pi} = 1$$

$$\rightarrow A = \sqrt{\frac{2\pi}{L}} \quad \left(= \sqrt{\frac{1}{R}} \right)$$

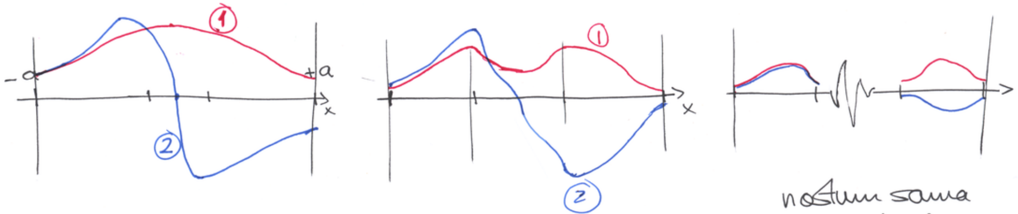
Lotubindingin kemur því í veg fyrir að bundnu ástöndin verði öll að vera einföld

En grunnástandið $\psi_{M=0}$ er einfalt!

2.47



a) $b = 0$ $\qquad$ $b \approx a$ $\qquad$ $b \gg a$



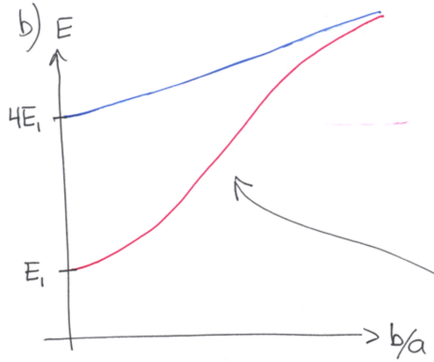
$$E_1 = \frac{\pi^2\hbar^2}{2m(2a)^2}$$

$$E_2 = \frac{4\pi^2\hbar^2}{2m(2a)^2}$$

$E_2 > E_1$

nokkurn sama orka fyrir 1 og 2

$E_1 \sim E_2$



c) Rafeindin lækkar orku kerfisins með því að draga brunnana saman

3.21

$$\hat{P} \equiv |\alpha\rangle\langle\alpha|$$

1 ef ástöndin stöðluð

$$\rightarrow \hat{P}^2 = |\alpha\rangle\langle\alpha|\alpha\rangle\langle\alpha| = |\alpha\rangle\langle\alpha| = \hat{P}$$

Eiginvigrar og gildi

$$\hat{P}|\mu\rangle = \lambda|\mu\rangle$$

Ef til er eiginvigur með $|\mu\rangle$ eigingildi λ

$$|\alpha\rangle\langle\alpha|\mu\rangle = |\mu\rangle$$

aðeins ef $\underline{|\mu\rangle = |\alpha\rangle}$ og eigingildið er $\underline{\lambda = 1}$

P3.22

$\{|1\rangle, |2\rangle, |3\rangle\}$ staðlaður grunnur

$$|\alpha\rangle = i|1\rangle - 2|2\rangle - i|3\rangle$$
$$|\beta\rangle = i|1\rangle + 2|3\rangle$$

a) Finna $\langle\alpha|$ og $\langle\beta|$

$$\langle\alpha| = \langle 1|(-i) - \langle 2|2 + \langle 3|i$$
$$\langle\beta| = \langle 1|(-i) + \langle 3|2$$

b) $\langle\alpha|\beta\rangle = \{\langle 1|(-i) - \langle 2|2 + \langle 3|i\}\{i|1\rangle + 2|3\rangle\}$

$$= \langle 1|1\rangle + 2i\langle 3|3\rangle = 1+2i$$

$$\langle\beta|\alpha\rangle = \{\langle 1|(-i) + \langle 3|2\}\{i|1\rangle - 2|2\rangle - i|3\rangle\}$$
$$= \langle 1|1\rangle - 2i\langle 3|3\rangle = 1-2i$$

$$\rightarrow \langle\beta|\alpha\rangle^* = \langle\alpha|\beta\rangle$$

---

c) finna 9 stök virkjans $\hat{A} \equiv |\alpha\rangle\langle\beta|$ í grunninum, $\hat{A} = \{i|1\rangle - 2|2\rangle - i|3\rangle\}\{\langle 1|(-i) + \langle 3|2\}$

$\langle 1|\hat{A}|1\rangle = 1$	$\langle 2|\hat{A}|1\rangle = 2i$
$\langle 1|\hat{A}|2\rangle = 0$	$\langle 2|A|2\rangle = 0$
$\langle 1|A|3\rangle = 2i$	$\langle 3|A|3\rangle = -2i$
$\langle 3|A|1\rangle = -1$	$\langle 3|A|2\rangle = 0$
	$\langle 2|A|3\rangle = -4$

$$A = \begin{pmatrix} 1 & 0 & 2i \\ 2i & 0 & -4 \\ -1 & 0 & -2i \end{pmatrix}$$

Ekki hermítt fylki

3.27

Virkinn $\hat{A}$ hefur tvö eiginástönd ψ_1 og ψ_2 með eigingildi a_1 og a_2

Virkinn $\hat{B}$ hefur tvö eiginástönd ϕ_1 og ϕ_2 með eigingildi b_1 og b_2

$$\psi_1 = \frac{3\phi_1 + 4\phi_2}{5}, \quad \psi_2 = \frac{4\phi_1 - 3\phi_2}{5}$$

5

a) A er mæld með niðurstöðu a_1, hvert er ástand kerfisins eftir mælinguna?

ψ_1

b) Ef B er mæld núna, hvaða niðurstöður fást, með hvaða líkum?

$$\psi_1 = \frac{3\phi_1 + 4\phi_2}{5}$$

b_1 með líkum $\frac{9}{25}$

b_2 —||— $\frac{16}{25}$

c) Eftir mælinguna með B er A mælt aftur hverjar eru líkurnar á að fá gildi a_1

'Ástandið er komið í ϕ_1 eða ϕ_2

$$\frac{1}{5}\begin{pmatrix}3 & 4\\ 4 & -3\end{pmatrix}\begin{pmatrix}\phi_1\\ \phi_2\end{pmatrix} = \begin{pmatrix}\psi_1\\ \psi_2\end{pmatrix}$$

6

Lausn gefur

$$\begin{pmatrix}\phi_1\\ \phi_2\end{pmatrix} = \frac{1}{5}\begin{pmatrix}3 & 4\\ 4 & -3\end{pmatrix}\begin{pmatrix}\psi_1\\ \psi_2\end{pmatrix}$$

→ líkindi fyrir því að mæla a_1 eru annað hvort $\left(\frac{3}{5}\right)^2$ eða $\left(\frac{4}{5}\right)^2$

1

4.27

$$\chi = A\begin{pmatrix}3i\\ 4\end{pmatrix}$$

a) Finna normunar fastann A

$$\chi^*\chi = |A|^2(-3i,4)\begin{pmatrix}3i\\ 4\end{pmatrix} = |A|^2\{9+16\} = |A|^2 25$$

$$\rightarrow A = \frac{1}{5}$$

b) finna væntigildi S_x, S_y og S_z í χ

$$\langle S_x\rangle_\chi = \frac{\hbar A^2}{2}(-3i,4)\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}\begin{pmatrix}3i\\ 4\end{pmatrix} = \frac{A^2\hbar}{2}(4,-3i)\begin{pmatrix}3i\\ 4\end{pmatrix} = \frac{A^2\hbar}{2}(12i-12i) = 0$$

2

$$\langle S_y\rangle_\chi = \frac{\hbar A^2}{2}(-3i,4)\begin{pmatrix}0 & -i\\ i & 0\end{pmatrix}\begin{pmatrix}3i\\ 4\end{pmatrix} = \frac{\hbar A^2}{2}(4i,-3)\begin{pmatrix}3i\\ 4\end{pmatrix} = \frac{\hbar A^2}{2}\{-12-12\}$$

$$= -\frac{6\hbar}{25}$$

$$\langle S_z\rangle = \frac{\hbar^2 A^2}{2}(-3i,4)\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}\begin{pmatrix}3i\\ 4\end{pmatrix} = \frac{\hbar A^2}{2}(-3i,-4)\begin{pmatrix}3i\\ 4\end{pmatrix} = \frac{\hbar A^2}{2}\{9-16\}$$

$$= -\frac{\hbar A^2}{2}7 = -\frac{\hbar 7}{50}$$

c)

$$\sigma_z^2 = \begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$$

$$\sigma_x^2 = \begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix} = \begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$$

$$\sigma_y^2 = \begin{pmatrix}0 & -i\\ i & 0\end{pmatrix}\begin{pmatrix}0 & -i\\ i & 0\end{pmatrix} = \begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$$

$$\rightarrow \langle\sigma_i^2\rangle_\chi = \frac{\hbar^2A^2}{4}(-3i,4)\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}\begin{pmatrix}3i\\ 4\end{pmatrix} = \frac{\hbar^2A^2}{4}(-3i,4)\begin{pmatrix}3i\\ 4\end{pmatrix} = \frac{\hbar^2A^2}{4}\{9+16\} = \frac{\hbar^2}{4}$$

3

$$\sigma_{S_x} = \sqrt{\langle S_x^2\rangle_\chi - \langle S_x\rangle_\chi^2} = \sqrt{\frac{\hbar^2}{4}} = \frac{\hbar}{2}$$

$$\sigma_{S_y} = \sqrt{\langle S_y^2\rangle_\chi - \langle S_y\rangle_\chi^2} = \sqrt{\frac{\hbar^2}{4} - \frac{\hbar^2 6^2}{25^2}} = \hbar\sqrt{\frac{1}{4} - \frac{6^2}{25^2}}$$

$$\simeq \hbar \cdot 0{,}43863$$

$$\sigma_{S_z} = \sqrt{\langle S_z^2\rangle_\chi - \langle S_z\rangle_\chi^2} = \sqrt{\frac{\hbar^2}{4} - \frac{\hbar^2 49}{(50)^2}} = \hbar \cdot 0.48$$

d) Nú verður að gilda að

$$\sigma_{S_x} \cdot \sigma_{S_y} \geq \frac{\hbar}{2}|\langle S_z\rangle|$$

$$\sigma_{S_x} \cdot \sigma_{S_y} = \frac{\hbar^2}{2} \cdot 0{,}43863 = \hbar^2 \cdot 0{,}2193$$

$$\frac{\hbar}{2}|\langle L_z\rangle| = \frac{\hbar^2}{2}\left|\frac{7}{50}\right| = \hbar^2 0{,}07$$

og áfram .....

4

4.29

a) Finna eigingildi og vigra $S_y = \frac{\hbar}{2}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$

Eigingildin eru $\pm\frac{\hbar}{2}$

með eiginvigra $\frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ i \end{pmatrix}$ og $\frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -i \end{pmatrix}$

b) S_y mælt fyrir almennt ástand $\chi = \begin{pmatrix} a \\ b \end{pmatrix}$

Til þess að sjá þarf að liða χ í eiginástöndunum

$$\chi = \frac{1}{\sqrt{2}}\left\{\frac{a+ib}{\sqrt{2}}\begin{pmatrix} 1 \\ -i \end{pmatrix} + \frac{a-ib}{\sqrt{2}}\begin{pmatrix} 1 \\ +i \end{pmatrix}\right\}$$

5

Mæligildið $+\frac{\hbar}{2}$ fæst með líkunum $\left|\frac{a-ib}{\sqrt{2}}\right|^2$

$$= \frac{1}{2}|a-ib|^2 =$$

Gildið $-\frac{\hbar}{2}$ fæst með líkunum $\left|\frac{a+ib}{\sqrt{2}}\right|^2 = \frac{1}{2}|a+ib|^2$

Heildarlíkurnar eru $\frac{1}{2}\left[|a-ib|^2 + |a+ib|^2\right]$

$$= \frac{1}{2}\left\{(a^*+ib^*)(a-ib) + (a^*-ib^*)(a+ib)\right\}$$

$$= \frac{1}{2}\left\{|a|^2 + |b|^2 + |a|^2 + |b|^2 \cancel{-iba^*} \cancel{+ib^*a} \cancel{-ib^*a} + iba^*\right\}$$

$$= \frac{\cancel{2}}{2}\left\{|a|^2 + |b|^2\right\} = |a|^2 + |b|^2 = 1$$

6

c) Ef S_y^2 er mælt, hvaða gildi fæst með hvaða líkum?

Á tvennan hátt má sjá svarið

T.d. $S_y^2 = \frac{\hbar^2}{4} I$ einingarfylkið

$\frac{\hbar^2}{4}$ með líkum 1

eða endurtekin

$\left.\begin{array}{l} (+\frac{\hbar}{2})(+\frac{\hbar}{2}) \\ (-\frac{\hbar}{2})(-\frac{\hbar}{2}) \end{array}\right\}$ með öllum líkum 1

(1)

3.39 Sýna að

a)

$$f(x+x_0) = \exp\left\{i\hat{p}\frac{x_0}{\hbar}\right\} f(x) \qquad x_0 \text{ er föst lengd}$$

$$= \sum_{n=0}^{\infty} \frac{\left(\frac{i\hat{p}x_0}{\hbar}\right)^n}{n!} f(x) = \sum_{n=0}^{\infty} \frac{(x_0\partial_x)^n}{n!} f(x)$$

$$= \sum_{n=0}^{\infty} \frac{x_0^n}{n!} f^{(n)}(x) = f(x+x_0)$$

$\frac{\hat{p}}{\hbar}$ er vaki hliðrunar í staðarrúminu

b) Ef H er ekki fall af t sýna að

$$\Psi(x,t+t_0) = \underbrace{\exp\left\{-i\hat{H}t_0/\hbar\right\}}_{\uparrow \text{ tímaþróunar virki}} \Psi(x,t)$$

$\hat{H}/\hbar$ er vaki tímahliðrunar

(2)

munum að $\quad i\hbar\partial_t\Psi = H\Psi$

$$\exp\left\{-i\hat{H}\frac{t_0}{\hbar}\right\}\Psi(x,t) = \sum_{n=0}^{\infty} \frac{\left(-i\hat{H}\frac{t_0}{\hbar}\right)^n}{n!}\Psi(x,t)$$

$$= \sum_{n=0}^{\infty} \frac{(t_0\partial_t)^n}{n!}\Psi(x,t) = \sum_{n=0}^{\infty}\frac{(t_0)^n}{n!}\Psi^{(n)}(x,t) = \Psi(x,t+t_0)$$

---

c) Sýna að

$$\langle Q\rangle_{t+t_0} = \langle\Psi(x,t)|e^{i\frac{Ht_0}{\hbar}}\hat{Q}(\hat{x},\hat{p},t+t_0)e^{-i\frac{Ht_0}{\hbar}}|\Psi(x,t)\rangle$$

$$\langle Q\rangle_{t+t_0} = \langle\Psi(x,t+t_0)|\hat{Q}(\hat{x},\hat{p},t+t_0)|\Psi(x,t+t_0)\rangle$$

$$= \langle\Psi(x,t)|e^{i\frac{Ht_0}{\hbar}}\hat{Q}(\hat{x},\hat{p},t+t_0)e^{-i\frac{Ht_0}{\hbar}}|\Psi(x,t)\rangle$$

(3)

Setjum $t_0 = dt$

$$\langle Q\rangle_{t+dt} \simeq \langle\Psi|\left\{1+\frac{iH}{\hbar}dt\right\}\hat{Q}(\hat{x},\hat{p},t+dt)\left\{1-\frac{iH}{\hbar}dt\right\}|\Psi\rangle$$

$$= \langle\Psi|\hat{Q}(\hat{x},\hat{p},t+dt)|\Psi\rangle$$

$$+ \frac{i}{\hbar}dt\langle\Psi|\left[\hat{H},\hat{Q}(\hat{x},\hat{p},t+dt)\right]|\Psi\rangle$$

$$\simeq \langle\Psi|\left\{\hat{Q}(\hat{x},\hat{p},t) + \partial_t\hat{Q}(\hat{x},\hat{p},t)\cdot dt\right\}|\Psi\rangle$$

$$+ \frac{i}{\hbar}dt\langle\Psi|\left[\hat{H},\hat{Q}(\hat{x},\hat{p},t)\right]|\Psi\rangle + o((dt)^2)$$

$$\rightarrow \quad d_t\langle\hat{Q}\rangle = \frac{i}{\hbar}\langle[\hat{H},\hat{Q}]\rangle + \langle\partial_t\hat{Q}\rangle$$

(4)

4.56 Sýna

$$f(\phi+\varphi) = \exp\left\{iL_z\varphi/\hbar\right\}f(\phi)$$

$$= \sum_{n=0}^{\infty}\frac{\left(\frac{iL_z\varphi}{\hbar}\right)^n}{n!}f(\phi) = \sum_{n=0}^{\infty}\frac{(\varphi\partial_\phi)^n}{n!}f(\phi)$$

$$= \sum_{n=0}^{\infty}\frac{\varphi^n}{n!}f^{(n)}(\phi) = f(\phi+\varphi)$$

$\frac{L_z}{\hbar}$ er vaki snúnings um z-ás $\{\phi$ er „miðbaugshornið"$\}$

Almennir snúningar fást með

$$\exp\left\{i\hat{L}\cdot\hat{n}\frac{\varphi}{\hbar}\right\}$$

(5)

fyrir spuna

$$\chi' = \exp\left\{\frac{i(\bar{\sigma}\cdot\hat{n})\varphi}{2}\right\}\chi$$

b) Búa til (2×2) fylki fyrir snúning um 180° um x-ás

$$\bar{\sigma}\cdot\hat{a}_x = \sigma_x$$

Reiknum því $\exp\left\{\frac{i\pi}{2}\sigma_x\right\}$, $\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

$$\exp\left\{\frac{i\pi}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\right\} = \begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix}$$

$$\exp\left\{\frac{i\pi}{2}\sigma_x\right\}\chi_+ = \begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ i \end{pmatrix} = i\chi_-$$

$$\exp\left\{\frac{i\pi}{2}\sigma_x\right\}\chi_- = \begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} i \\ 0 \end{pmatrix} = i\chi_+$$

(6)

c) $\underline{90^\circ \text{ um y-ás}}$

$$\exp\left\{\frac{i\pi}{4}\sigma_y\right\} = \exp\left\{\frac{i\pi}{4}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}\right\} = \exp\left\{\frac{\pi}{4}\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\right\}$$

$$= \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}$$

$$\rightarrow \exp\left\{\frac{i\pi}{4}\sigma_y\right\}\chi_+ = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \end{pmatrix} = \frac{1}{\sqrt{2}}(\chi_+ - \chi_-)$$

$= \chi_-^{(x)}$ eins og var sýnt í spuna kaflanum (4.151) í bók

d) 360 gráður um z-ás

$$\exp\{i\pi\sigma_z\} = \exp\left\{\begin{pmatrix} i\pi & 0 \\ 0 & -i\pi \end{pmatrix}\right\} = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\rightarrow \exp\{i\pi\sigma_z\}\chi_+ = -\chi_+ \quad , \quad \exp\{i\pi\sigma_z\}\chi_- = -\chi_-$$

(7)

360° snúningur um z-ás skiptir um formerki

Þetta er oft túlkað þ.a. 2π-snúningur geti formerkjabreytingu
→ þarf 4π snúning til þess að fá sama ástand,
en gleymum ekki að fasastuðull (heildar) skiptir ekki
máli.

e)

$$\exp\left\{\frac{i\varphi}{2}(\bar{\sigma}\cdot\hat{n})\right\} = \sum_{n=0}^{\infty}\frac{\left(\frac{i\varphi}{2}\right)^n(\bar{\sigma}\cdot\hat{n})^n}{n!}$$

Hér ætla ég að leyfa mér að nota

$$(\bar{\sigma}\cdot\bar{A})(\bar{\sigma}\cdot\bar{B}) = \bar{A}\cdot\bar{B} + i\bar{\sigma}\cdot(\bar{A}\times\bar{B})$$

$$\rightarrow (\bar{\sigma}\cdot\hat{n})^2 = \hat{n}^2 = I \qquad \rightarrow (\bar{\sigma}\cdot\hat{n})^n = \begin{cases} I & \text{ef } n \text{ er jöfn} \\ \bar{\sigma}\cdot\hat{n} & \text{ef } n \text{ er oddatala} \end{cases}$$

(8)

$$\rightarrow \sum_{n=0}^{\infty}\frac{\left(\frac{i\varphi}{2}\right)^n(\bar{\sigma}\cdot\hat{n})^n}{n!} = \cos\left(\frac{\varphi}{2}\right)\cdot I + i(\bar{\sigma}\cdot\hat{n})\sin\left(\frac{\varphi}{2}\right)$$

5.4 (1)

$$\psi_\pm(\bar{x},\bar{y}) = A\{\psi_a(\bar{x})\psi_b(\bar{y}) \pm \psi_b(\bar{x})\psi_a(\bar{y})\}$$

a) Ef ψ_i eru stöðluð, hver er stuðullinn A?

$$1 = \int d\bar{x}d\bar{y}\,|\psi_\pm(\bar{x},\bar{y})|^2 = A^2\int d\bar{x}d\bar{y}\,\{|\psi_a(\bar{x})|^2|\psi_b(\bar{y})|^2 + |\psi_b(\bar{x})|^2|\psi_a(\bar{y})|^2$$

$$\left.\begin{array}{l}\pm\,\psi_a^*(\bar{x})\psi_b(\bar{x})\,\psi_b^*(\bar{y})\,\psi_a(\bar{y}) \\ \pm\,\psi_b^*(\bar{x})\,\psi_a(\bar{x})\,\psi_a^*(\bar{y})\psi_b(\bar{y})\end{array}\right\}$$

hverfa í heildun þar sem ástöndin eru hornrétt

$$= A^2\{1+1\} \rightarrow A = \frac{1}{\sqrt{2}}$$

(2)

b) Ef $\psi_a = \psi_b$ (aðeins fyrir bóseindir)

$$\psi_+(\bar{x},\bar{y}) = A\cdot 2\cdot \psi_a(\bar{x})\psi_a(\bar{y})$$

$$1 = \int d\bar{x}d\bar{y}\,|\psi_+(\bar{x},\bar{y})|^2 = 4|A|^2\int d\bar{x}d\bar{y}\,|\psi_a(\bar{x})|^2|\psi_a(\bar{y})|^2$$

$$= 4|A|^2 \rightarrow A = \frac{1}{2}$$

5.20 (3)

Hvað gerist með Dirac greiðuna ef í stað toppa koma „dalir"

$$V(x) = -\alpha\sum_{j=0}^{N=1}\delta(x-ja)$$

Fyrir jákvæðu lausnirnar, $E \geq 0$ var óbeina jafnan

$$\cos(Ka) = \cos(ka) + \beta\frac{\sin(ka)}{ka}, \quad \beta = \frac{\alpha}{aE_1} \qquad (1)$$

hér breytist $\beta = -\dfrac{\alpha}{aE_1}$

(4)

Fyrir neikvæðu lausnirnar, $E < 0$

fæst á bilinu $0<x<a$ lausnin

$$\psi(x) = A\cosh(\varkappa x) + B\sinh(\varkappa x)$$

með $\varkappa = \sqrt{-\dfrac{2mE}{\hbar^2}}$

á jöfnunni

$$d_x^2\psi = \varkappa^2\psi$$

því fæst jafna, óbein fyrir orkugildin

$$\cos(Ka) = \cosh(\varkappa a) + \beta\frac{\sinh(\varkappa a)}{\varkappa a}$$

með $\beta = -\dfrac{\alpha}{aE_1} \qquad (2)$

Jöfnur (1) og (2) sýna að lægsti borðinn er með N ástönd

1

6-4

a) Óendanlegur brunnur $V(x) = \begin{cases} 0 & \text{ef } 0<x<a \\ \infty & \text{annars} \end{cases}$

$$H' = \alpha\delta(x - \tfrac{a}{2}) \qquad E_n^0 = \frac{n^2\pi^2\hbar^2}{2ma^2}, \quad \phi_n(x) = \sqrt{\frac{2}{a}}\sin\left(\frac{n\pi x}{a}\right)$$

Reiknum $E_n^2 = \sum_{m\neq n} \frac{|\langle m|H'|n\rangle|^2}{E_n^0 - E_m^0}$ $\qquad E_n^0 = E_1^0 \cdot n^2$

$$\langle m|H'|n\rangle = \alpha\int_0^a dx\, \phi_m^*(x)\,\delta(x-\tfrac{a}{2})\,\phi_n(x)$$

$$= \frac{2}{a}\alpha\int_0^a dx\, \sin\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi x}{a}\right)\delta(x-\tfrac{a}{2})$$

$$= \frac{2}{a}\alpha\sin\left(\frac{m\pi}{2}\right)\sin\left(\frac{n\pi}{2}\right) = \begin{cases} \pm\frac{2\alpha}{a} & \text{ef } n \text{ og } m \text{ odd} \\ 0 & \text{annars} \end{cases}$$

2

$$E_n^2 = \left(\frac{2\alpha}{a}\right)^2\frac{1}{|E_1^0|}\sum_{\substack{m\neq n\\ \text{odd}}}\frac{1}{n^2-m^2}$$

þessa summu þarf að þátta

$$\frac{1}{n^2-m^2} = \frac{1}{2n}\left\{\frac{1}{m+n} - \frac{1}{m-n}\right\}$$

Allir liðir munu styttast út í summunni, nema lægsti liðurinn hér

staki liðurinn er $\frac{1}{2n}\left(-\frac{1}{2n}\right)$

$$\rightarrow E_n^2 = \left(\frac{2\alpha}{a}\right)^2\frac{1}{(E_1^0)^2}\cdot\frac{-1}{4n^2} \quad \text{ef } n = \text{odd, annars } 0$$

$$= \begin{cases} -\left(\frac{\alpha}{aE_1^0}\right)^2\frac{1}{n^2} & \text{ef } n = \text{odd} \\ 0 & \text{annars} \end{cases}$$

1. stigs liður er jákvæður

2. stigs liður er neikvæður

3

b) Við höfðum í (6-2) reiknað fyrir hreintóna sveifil hvað gerist þegar $k \rightarrow (1+\epsilon)k$

$$E_n^1 \simeq E_n^0\cdot\left\{1 + \frac{\epsilon}{2} - \frac{\epsilon^2}{8} + \dots\right\}$$

$$E_n^1 = E_n^0\cdot\sqrt{1+\epsilon}$$

getum við sannreynt 2. stigs liðinn?

Munum að

$$H' = \frac{\epsilon}{2}kx^2$$

$$E_n^2 = \sum_{m\neq n}\frac{|\langle m|H'|n\rangle|^2}{E_n^0 - E_m^0}$$

$a_+|n\rangle = \sqrt{n+1}\,|n+1\rangle$

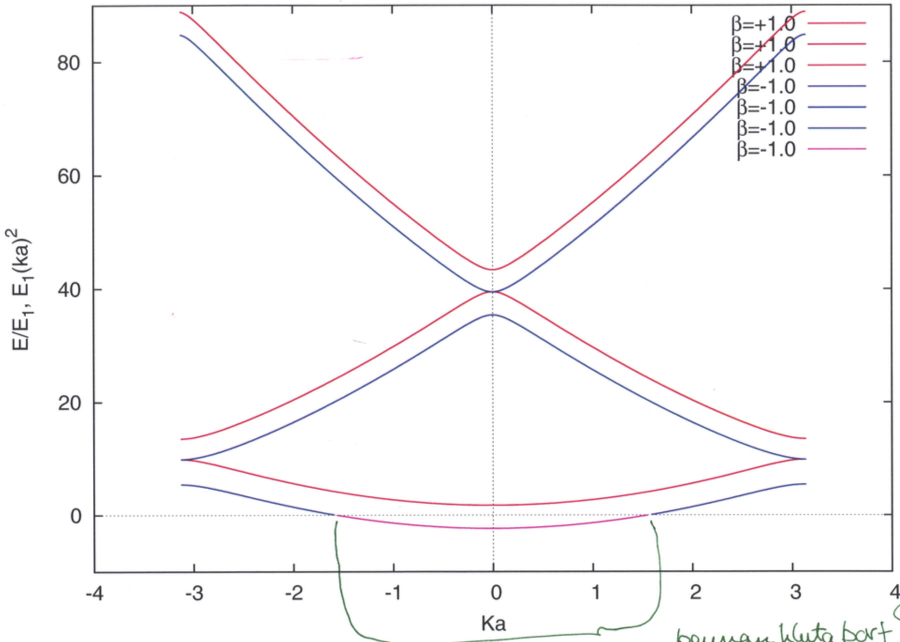
$a_-|n\rangle = \sqrt{n}\,|n-1\rangle$

5

Lausnin fyrir $\alpha<0$ hliðrast niður í orku um fasta

$|\beta| = \frac{\alpha}{aE_1} = 1.0$

Sem $-E_1(ka)^2$ fyrir jöfnu (2)



þennan hluta þarf að reikna sér með jöfnu (2)

Notum aftur

$$x^2 = \frac{a^2}{2}(a_+ + a_-)^2 = \frac{a^2}{2}\left\{a_+a_+ + a_-a_- + a_+a_- + a_-a_+\right\}$$

og reiknum

$$\langle m|H'|n\rangle = \frac{\epsilon k}{2}\langle m|\left\{a_+a_+ + a_-a_- + a_+a_- + a_-a_+\right\}|n\rangle \cdot \frac{a^2}{2}$$

$$= \frac{\epsilon k}{2}\Big\{\sqrt{(n+1)(n+2)}\,\langle m|n+2\rangle + \sqrt{n(n-1)}\,\langle m|n-2\rangle + n\langle m|n\rangle + (n+1)\langle m|n\rangle\Big\}\cdot\frac{a^2}{2}$$

$$= \frac{a^2\cdot\epsilon k}{2\cdot 2}\Big\{\sqrt{(n+1)(n+2)}\,\delta_{m,n+2} + \sqrt{n(n-1)}\,\delta_{m,n-2} + n\,\delta_{m,n} + (n+1)\,\delta_{m,n}\Big\}$$

en munum að í summunni er $m \neq n$

$$E_n^2 = \frac{\epsilon^2 k^2\cdot a^4}{4\hbar\omega\cdot 4}\sum_{m\neq n}\frac{\left|\sqrt{(n+1)(n+2)}\,\delta_{m,n+2} + \sqrt{n(n-1)}\,\delta_{m,n-2}\right|^2}{(n+\frac12)-(m+\frac12)}$$

nota

$$= \epsilon^2\hbar\omega\,\frac{1}{16}\sum_{m\neq n}\frac{(n+1)(n+2)\,\delta_{m,n+2} + n(n-1)\,\delta_{m,n-2}}{n-m}$$

$$= \epsilon^2\hbar\omega\,\frac{1}{16}\left\{\frac{(n+1)(n+2)}{n-(n+2)} + \frac{n(n-1)}{n-(n-2)}\right\}$$

$$= \epsilon^2\hbar\omega\,\frac{1}{16}\left\{-\frac12(n+1)(n+2) + \frac12 n(n-1)\right\}$$

$$= \epsilon^2\hbar\omega\,\frac{1}{32}\left\{-\cancel{n^2}-3n-2 + \cancel{n^2}-n\right\} = -\epsilon^2\frac18\hbar\omega(n+\tfrac12)$$

$= -\epsilon^2\frac18 E_n^0$ eins og áður

6-30

3D - hreintóna sveifill (einsleitur)

$$H' = \lambda x^2 y z$$

a) Reikna E_0^1 1. stigs truflun grunnástands

Grunnástandið er eins og 3 hreintóna sveiflar, óháðir, í 3 höfuðstefnum í grunnástandi

$$\to E_0 = \hbar\omega\cdot\frac32$$

$$E_0^1 = \langle 0|H'|0\rangle = \lambda\langle 0|x^2|0\rangle\underbrace{\langle 0|y|0\rangle}_{=0}\underbrace{\langle 0|z|0\rangle}_{=0} = 0$$

b) þrefalda lægsta örvaða ástandið

$|1\rangle = |1,0,0\rangle$ - p.s. ástöndin eru $|n_x, n_y, n_z\rangle$

$|2\rangle = |0,1,0\rangle$

$|3\rangle = |0,0,1\rangle$

$$\langle 3|H'|3\rangle = \lambda\langle 0|x^2|0\rangle\langle 0|y|0\rangle\langle 1|z|1\rangle = 0$$

$$\langle 3|H'|2\rangle = \lambda\langle 0|x^2|0\rangle\langle 0|y|1\rangle\langle 1|z|0\rangle \neq 0$$

$$\langle 3|H'|1\rangle = \lambda\langle 0|x^2|1\rangle\langle 0|y|0\rangle\langle 1|z|0\rangle = 0$$

Öll önnur nema $\langle 2|H'|3\rangle$ gefa líka 0

$$\langle 3|H'|2\rangle = \lambda \langle 0|x^2|0\rangle \langle 0|y|1\rangle \langle 1|z|0\rangle$$

$$= \lambda \langle 0|x^2|0\rangle \, |\langle 0|x|1\rangle|^2$$

ef við notum að þetta er einsleitur hreintóna sveifill og við notum fylkjastökin fyrir einvíðan hreintóna sveifil

Munum að

$$\langle 0|x^2|0\rangle = \frac{a^2}{2}$$

$$\langle 0|x|1\rangle = \frac{a}{\sqrt{2}}\langle 0|(a_+ + a_-)|1\rangle = \frac{a}{\sqrt{2}}\langle 0|a_-|1\rangle$$

$$= \frac{a}{\sqrt{2}}\langle 0|0\rangle \cdot 1 = \frac{a}{\sqrt{2}}$$

$$\langle 3|H'|2\rangle = \lambda \frac{a^2}{2}\cdot\frac{a^2}{2} = \lambda\frac{a^4}{4}$$

þess vegna verður fylkið

$$W = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \frac{\lambda a^4}{4}$$

með eigingildi $0, \pm \frac{\lambda a^4}{4}$

þríklofnun

6-32 Hamiltonvirki einhvers kerfis er fall af stikanum λ, því köllum við

$H(\lambda)$, $E_n(\lambda)$ og $|n(\lambda)\rangle$ — eigingildi og eiginástönd $H(\lambda)$

Feynman-Hellmann setningin er þá

$$\partial_\lambda E_n(\lambda) = \langle n(\lambda)|\partial_\lambda H(\lambda)|n(\lambda)\rangle$$

þ.s. $E_n(\lambda)$ er annaðhvort einfalt eða „góð" samantekt margfaldra ástanda

a) Sýna fram á

$$\frac{\partial}{\partial\lambda} H(\lambda) = \frac{H(\lambda+d\lambda) - H(\lambda)}{d\lambda}$$

$E_n(\lambda) = \langle n(\lambda)|H(\lambda)|n(\lambda)\rangle$ nákvæmt

$E_n(\lambda+d\lambda) = \langle n(\lambda+d\lambda)|H(\lambda+d\lambda)|n(\lambda+d\lambda)\rangle$ nákvæmt

1. stigs truflun gefur

$$E_n(\lambda+d\lambda) \approx \langle n(\lambda)|H(\lambda+d\lambda)|n(\lambda)\rangle + O(\lambda^2)$$

$$dE_n(\lambda) = E_n(\lambda+d\lambda) - E_n(\lambda) \approx \langle n(\lambda)|(H(\lambda+d\lambda) - H(\lambda))|n(\lambda)\rangle$$

$$H(\lambda+d\lambda) - H(\lambda) = \frac{\partial H}{\partial\lambda} d\lambda$$

3

$$\rightarrow \frac{\partial E_n(\lambda)}{\partial \lambda} = \langle n(\lambda)| \frac{\partial H}{\partial \lambda} | n(\lambda)\rangle$$

nákvæmlega, því þegar $d\lambda \to 0$ verður 1. stigs truflun nákvæm

b) 1D-H.O. notum, $E_n = \hbar\omega(n+\frac{1}{2})$, $H = \frac{p^2}{2m} + \frac{1}{2}m\omega^2 x^2$

i) Reiknum $\lambda = \omega$

$$\frac{\partial E_n}{\partial \omega} = \frac{E_n}{\omega}, \quad \langle n| \frac{\partial H}{\partial \omega}|n\rangle = \langle n| m\omega x^2|n\rangle$$

$$= m\omega \langle n|x^2|n\rangle$$

$$\rightarrow \langle n|x^2|n\rangle = \frac{E_n}{m\omega^2} = (n+\tfrac{1}{2})\frac{\hbar}{m\omega} = a^2(n+\tfrac{1}{2})$$

Sem líka $\langle n|V|n\rangle = \frac{1}{2}m\omega^2\langle n|x^2|n\rangle = \frac{1}{2}(n+\frac{1}{2})\hbar\omega = \frac{E_n}{2}$

4

ii) $\lambda = \hbar \rightarrow \frac{\partial E_n}{\partial \hbar} = \frac{E_n}{\hbar}$, $T = -\frac{\hbar^2}{2m}\partial_x^2$

$$\langle n|\frac{\partial H}{\partial \hbar}|n\rangle = \langle n|\frac{\partial T}{\partial \hbar}|n\rangle = \frac{2}{\hbar}\langle n|T|n\rangle$$

$\rightarrow \langle n|T|n\rangle = \frac{E_n}{2}$, sem $\frac{1}{2m}\langle n|p^2|n\rangle = \frac{E_n}{2}$

$$\rightarrow \langle n|p^2|n\rangle = E_n \cdot m$$

iii) $\lambda = m$, $\frac{\partial}{\partial m}E_n = 0$

$$\langle n|\frac{\partial H}{\partial m}|n\rangle = \langle n|\left\{-\frac{T}{m} + \frac{V}{m}\right\}|n\rangle$$

$\rightarrow \langle n|T|n\rangle = \langle n|V|n\rangle$ eins og bæði er að koma í ljós í i) og ii)

5

6.33 Nota Feynman-Hellmann til þess að reikna $\langle \frac{1}{r}\rangle$ og $\langle \frac{1}{r^2}\rangle$ fyrir vetni

Virka H fyrir r-hlutann er

$$H = -\frac{\hbar^2}{2m}\frac{d^2}{dr^2} + \frac{\hbar^2 l(l+1)}{2mr^2} - \frac{e^2}{4\pi\epsilon_0 r}$$

og eigin gildin

$$E_n = -\frac{\overbrace{me^4}^{R_y}}{32\pi^2\epsilon_0^2\hbar^2\underbrace{(j_{max}+l+1)^2}_{n^2}}$$

$E_n = -\frac{R_y}{n^2}$ en þetta form er heppilegt fyrir hið b)

a) Nota $\lambda = e$ til þess að finna $\langle \frac{1}{r}\rangle$

6

$$\langle \psi|\frac{\partial H}{\partial \lambda}|\psi\rangle = -\frac{2e}{4\pi\epsilon_0}\langle\psi|\frac{1}{r}|\psi\rangle$$

$$\frac{\partial E_n}{\partial e} = -\frac{4me^3}{32\pi^2\epsilon_0^2\hbar^2(j_{max}+l+1)^2} = \frac{4E_n}{e}$$

$$\rightarrow \boxed{\frac{4E_n}{e} = -\frac{2e}{4\pi\epsilon_0}\langle\psi|\frac{1}{r}|\psi\rangle}, \quad E_n = -R_y\frac{1}{n^2}$$

Munum að

$$R_y = \frac{\hbar^2}{2ma^2} = \frac{me^4}{\hbar^2 32\pi^2\epsilon_0^2}, \quad a = \frac{4\pi\epsilon_0\hbar^2}{me^2}$$

$$\frac{4R_y}{en^2} = \frac{2e}{4\pi\epsilon_0}\langle\frac{1}{r}\rangle \rightarrow \langle\frac{1}{r}\rangle = \frac{8\pi\epsilon_0 R_y}{e^2 n^2}$$

$\langle \frac{1}{r}\rangle = \frac{1}{n^2 a}$ eins og gefið var áður í bók

b) nota $\lambda = \ell$

$$\frac{\partial E_n}{\partial \ell} = \frac{2me^4}{32\pi^2\epsilon_0^2\hbar^2(j_{max}+\ell+1)^3} = -\frac{2E_n}{n}$$

$$\frac{\partial H}{\partial \ell} = \frac{\hbar^2(2\ell+1)}{2mr^2} \rightarrow \frac{\hbar^2(2\ell+1)}{2m}\left\langle\frac{1}{r^2}\right\rangle = -\frac{2E_n}{n}$$

$$\rightarrow \left\langle \frac{1}{r^2}\right\rangle = -\frac{4mE_n}{\hbar^2(2\ell+1)\hbar^2\cdot n} = +\frac{4mR_y}{\hbar^2(2\ell+1)\hbar^2 n^3}$$

$$= \frac{1}{n^3(\ell+\frac{1}{2})a^2}$$ eins og áður

9.2

$$\dot{C}_a = -\frac{i}{\hbar}H'_{ab}e^{-i\omega_0 t}C_b \qquad \bar{C}(0) = \begin{pmatrix}C_a(0)\\C_b(0)\end{pmatrix} = \begin{pmatrix}1\\0\end{pmatrix}$$

$$\dot{C}_b = -\frac{i}{\hbar}H'_{ba}e^{i\omega_0 t}C_a \qquad H_{ba} \text{ óháð } t$$

Sama loysnir sum fyrr og í (9.7), nema $\Delta\omega = -\omega_0$ ettir ω hér

Því fæst loysnin

$$C_b(t) = -i\frac{2H'_{ba}}{\hbar\omega_r}e^{\frac{i\omega_0 t}{2}}\sin\left(\frac{\omega_r t}{2}\right)$$

$$C_a(t) = e^{-\frac{i\omega_0 t}{2}}\left[\cos\left(\frac{\omega_r t}{2}\right) + \frac{i\omega_0}{\omega_r}\sin\left(\frac{\omega_r t}{2}\right)\right]$$

við $\omega_r = \sqrt{\omega_0^2 + \frac{4|H'_{ab}|^2}{\hbar^2}}$

Hóast vegna á alveg sama hátt fæst at $|C_a|^2+|C_b|^2=1$

Ef við hugsum okkur at truflan byrji klokkan $t=0$ tá er hægt at siga at $|a\rangle$ og $|b\rangle$ hafi verið ástøður kerfisins fyrir truflan.

Við sjáum Rabi sveiflur við tíðni $\omega_r \neq \omega_0$ ef $|H'_{ab}| \neq 0$

Áhugavert at Rabi sveiflurnar fyrir [illegible] sveiflur vera óháðar, en bjagaðar vegna forskotið til eftir ástanda

nákvæma loysnin í fyrilestrinum

9.11

Við fórum ikki í lítt tíma hugmyndir Griffiths, en skoðum fylkjastøkin

Dømi 9.1 gaf okkur $\langle 100|z|210\rangle = \frac{a}{\sqrt{18}}\frac{256}{81}$

$$\simeq 0{,}74494\, a$$

$$x = r\sin\theta\cos\phi = r\sqrt{\frac{8\pi}{3}}\left(-Y_{1+1}+Y_{1-1}\right)\frac{1}{2}$$

$$y = r\sin\theta\sin\phi = r\sqrt{\frac{8\pi}{3}}\left(-Y_{1+1}-Y_{1-1}\right)\frac{1}{2}$$

Y_{00} er fasti því eru eins liðini $\langle 100|z|210\rangle$ og $\langle 100|x|21\pm1\rangle$ og $\langle 100|y|21\pm1\rangle$ Sum ikki eru núll