6.5 Veikt rafsvið lagt á hreintóna sveifil

$$H' = -qEx$$

náttúruleg lengd $a = \sqrt{\frac{\hbar}{m\omega}}$

a) sýna að fyrsta stigs truflun hverfi

Notum úr dæmi 6.2

$$x = \frac{a}{\sqrt{2}}(a_+ + a_-) \quad , \quad E_n^1 = \langle n|H'|n\rangle = -qE\langle n|x|n\rangle$$

$$= -qE\langle n|(a_+ + a_-)|n\rangle \frac{a}{\sqrt{2}}$$

$= 0$ því $a_\pm$ hækka eða lækka n

og $\langle n|n\pm 1\rangle = 0$

$$a_+|n\rangle = \sqrt{n+1}\,|n+1\rangle$$

$$a_-|n\rangle = \sqrt{n}\,|n-1\rangle$$

Reikna 2. stigstruflun ástandanna

$$E_n^2 = \sum_{m\neq n} \frac{|\langle m|H'|n\rangle|^2}{E_n^0 - E_m^0} = q^2E^2 \sum_{m\neq n}\frac{|\langle m|x|n\rangle|^2}{\hbar\omega(n-m)}$$

$$\langle m|x|n\rangle = \frac{a}{\sqrt{2}}\langle m|(a_+ + a_-)|n\rangle = \frac{a}{\sqrt{2}}\left\{\langle m|\sqrt{n+1}|n+1\rangle + \langle m|\sqrt{n}|n-1\rangle\right\}$$

$$= \frac{a}{\sqrt{2}}\left\{\sqrt{n+1}\,\delta_{m,n+1} + \sqrt{n}\,\delta_{m,n-1}\right\}$$

$$E_n^2 = \frac{q^2E^2a^2}{2\hbar\omega}\sum_{m\neq n}\frac{|\sqrt{n+1}\,\delta_{m,n+1} + \sqrt{n}\,\delta_{m,n-1}|^2}{n-m}$$

$$= \frac{q^2E^2a^2}{2\hbar\omega}\left\{\frac{n+1}{n-(n+1)} + \frac{n}{n-(n-1)}\right]$$

3

$$E_n^2 = \frac{q^2E^2a^2}{2\hbar\omega}\left\{\frac{n+1}{-1} + \frac{n}{+1}\right\} = -\frac{q^2E^2a^2}{2\hbar\omega} = -\frac{q^2E^2\hbar}{2\hbar\omega\, m\omega}$$

$$= -\frac{q^2E^2}{2m\omega^2}$$

og ~~poss~~vegner

$$E_n = \hbar\omega\left(n+\tfrac{1}{2}\right) - \frac{q^2E^2}{2m\omega^2} = \hbar\omega\left\{\left(n+\tfrac{1}{2}\right) - \frac{q^2E^2}{2m\omega^2\hbar\omega}\right\}$$

$$= \hbar\omega\left\{\left(n+\tfrac{1}{2}\right) - \frac{q^2E^2a^2}{2(\hbar\omega)^2}\right\}$$

↑ Stark klotrum orturöts

b) Finna nákvæmu lausnina

Hamilton virkinn var

$$\frac{p^2}{2m} + \frac{1}{2}m\omega^2 x^2$$

með rafsviðinu verður hann

$$\frac{p^2}{2m} + \frac{1}{2}m\omega^2 x^2 - qEx$$

Getum við umritað H yfir í hreintóna sveifil eftir

$$\frac{p^2}{2m} + \frac{1}{2}m\omega^2\left(x^2 - \frac{qEx2}{m\omega^2}\right)$$

$\longrightarrow$

$$\frac{p^2}{2m} + \frac{1}{2}m\omega^2\left(x - \frac{qE}{m\omega^2}\right)^2 - \frac{q^2E^2}{m^2\omega^4}\frac{m\omega^2}{2}$$

$$= \frac{p^2}{2m} + \frac{1}{2}m\omega^2\left(x - \frac{qE}{m\omega^2}\right)^2 - \frac{q^2E^2}{2m\omega^2}$$

$$= \frac{p^2}{2m} + \frac{1}{2}m\omega^2\left(x - x_0\right)^2 - \frac{q^2E^2}{2m\omega^2}$$

$$x_0 = \frac{qE}{m\omega^2}$$

hliðruð hreintóna sveifils með lækkaða orku

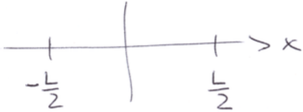
Í fyrsta rætsviði er loysn heimtóna sveiggjurs eins og loysnin fyrir heiðraðan heimtónasveiggj til með lokkaða orku

2. stigs nærkanin er líka nakað loysn

stark-hit heimtóna sveiggjurs

6.7 Eind með massa m á bili með lengd L
(lotubundið 1D, t.d. hringur)

a) Eigin föll og róf



$$H = \frac{p^2}{2m}, \quad \psi\left(-\tfrac{L}{2}\right) = \psi\left(\tfrac{L}{2}\right)$$

$$-\frac{\hbar^2}{2m} d_x^2 \psi = E\psi$$

Reynum lausn með

$$\psi = A e^{i\alpha x}$$

jaðar skilyrði

$$A e^{-i\alpha \frac{L}{2}} = A e^{+i\alpha \frac{L}{2}}$$

eða

$$1 = e^{i\alpha L}$$

$$\rightarrow \alpha = \frac{2\pi n}{L}, \quad n \in \mathbb{Z}$$

$$\psi = A e^{2\pi i n \frac{x}{L}}$$

finnum A

$$1 = |A|^2 \int_{-L/2}^{+L/2} dx\, |\psi|^2 = |A|^2 L$$

því fæst t.d. $A = \frac{1}{\sqrt{L}}$

og orkan fæst með innsetningu í jöfnu Schrödingers

$$E_n = \frac{\hbar^2}{2m}\frac{4\pi^2 n^2}{L^2}$$

$$= \underbrace{\left(\frac{\hbar^2}{2mL^2}\right)}_{\text{með vídd orku}} 4\pi^2 n^2$$

Öll ástöndin eru tvöföld nema $n = 0$

köllum þau $|n\rangle$

b) Bætum við truflun

$$H' = -V_0 e^{-\frac{x^2}{a^2}}, \quad a \ll L$$

Finna 1. stigs truflun rófsins

$$E_n^1 = \langle n|H'|n\rangle$$

$$= -V_0 \frac{1}{L}\int_{-L/2}^{+L/2} dx\, |\Phi_n|^2 e^{-\frac{x^2}{a^2}}$$

$$= -\frac{V_0}{L}\int_{-L/2}^{+L/2} dx\, e^{-\frac{x^2}{a^2}} = -V_0\frac{a}{L}\int_{-\frac{L}{2a}}^{+\frac{L}{2a}} \frac{dx}{a} e^{-\frac{x^2}{a^2}}$$

$$= -V_0\frac{a}{L}\int_{-\frac{L}{2a}}^{+\frac{L}{2a}} du\, e^{-u^2}$$

$a \ll L \rightarrow \frac{L}{a} \rightarrow \infty$ og

$$E_n^1 \simeq -V_0 \frac{a}{L} \int_{-\infty}^{\infty} du\, e^{-u^2} = -V_0 \sqrt{\pi} \frac{a}{L}$$

Sem er notanlegt fyrir einfalda ástandið $n=0$

$$\boxed{E_0^1 \simeq -V_0 \sqrt{\pi} \frac{a}{L}}$$

fyrir tvö földu pörin $n \neq 0$ verðum við að nota truflanareikning fyr tvö föld ástönd með

$$W_{nn} = W_{aa} = W_{bb} = -V_0 \sqrt{\pi} \frac{a}{L}$$

$$W_{n,-n} = W_{ab} = \langle n | V | -n \rangle = -V_0 \frac{1}{L} \int_{-\frac{L}{2}}^{+\frac{L}{2}} dx\, e^{-\frac{x^2}{a^2}} e^{4\pi n i \frac{x}{L}}$$

$$= -V_0 \frac{a}{L} \int_{-\infty}^{\infty} du\, e^{-u^2} \exp\left\{4\pi n i\, u \frac{a}{L}\right\} = -V_0 \frac{a}{L} \sqrt{\pi}\, e^{-(2\pi n \frac{a}{L})^2}$$

$$E^1_{\pm} = \frac{1}{2}\left\{\underbrace{W_{aa} + W_{bb}}_{2W_{aa}} \pm \sqrt{\underbrace{(W_{aa} - W_{bb})^2}_{0} + 4|W_{ab}|^2}\right\}$$

$$= W_{aa} \pm |W_{ab}| = -V_0 \sqrt{\pi}\left(\frac{a}{L}\right)\left\{1 \mp e^{-(2\pi n \frac{a}{L})^2}\right\}$$

Tvö földu orkustigin klofna

$n \neq 0$

c) Hvaða samantekt $|+|n|\rangle$ og $|-|n|\rangle$ er góð samantekt fyrir venjulegan 1. stigs truflunar reikn.

Aðeins 2 ástönd, svo ég gista á

$$|+\rangle = \frac{1}{\sqrt{2}}\left\{|+|n|\rangle + |-|n|\rangle\right\} \to \psi_+ = \sqrt{\frac{2}{L}}\cos\left(2\pi n \frac{x}{L}\right)$$

$$|-\rangle = \frac{1}{\sqrt{2}}\left\{|+|n|\rangle - |-|n|\rangle\right\} \to \psi_- = i\sqrt{\frac{2}{L}}\sin\left(2\pi n \frac{x}{L}\right)$$

Reynum

$$E_+^1 = \langle +|H'|+\rangle = -V_0 \frac{2}{L}\int_{-L/2}^{L/2} dx\, e^{-\frac{x^2}{a^2}} \cos^2\left(2\pi n \frac{x}{L}\right)$$

$$\approx -V_0 \frac{2a}{L}\int_{-\infty}^{\infty} du\, e^{-u^2} \cos^2\left(2\pi n u \frac{a}{L}\right)$$

$$= -V_0 \frac{2a}{L} \frac{\sqrt{\pi}}{2}\left\{\exp\left(-\left(\frac{2\pi n a}{L}\right)^2\right)+1\right\} = -V_0\sqrt{\pi}\left(\frac{a}{L}\right)\left\{1+e^{-\left(2\pi n \frac{a}{L}\right)^2}\right\}$$

$$E^1_- = \langle - | H' | - \rangle = -V_0 \frac{2}{L}\int_{-L/2}^{L/2} dx\, e^{-\frac{x^2}{a^2}} \sin^2\left(2\pi n \frac{x}{L}\right)$$

$$\simeq -V_0 \frac{2a}{L}\int_{-\infty}^{\infty} du\, e^{-u^2} \sin^2\left(2\pi n u \frac{a}{L}\right) = -V_0\sqrt{\pi}\left(\frac{a}{L}\right)\left\{1-e^{-\left(2\pi n \frac{a}{L}\right)^2}\right\}$$

þannig að við þekkjum hér eftir orku stigin þ.a

$|+\rangle$ hefur orkuna sem við ve fundum áður E^1_-

$|-\rangle$ — // — E^1_+

d) $|+\rangle$ er jamstøtt fall

$|-\rangle$ er odd støtt fall

Speglunarvirkin P þ.a. $P\psi(x) = \psi(-x)$

hefur mismunandi eigingildi fyrir $|+\rangle$ og $|-\rangle$

P víxlast við H^0 og H'

Mér datt líka í hug L_z ef ég let $2\pi\frac{x}{L} = \phi$, en

sá virki víxlast ekki við H'