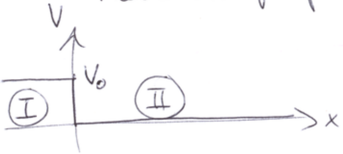
(1)

Mættisþrep



I

$$\left\{-\frac{\hbar^2}{2m}\partial_x^2 + V_0\right\}\psi = E\psi$$

lausn $\psi(x) = e^{ikx} + Be^{-ikx}$

og því

$$\frac{\hbar^2 k^2}{2m} + V_0 = E$$

$$k^2 = \underbrace{\frac{2m}{\hbar^2}(E-V_0)}_{\text{jákvæð tala}}$$

$E > V_0$

$\rightarrow k \in \mathbb{R}$ og lausn

$$\psi(x) = e^{ikx} + Be^{-ikx}, \quad k = \sqrt{\frac{2m}{\hbar^2}(E-V_0)}$$

II

$$-\frac{\hbar^2}{2m}\partial_x^2\psi = E\psi$$

lausn $\psi(x) = Ce^{iqx}$ bara bylgja til hægri

með $\dfrac{\hbar^2 q^2}{2m} = E > 0 \rightarrow q = \sqrt{\dfrac{2mE}{\hbar^2}}$

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lausn samfelld í $x=0$

$$\psi^{I}(0) = \psi^{II}(0)$$

$$1 + B = C \quad (1)$$

Afleiða samfeld

$$ik - ikB = iqC$$

$$\rightarrow k(1-B) = qC \quad (2)$$

Umritum

$$B - C = -1$$

$$kB + qC = k$$

$$\begin{pmatrix} 1 & -1 \\ k & q \end{pmatrix} \begin{pmatrix} B \\ C \end{pmatrix} = \begin{pmatrix} -1 \\ k \end{pmatrix}$$

Með lausn

$$B = -\frac{q-k}{q+k}$$

$$C = \frac{2k}{q+k}$$

Athugum líkinda straumþéttleika

$$J(x,t) = \frac{i\hbar}{2m}\left\{(\partial_x\Psi)^*\Psi - \Psi^*\partial_x\Psi\right\}$$

$$= \frac{i\hbar}{2m}|A|^2\left\{-ik - ik\right\} = \frac{\hbar k}{m}|A|^2$$

ef bylgjufallið væri

$$\Psi_k(x,t) = A\exp\left\{i(kx - \omega_k t)\right\}$$

líkindastreymur innbylgju er þá

$$J_{inn} = \frac{\hbar k}{m}$$

líkindastreymur endurkasts

$$J_R = -\frac{\hbar k}{m}|B|^2$$

og frambylgju

$$J_T = \frac{\hbar q}{m}|C|^2$$

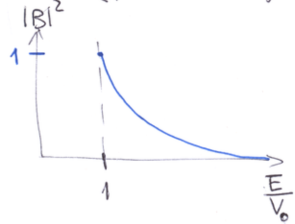
líkindi endurkasts eru

a)

$$\left|\frac{J_R}{J_{inn}}\right|^2 = |B|^2 = R$$

$$= \frac{(q-k)^2}{(q+k)^2} = \frac{\left(\sqrt{E}-\sqrt{E-V_0}\right)^2}{\left(\sqrt{E}+\sqrt{E-V_0}\right)^2}$$

$$= \frac{\left(\sqrt{1-\frac{V_0}{E}}-1\right)^2}{\left(\sqrt{1-\frac{V_0}{E}}+1\right)^2}$$

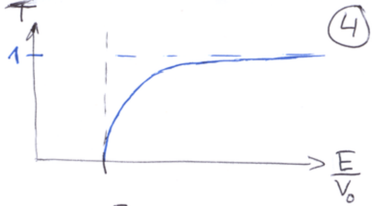


b)

líkindi framferðar eru

$$\left|\frac{J_T}{J_{inn}}\right| = \frac{q}{k}|C|^2 = \frac{4qk}{(q+k)^2} = T$$

$$T = 4\frac{\sqrt{(E-V_0)E}}{\left(\sqrt{E}+\sqrt{E-V_0}\right)^2} = \frac{4\sqrt{\left(1-\frac{V_0}{E}\right)}}{\left(\sqrt{1-\frac{V_0}{E}}+1\right)^2}$$

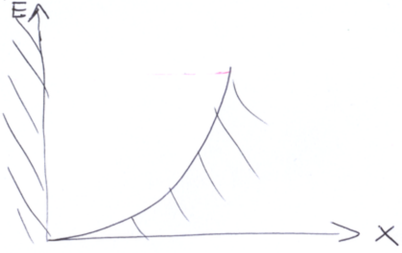


c)

$$R + T = \frac{(q-k)^2}{(q+k)^2} + \frac{4qk}{(q+k)^2} = \frac{(q+k)^2}{(q+k)^2} = 1$$

þannig að inn streymir jafnmikið og út

② Hálfur hreintóna sveifill



Jafna Schrödingers er sú sama, en gildir aðeins fyrir bilið $x > 0$ núna

Vegna veggsins í $x = 0$ verður að gilda $\psi(0) = 0$

$\rightarrow$ hér eru allar oddstæðu lausnir hreintóna sveifils mögulegar og engar aðrar

$$E_n = \hbar\omega\left(n + \frac{1}{2}\right) \quad n = 1, 3, 5, 7, \ldots\ldots$$