(1)

Problem 1: Estimate surface area A of human being with mass M and height h

Cylindrical human should give us lower bounds for A, ("spherical cow" :))

$$M = \rho V = \rho \cdot h \cdot \pi r^2, \quad V: \text{volume}, \ \rho: \text{density}, \ r: \text{radius}$$

$$A = \underbrace{h \cdot 2\pi r}_{\text{side}} + \underbrace{2 \cdot \pi r^2}_{\text{top + bottom}}$$

$$r^2 = \frac{M}{\rho h \pi} \rightarrow r = \sqrt{\frac{M}{\rho h \pi}}$$

$$A = h 2\pi \sqrt{\frac{M}{\rho h \pi}} + \frac{2\pi M}{\rho h \pi} = 2\sqrt{\frac{h M \pi}{\rho}} + \frac{2M}{\rho h}$$

(2)

here is good to check the dimension

$$[A] = L^2 = \sqrt{\frac{LML^3}{M}} + \frac{ML^3}{ML} = L^2$$

For fun we test numbers

$$M = 80 \text{ kg}, \ h = 1.90 \text{ m}, \ \rho \approx 1000 \ \text{kg}/\text{m}^3$$

$$\rightarrow A = \left\{ 2\sqrt{\frac{1.90 \cdot 80 \cdot \pi}{1000}} + \frac{2 \cdot 80}{1900} \right\} \text{m}^2$$

$$\approx \{1{,}382 + 0{,}084\} \text{m}^2 \approx 1{,}5 \ \text{m}^2$$

(3)

Problem 2: (1-01-70)

72.0 beats/min

a) Beats in 2.0 yr

$$\{2 \text{ yr} \approx 2 \cdot 365 \cdot 24 \cdot 60 \text{ min} = 1.0512 \cdot 10^6 \text{ min}\}$$

$$2.0 \text{ yr} \rightarrow N = 7.20 \cdot 10^1 \ \text{beat}/\text{min} \cdot 1.1 \cdot 10^6 \text{ min}$$

$$\approx 7.6 \cdot 10^7$$

b) 2.00 yr

$$N = 7.20 \cdot 10^1 \ \text{beats}/\text{min} \cdot 1.0512 \cdot 10^7 \text{ min}$$

$$\approx 7.57 \cdot 10^7$$

c) 2.000 yr

even though the time is known with more accuracy, the heart rate only has 3 significant digits $\rightarrow$ $N \simeq 7.57 \cdot 10^7$

(4)

Problem 3: (1-01-84)

Box:

$a = 1.80 \pm 0.1$ cm

$b = 2.05 \pm 0.02$ cm

$c = 3.1 \pm 0.1$ cm

discard terms with $\Delta \cdot \Delta \cdots$ and $\Delta \cdot \Delta \cdot \Delta$

$$V = abc = (a \pm \Delta a)(b \pm \Delta b)(c \pm \Delta c)$$

$$= abc\left(1 \pm \frac{\Delta a}{a}\right)\left(1 \pm \frac{\Delta b}{b}\right)\left(1 \pm \frac{\Delta c}{c}\right)$$

$$\simeq abc\left\{1 \pm \frac{\Delta a}{a} \pm \frac{\Delta b}{b} \pm \frac{\Delta c}{c}\right\}$$

$$= abc \pm \{bc\Delta a + ac\Delta b + ab\Delta c\}$$

$$= \{1.1 \cdot 10^1 \pm 2\} \text{cm}^3 = \{11 \pm 2\} \text{cm}^3$$

Problem 4: (1-02-72) ⑤

Cartesian $(2, y)$ — Find y and r

Polar $(r, \frac{\pi}{6})$ — Many ways, I use:

$x = 2$ $\quad x = r\cos\varphi$ $\quad r = \sqrt{x^2 + y^2}$

$\varphi = \frac{\pi}{6}$ $\quad y = r\sin\varphi$

$r = \sqrt{4 + y^2}$ and $y = \frac{r}{2}$, as $\sin\frac{\pi}{6} = \frac{1}{2}$

$\rightarrow r = \sqrt{4 + \frac{r^2}{4}} \rightarrow r^2 = 4 + \frac{r^2}{4}$

$\rightarrow \left\{1 - \frac{1}{4}\right\} r^2 = 4 \rightarrow r^2 = \frac{4}{1 - \frac{1}{4}} = \frac{16}{3}$

⑥

$\rightarrow \boxed{r = \frac{4}{\sqrt{3}}} \rightarrow \boxed{y = \frac{2}{\sqrt{3}}}$, as $y = \frac{r}{2}$

Test, we had

$x = r\cos\varphi = \frac{4}{\sqrt{3}} \cdot \underbrace{\frac{\sqrt{3}}{2}}_{\cos(\frac{\pi}{6})} = 2$ as was given initially