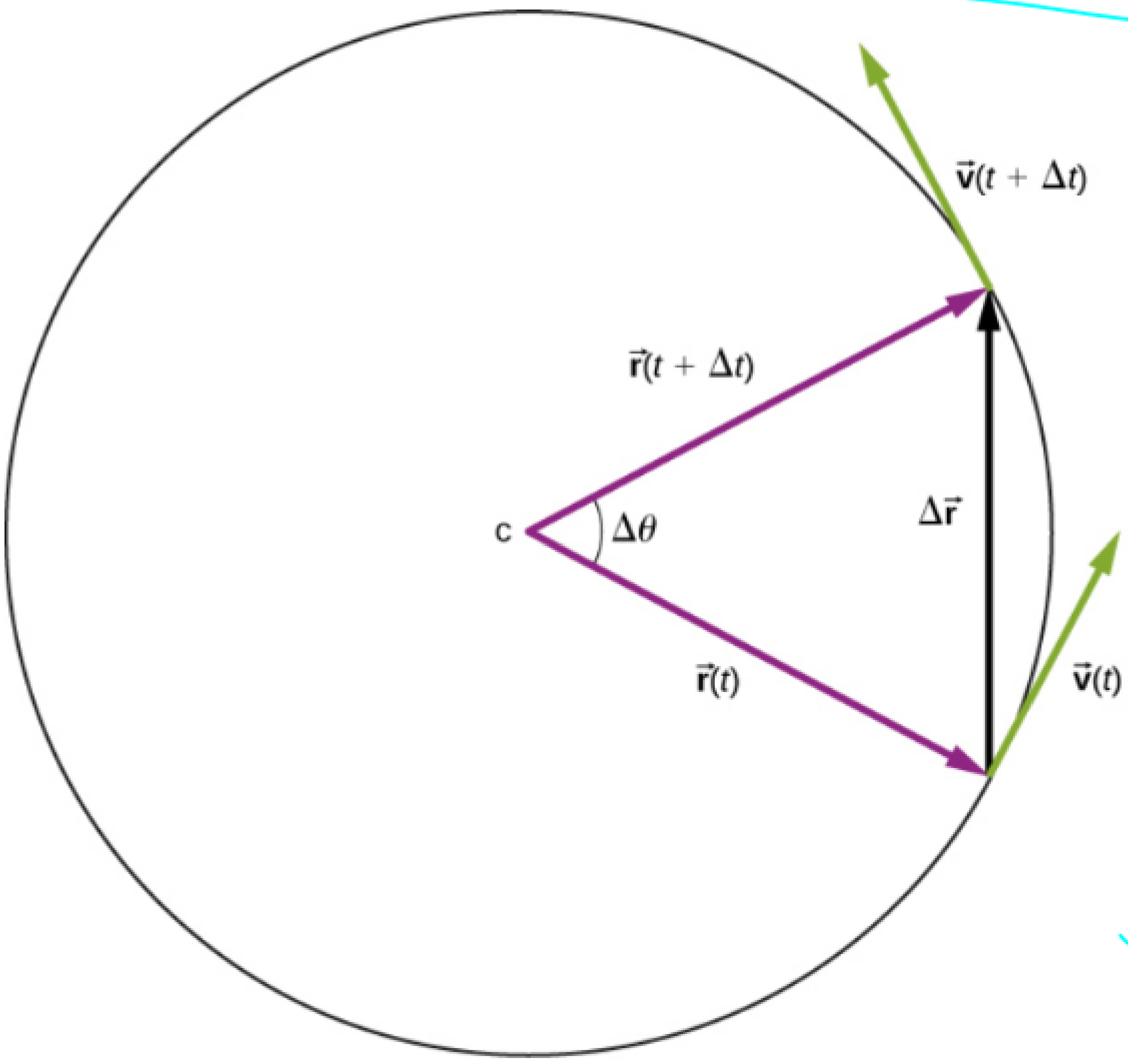
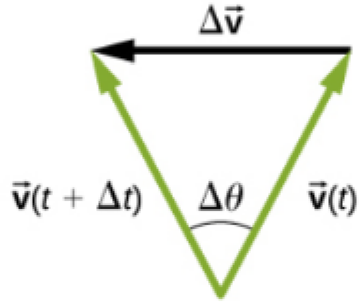
Stöðug hringhreyfing

$$|\vec{v}(t)| = |\vec{v}(t')|$$
$$|\vec{r}(t)| = |\vec{r}(t')|$$
fyrir öll t og t'



$$|\vec{r}(t)| = |\vec{r}(t+\Delta t)|$$
$$|\vec{v}(t)| = |\vec{v}(t+\Delta t)|$$



Jafnarma einslaga þríhyrningar

$$\frac{\Delta v}{v} = \frac{\Delta r}{r} \longrightarrow \Delta v = \left(\frac{v}{r}\right)\Delta r$$

openstax

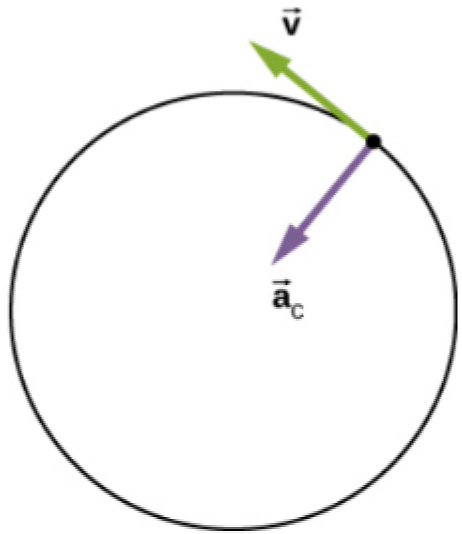
Radialhröðun -- hröðun útpáttar

$$\boxed{a = \lim_{\Delta t \to 0} \left(\frac{\Delta v}{\Delta t}\right) = \lim_{\Delta t \to 0} \left(\frac{v}{r}\frac{\Delta r}{\Delta t}\right)}$$

$$= \frac{v}{r}\lim_{\Delta t \to 0}\left(\frac{\Delta r}{\Delta t}\right) = \frac{v}{r} v = \boxed{\frac{v^2}{r}}$$

fyrir jafna hringhreyfingu verður að vera föst miðsóknarhröðun

$$\boxed{a_c = \frac{v^2}{r}}$$



openstax

að miðju hringbrautar

Dæmi um stærð miðsóknarhröðunar

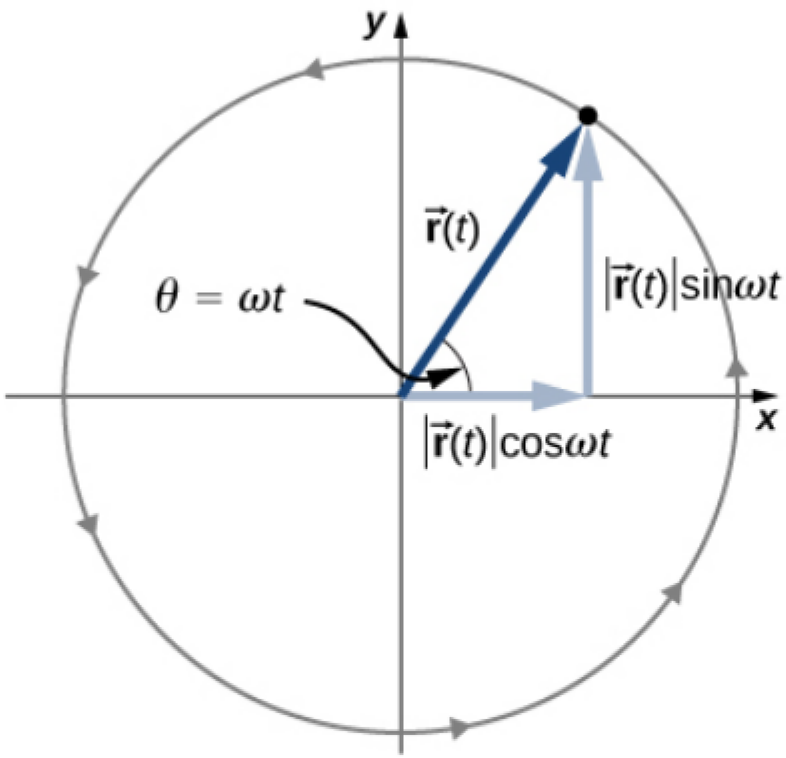
Object	Centripetal Acceleration (m/s^2 or factors of *g*)
Earth around the Sun	5.93×10^{-3}
Moon around the Earth	2.73×10^{-3}
Satellite in geosynchronous orbit	0.233
Outer edge of a CD when playing	5.78
Jet in a barrel roll	$(2–3\ g)$
Roller coaster	$(5\ g)$
Electron orbiting a proton in a simple Bohr model of the atom	9.0×10^{22}

Table 4.1 Typical Centripetal Accelerations

openstax

Lýsing jafnrar brautarhreyfingar

Hér væri hægt að nota pólhnit, en ....
Byrjum með kartísk hnit



openstax

Ef $A = |\vec{r}(t)| \rightarrow$

$$\boxed{\vec{r}(t) = A\cos(\omega t)\,\hat{i} + A\sin(\omega t)\,\hat{j}}$$

$\theta = \omega t$, ω horntíðni

$T = \frac{2\pi}{\omega}$, Lota

Í kartískum hnitum fæst

$$\vec{\mathbf{v}}(t) = \frac{d\vec{\mathbf{r}}(t)}{dt} = -A\omega \sin \omega t\,\hat{\mathbf{i}} + A\omega \cos \omega t\,\hat{\mathbf{j}}.$$

og

openstax

$$\vec{\mathbf{a}}(t) = \frac{d\vec{\mathbf{v}}(t)}{dt} = -A\omega^2 \cos \omega t\,\hat{\mathbf{i}} - A\omega^2 \sin \omega t\,\hat{\mathbf{j}}.$$

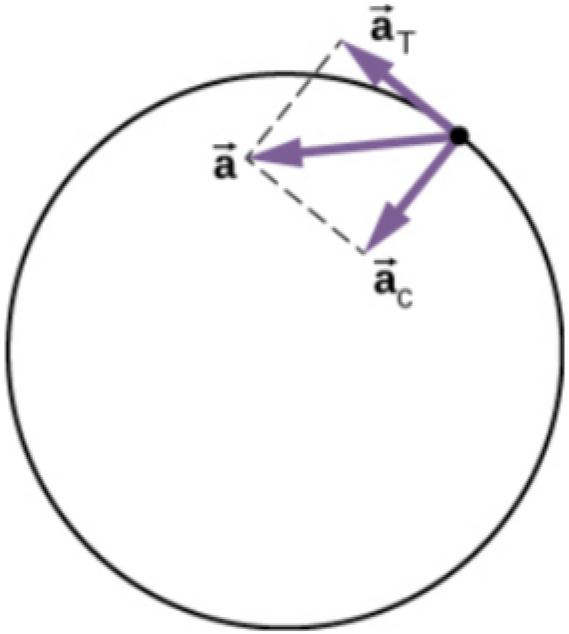
því sést líka að

$$\boxed{\bar{a}(t) = -\omega^2 \bar{r}(t)}$$

Í pólhnitum er erfiðara að finna afleiðurnar því einingarvigrarnir eru líka háðir tíma. Svo er ekki í kartískum hnitum

Ójöfn hringhreyfing

Til viðbótar við miðsóknarhröðunina birtist snertilhröðun



$$\boxed{a_T = \frac{d}{dt}|\overline{v}(t)|}$$

og heildarhröðunin verður

$$\boxed{\overline{a} = \overline{a}_c + \overline{a}_T}$$

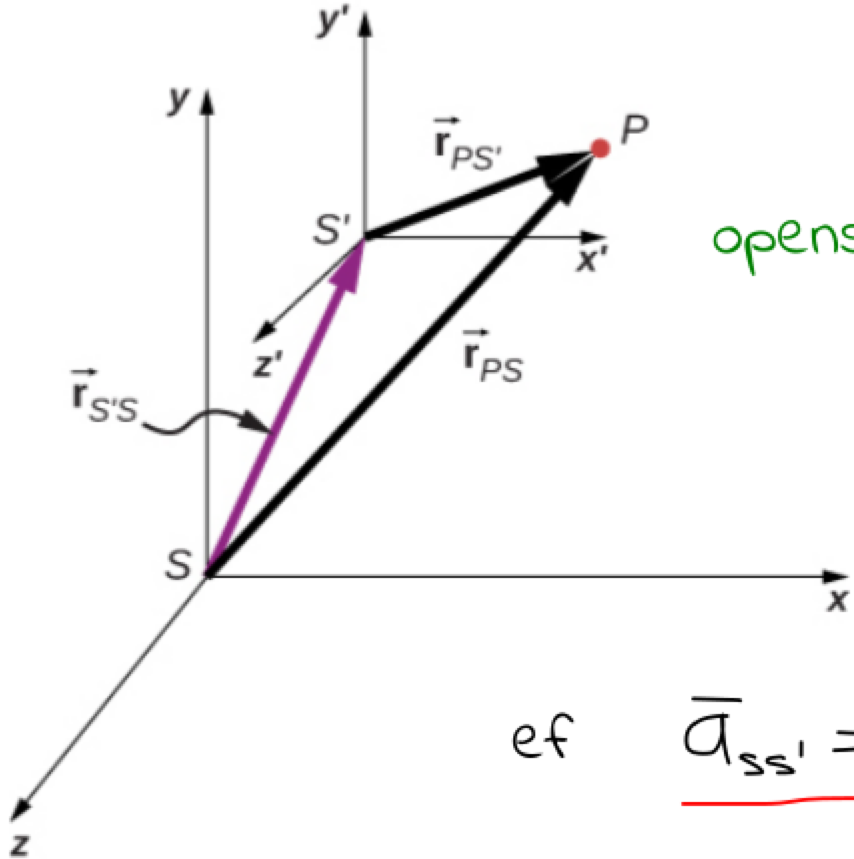
openstax

þar sem miðsóknarhröðunina má reikna á sama hátt og áður

Afstæður hraði

T.d. hraði flugvélar miðað við jörð eða loft

Tvö viðmiðunarkerfi S og S'



$$\vec{\mathbf{r}}_{PS} = \vec{\mathbf{r}}_{PS'} + \vec{\mathbf{r}}_{S'S}$$

openstax

$$\vec{\mathbf{v}}_{PS} = \vec{\mathbf{v}}_{PS'} + \vec{\mathbf{v}}_{S'S}$$

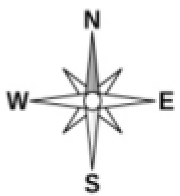
ef $\bar{a}_{ss'} = 0$ $\rightarrow$ $\vec{\mathbf{a}}_{PS} = \vec{\mathbf{a}}_{PS'}$

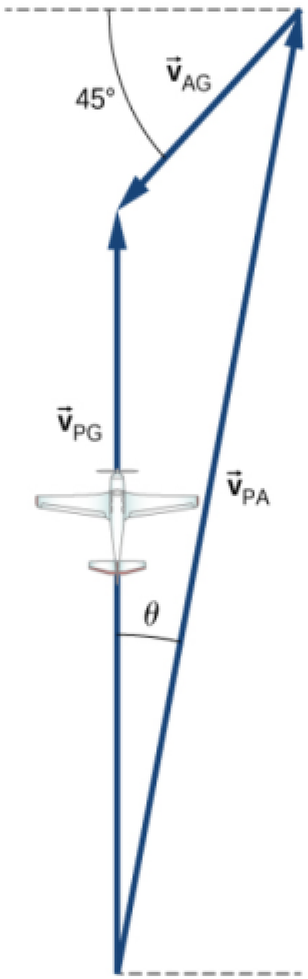
Figure 4.26 The positions of particle P relative to frames S and S' are $\vec{\mathbf{r}}_{PS}$ and $\vec{\mathbf{r}}_{PS'}$, respectively.

EXAMPLE 4.14

Flying a Plane in a Wind

A pilot must fly his plane due north to reach his destination. The plane can fly at 300 km/h in still air. A wind is blowing out of the northeast at 90 km/h. (a) What is the speed of the plane relative to the ground? (b) In what direction must the pilot head her plane to fly due north?





openstax

Jörð (G)

Flugkona (P)

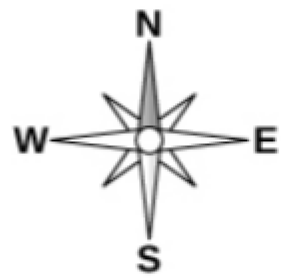
Loft (A)

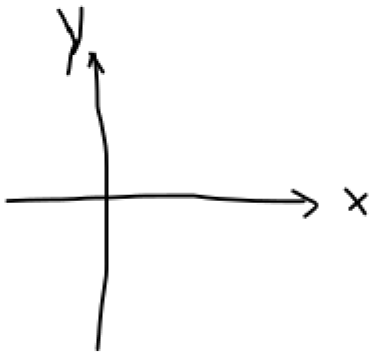
$\bar{v}_{AG}$: vindhraði miðað við jörð

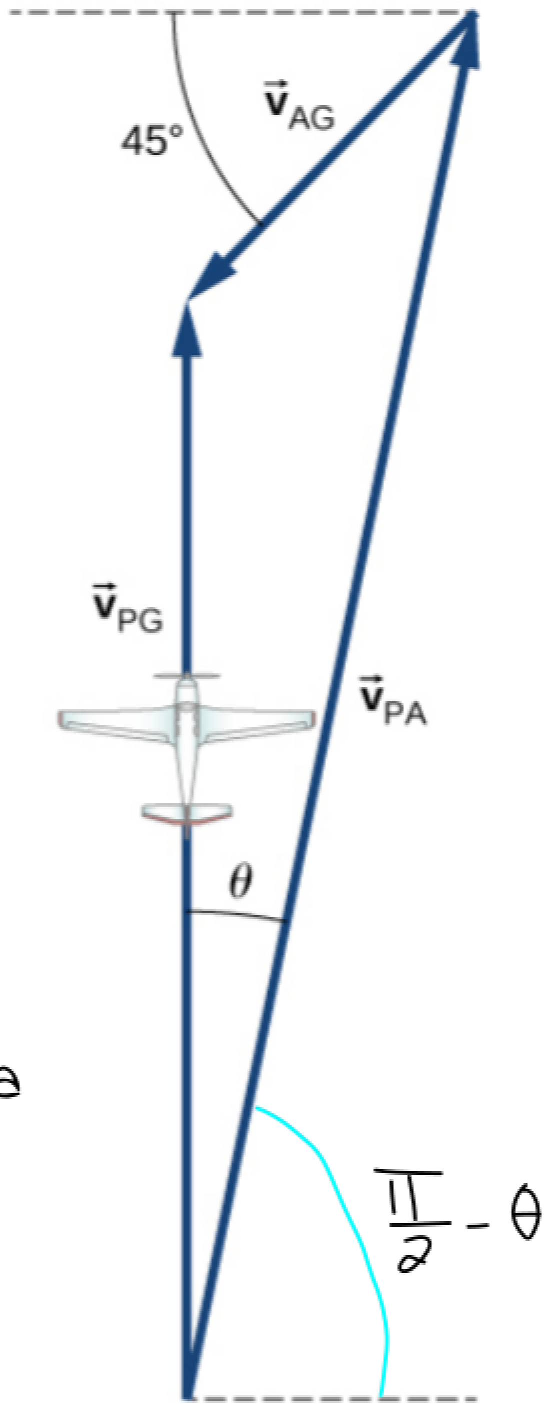
$\bar{v}_{PG}$: hraði vélar miðað við jörð

$\bar{v}_{PA}$: hraði vélar miðað við loft

Þekkjum ekki θ og $|\bar{v}_{PG}|$,
en vitum $|\bar{v}_{PA}| = 300\ km/s$







$$\bar{v}_{PG} = \bar{v}_{PA} + \bar{v}_{AG}$$

notum kartísku hnitin til að leggja saman hraðavigrana

$$\bar{v}_{PG} = (0, v_{PG})$$

$$\bar{v}_{PA} = v_{PA}\left(\cos\left(\frac{\pi}{2} - \theta\right), \sin\left(\frac{\pi}{2} - \theta\right)\right)$$

$$\bar{v}_{AG} = v_{AG}\left(\cos\left(\frac{5\pi}{4}\right), \sin\left(\frac{5\pi}{4}\right)\right)$$

því umskrifast

$$\bar{v}_{PG} = \bar{v}_{PA} + \bar{v}_{AG}$$

sem

$$(0, v_{PA}) = \left(v_{PA}\cos\left(\frac{\pi}{2}-\theta\right) + v_{AG}\cos\left(\frac{5\pi}{4}\right),\right.$$

$$\left. v_{PA}\sin\left(\frac{\pi}{2}-\theta\right) + v_{AG}\sin\left(\frac{5\pi}{4}\right)\right)$$

notum

$$\cos\left(\frac{\pi}{2}-\theta\right) = \sin\theta, \quad \sin\left(\frac{5\pi}{4}\right) = -\frac{1}{\sqrt{2}}$$

$$\sin\left(\frac{\pi}{2}-\theta\right) = \cos\theta, \quad \cos\left(\frac{5\pi}{4}\right) = -\frac{1}{\sqrt{2}}$$

þá fæst fyrir x-hnitið

$$0 = v_{PA} \sin\theta - \frac{v_{AG}}{\sqrt{2}} \qquad 1$$

og fyrir y-hnitið

$$v_{PG} = v_{PA} \cos\theta - \frac{v_{AG}}{\sqrt{2}} \qquad 2$$

Við þekkjum $v_{PA} = 300$ km/klst og $v_{AG} = 90$ km/klst, en viljum finna hornið θ og ferðina v_{PG}

1 $\rightarrow$ $$\sin\theta = \left(\frac{v_{AG}}{v_{PA}} \frac{1}{\sqrt{2}}\right)$$

$\rightarrow$ $$\theta = \arcsin\left(\frac{v_{AG}}{v_{PA}} \frac{1}{\sqrt{2}}\right) \approx \underline{0{,}2138 \text{ rad}}$$

$$\approx \underline{12{,}2^{\circ}}$$

austur af norður

2 $\rightarrow$ $$v_{PG} = v_{PA} \cos\theta - \frac{v_{AG}}{\sqrt{2}}$$

$$\simeq \underline{230 \text{ km/klst}}$$