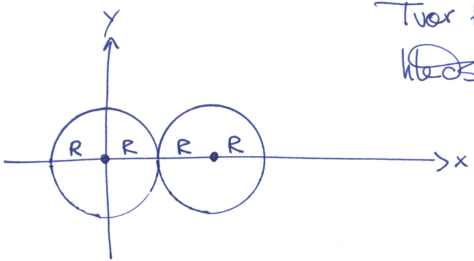
1



Tvær kúlur með geisla R og hleðslu Q

$\to$ hleðslu þéttleiki

$$\rho = \frac{Q}{V} = \frac{Q}{\frac{4}{3}\pi R^3} = \frac{3Q}{4\pi R^3}$$

finnum $\bar{E}$ innan kúlu með lögmáli Gauß

$$\oint \bar{E}\cdot d\bar{A} = \frac{Q'}{\epsilon_0}$$

Kúlusamhverfa $\to$ Kúlu Gauß flötur með geisla $r < R$

$$4\pi r^2 E = \frac{\frac{4\pi}{3} r^3 \cdot \rho}{\epsilon_0} = \frac{r^3}{R^3}\frac{Q}{\epsilon_0}$$

$$\to E = \frac{Q}{4\pi\epsilon_0}\frac{r}{R^3} \quad \text{ef } r < R$$

og

$$\bar{E} = \frac{Q}{4\pi\epsilon_0}\frac{r}{R^3}\hat{r}$$

einingarvigur í radial eða út átt

utan kúlu gefur Gauß

$$\bar{E} = \frac{Q}{4\pi\epsilon_0 r^2}\,\hat{r} \qquad \text{ef} \quad r > R$$

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a) Finna $\bar{E}$ í $x = 0$

Rafsvið frá báðum kúlum

$$\bar{E} = \bar{E}_1 + \bar{E}_2 = \frac{Q}{4\pi\epsilon_0}\frac{0}{R^3}\,\hat{r}_1 + \frac{Q}{4\pi\epsilon_0 (2R)^2}\,\hat{r}_2$$

$$= 0 + \frac{Q}{4\pi\epsilon_0 (2R)^2}(-\hat{x})$$

$$= -\frac{Q}{16\pi\epsilon_0 R^2}\,\hat{X}$$

b) $\bar{E}$ @ $x = \frac{R}{2}$

$$\bar{E} = \bar{E}_1 + \bar{E}_2 = \frac{Q}{4\pi\epsilon_0} \frac{R/2}{R^3} \hat{r}_1 + \frac{Q}{4\pi\epsilon_0 \left(\frac{3R}{2}\right)^2} \hat{r}_2$$

$$= \frac{Q}{4\pi\epsilon_0} \frac{1}{2R^2} (\hat{x}) + \frac{Q}{4\pi\epsilon_0} \frac{4}{9R^2} (-\hat{x})$$

$$= \frac{Q}{4\pi\epsilon_0} \hat{x} \left\{ \frac{1}{2R^2} - \frac{4}{9R^2} \right\} = \frac{Q}{4\pi\epsilon_0 R^2} \hat{x} \left\{ \frac{1}{2} - \frac{4}{9} \right\}$$

$$= \frac{Q}{72\pi\epsilon_0 R^2}$$

c) $x = R$

Hér verða $|\bar{E}_1| = |\bar{E}_2|$ en í gagnstæða stefnu

$\rightarrow \bar{E} = 0$

d) $x = 3R$

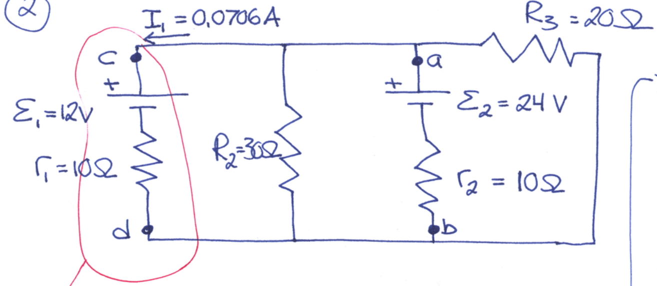
$$\bar{E} = \bar{E}_1 + \bar{E}_2 = \frac{Q}{4\pi\epsilon_0 (3R)^2}\hat{r}_1 + \frac{Q}{4\pi\epsilon_0 R^2}\hat{r}_2$$

$$= \frac{Q}{4\pi\epsilon_0 (3R)^2}\hat{x} + \frac{Q}{4\pi\epsilon_0 R^2}\hat{x}$$

$$= \frac{Q}{4\pi\epsilon_0 R^2}\hat{x}\left\{\frac{1}{9} + 1\right\} = \frac{Q}{4\pi\epsilon_0 R^2}\hat{x}\,\frac{10}{9}$$

$$= \frac{5Q}{18\pi\epsilon_0 R^2}\hat{x}$$

2



finna V_{ab}

Straumurinn hér er þekktur, líta $\mathcal{E}_1$ og r_1 -> því má reikna

V_{cd}: $V_d + I_1 r_1 + \mathcal{E}_1 = V_c$

-> $V_c - V_d = I_1 r_1 + \mathcal{E}_1$

-> Þetta sama spennufall verður að vera yfir R_2, R_3 og $V_a - V_b$

-> $V_a - V_b = I_1 r_1 + \mathcal{E}_1$

$$= 0{,}0706\,A \cdot 10\,\Omega + 12\,V \simeq 12.7\ V$$

Straumurinn í gegnum 24-V rafhlöðuna? Köllum hann I_2

$I_2 r_2 + \mathcal{E}_2 = V_a - V_b = V_c - V_d$

hér geri ég ráð fyrir að hann sé frá a til b

$$\rightarrow \quad I_2 r_2 = V_c - V_d - \Sigma_2 \rightarrow \quad I_2 = \frac{V_c - V_d - \Sigma_2}{r_2}$$

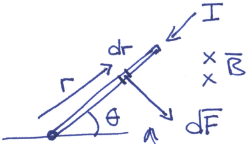
$$\rightarrow \quad I_2 = \frac{I_1 r_1 + \Sigma_1 - \Sigma_2}{r_2} = \frac{(12{,}7 - 24)\,V}{10\,\Omega} \approx -1.13\,A$$

$$\rightarrow \quad I_2 = 1.13\,A$$ frå b til a

③

Í jafnvægi er vægi gormsins jafnt vægi segulsviðsins en í andstæða átt.

a)



$$\underline{d\bar{F} = I\, d\bar{l} \times \bar{B}}$$

$$\bar{\tau} = \bar{r} \times \bar{F} \longrightarrow d\bar{\tau} = \bar{r} \times d\bar{F}$$

rétt horn milli $\bar{r}$ og $\bar{F}$

$$\hookrightarrow d\tau = r\,dF = IB\,r\,dr$$

$$\rightarrow \tau = IB \int_0^L r\,dr = \frac{1}{2} IBL^2$$

fastar

Segulvægið á stöngina er réttsælis

→ það togar á gorminum

b)

c) Jafnvægi → $\bar\tau_g + \bar\tau_B = 0$

$$\bar\tau = \bar r \times \bar F$$
$$rF\sin\theta\,\hat\theta$$

$$\tau_g = \frac{1}{2} IBL^2$$

En við vitum líka $F_g = kx$

$$\rightarrow \tau_g = L F_g \sin\theta = Lkx\sin\theta = \frac{1}{2} IBL^2$$

$$\rightarrow x = \frac{IBL^2}{2Lk\sin\theta} = \frac{IBL}{2k\sin\theta}$$

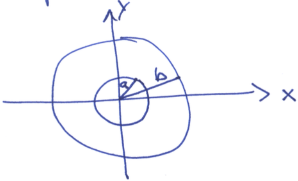
svo mikið teygist gormurinn

Stöðuorka gorms

$$U = \frac{1}{2} k x^2$$

$$\rightarrow \quad U = \frac{1}{2} k \left(\frac{IBL}{2k\sin\theta} \right)^2 = \frac{I^2 B^2 L^2}{8k\sin^2\theta}$$

4 þversnið leiðara, um hann flýtur straumur I



Hér er hægt að nota Lögmál Ampers

$$\oint \bar{B} \cdot d\bar{l} = \mu_0 I'$$

a) Innan leiðara $r<a$

Enginn straumur

$\rightarrow \bar{B} = 0$ þ. $r<a$

í holi hans

c) utan leiðara $r>b$

notum hringsamhverfu

$$2\pi r B = \mu_0 I$$

$$\rightarrow B = \frac{\mu_0 I}{2\pi r}$$

b) Innan leiðarans sjálfs

$$a < r < b$$

þverskurðar flötur leiðarans er $A = \pi b^2 - \pi a^2$

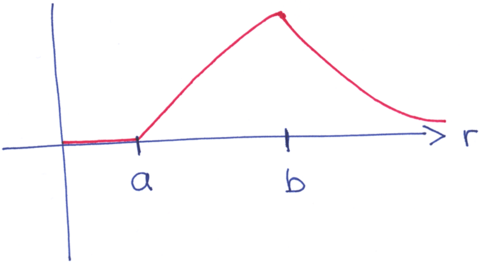
$$\rightarrow A(r) = \pi r^2 - \pi a^2$$

og
$$I'(r) = \frac{A(r)}{A} I = \frac{r^2-a^2}{b^2-a^2}$$

$$\oint \bar{B}\cdot d\bar{l} = \mu_0 I'(r)$$

$$\rightarrow 2\pi r B = \mu_0 \frac{r^2-a^2}{b^2-a^2} I$$

$$\rightarrow B = \frac{\mu_0 I}{2\pi r} \frac{r^2-a^2}{b^2-a^2}$$



Orka í þétti $U_c = \frac{1}{2} C V^2$ gefin

$\rightarrow \boxed{C = 2U_c / V^2}$ og við vitum $V = 12\ V$

$f_a = 3500\ Hz \rightarrow \omega_a = 2\pi f_a$

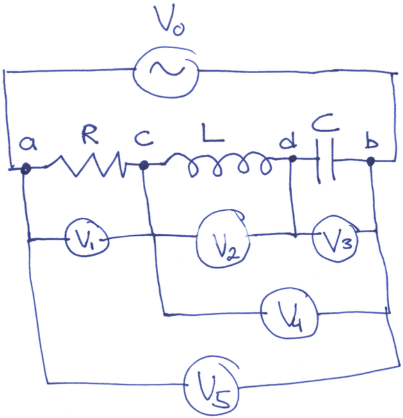
og hermitíðni $\omega_a = \frac{1}{\sqrt{LC}} \rightarrow 2\pi f_a = \frac{1}{\sqrt{LC}}$

$\rightarrow 4\pi^2 f_a^2 = \frac{1}{LC} \rightarrow L = \frac{1}{4\pi^2 f_a^2 C}$

$$\rightarrow \boxed{L = \frac{V^2}{4\pi^2 f_a^2 \, 2U_c}}$$

Allt gefnar stærðir

6



Alt rms-målar

rødtengd rås

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$I_{rms} = \frac{V_{rms}}{Z}$$

$$V_{R\,rms} = I_{rms} R$$

$$V_{C\,rms} = I_{rms} X_C$$

$$V_{L\,rms} = I_{rms} X_L$$

$$X_L = \omega L$$

$$X_C = \frac{1}{\omega C}$$

L, C, ω gefin āsamt V_0

$\hookrightarrow X_L, X_C, Z$ ūtreitnanbg

$$\boxed{V_{rms} = \frac{V_0}{\sqrt{2}}} = I_{rms} Z \rightarrow \boxed{I_{rms} = \frac{V_0}{\sqrt{2}} \frac{1}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}}$$

↑ ūtreitnanbgt

$$\boxed{V_1 = V_{R\,rms} = I_{rms} R = \frac{V_0 R}{\sqrt{2}} \frac{1}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}}$$

$$\boxed{V_2 = V_{L\,rms} = I_{rms} X_L = \frac{V_0}{\sqrt{2}} \frac{\omega L}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}}$$

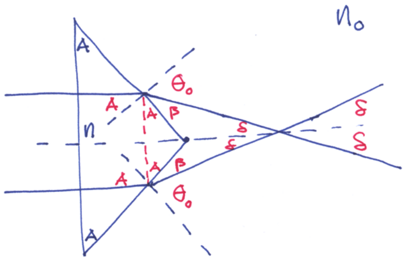
$$V_3 = V_{C\,rms} = I_{rms} X_C = \frac{V_0}{\sqrt{2}} \frac{1}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}\,(\omega C)}$$

$$V_4 = \left| V_{L\,rms} - V_{C\,rms} \right| = I_{rms} \left| \omega L - \frac{1}{\omega C} \right|$$

$$= \frac{V_0}{\sqrt{2}} \frac{\left| \omega L - \frac{1}{\omega C} \right|}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

$$V_5 = V_{rms} = \frac{V_0}{\sqrt{2}}$$

7



$$n \sin A = n_0 \sin\theta_0 \quad \rightarrow \quad \sin\theta_0 = \overbrace{\frac{n}{n_0}\sin A}^{\text{gefit}}$$

$$\rightarrow \quad \theta_0 = \arcsin\left\{\frac{n}{n_0}\sin A\right\}$$

$$\beta = \frac{\pi}{2} - \theta_0 = \frac{\pi}{2} - \arcsin\left\{\frac{n}{n_0}\sin A\right\}$$

$$\delta = \frac{\pi}{2} - A - \beta = \frac{\pi}{2} - A - \frac{\pi}{2} + \theta_0 = \theta_0 - A$$

homm nilli gestanna tueggjá ar

$$2\delta = \left[\theta_0 - A\right] \cdot 2$$

$$= 2\left\{\arcsin\left(\frac{n}{n_0}\sin A\right) - A\right\}$$