Nokkur dæmi um rafsegulfræði í efni

Óhemju vítt svið sem við getum aðeins tæpt á.

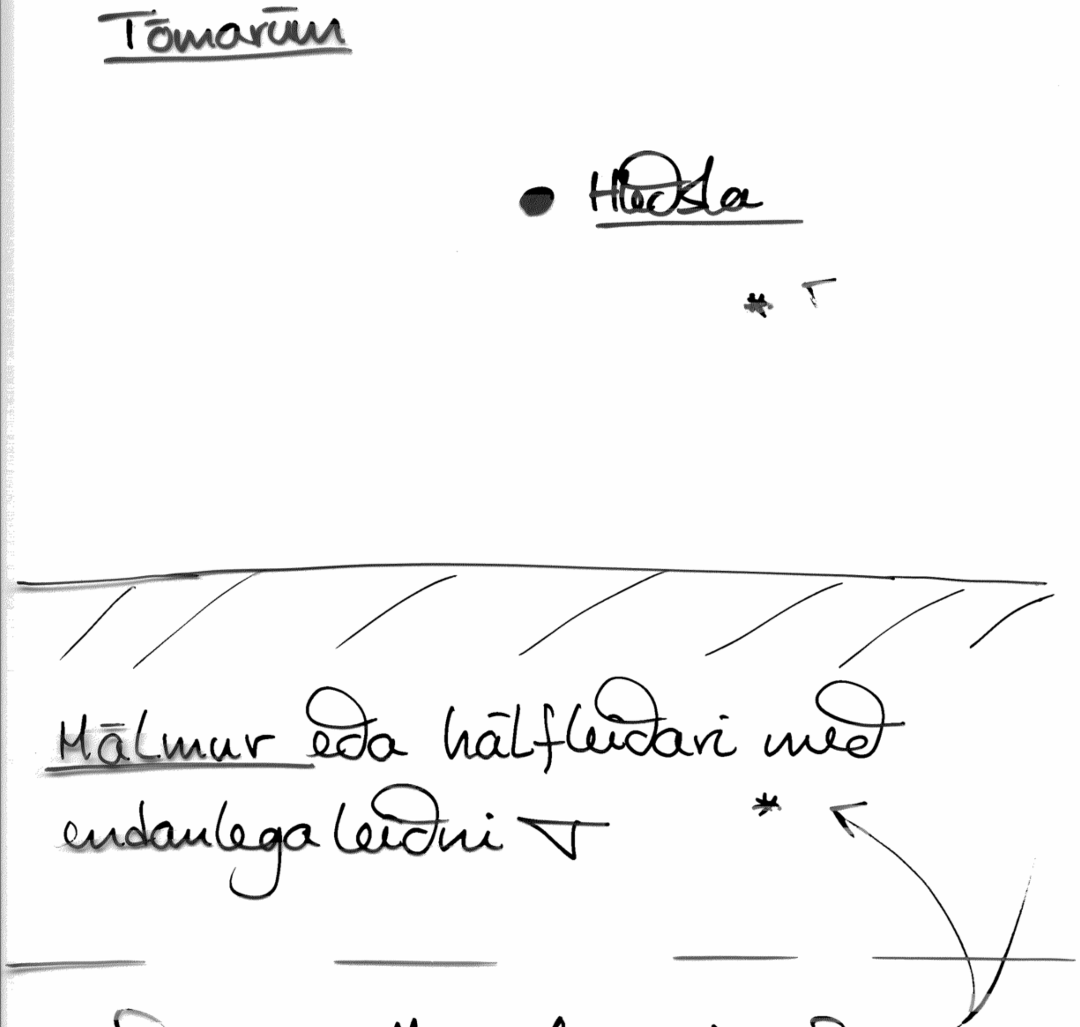
Í námskeiðinu var hingað til aðallega fjallað um rafsegulfræði í tómarúmi, þó var aðeins litið á rafsvara og seguleiginleika efna.

Eins komu fyrir straumar og hleðslur í jöfnum Maxwells:

$$\boxed{\begin{array}{ll} \bar{\nabla}\cdot\bar{E} = \dfrac{\rho}{\epsilon_0} & \bar{\nabla}\cdot\bar{B} = 0 \\[2ex] \bar{\nabla}\times\bar{E} = -\dfrac{\partial \bar{B}}{\partial t} & \bar{\nabla}\times\bar{B} = \mu_0\left(\bar{J} + \epsilon_0\dfrac{\partial \bar{E}}{\partial t}\right) \end{array}} \quad (1)$$

En hvaða hleðslur og strauma er átt við hér, hvaða rafsvið .... ?

Dæmi



Hvaða rafmætti mæli ég hér?
Hvaða hleðslur valda því?
Heildarmætti, mætti hleðslu, ...

Til þess að skilja þetta betur verður litið á:

Rafsvörun
Málmar
Hálfleiðarar

- **óháð tíma**
  skýling
  ϵ, ∇
- **háð tíma**
  rafgasbylgjur

- **Ofurleiðni**
  Lýsing
  Segul eiginleikar
  $\longrightarrow$ **Meissner hrif**

Til þess að einfalda framsetningu verður smyglað inn örfáum eiginleikum Fourierummyndana.

Jafna Poissons

Ein Maxwell jafnan $\bar{\nabla}\cdot\bar{E} = \frac{\rho}{\epsilon_0}$ tengir rafsvið og hleðslu. Við höfum síðan fundið rafstöðumættið ϕ með vegheildum á $\bar{E}$, því $\bar{E} = -\bar{\nabla}\phi$

Hentugra er að tengja þessar 2 jöfnur í eina jöfnu fyrir mættið og hleðsluna

$$\bar{\nabla}\cdot\left(-\bar{\nabla}\phi\right) = \frac{\rho}{\epsilon_0}$$

eða

$$\boxed{-\nabla^2\phi(\bar{r}t) = \frac{\rho(\bar{r}t)}{\epsilon_0}} \qquad (2)$$

Í Rafsegulfræði 2 fáið þið hjálp við að leysa jöfnu Poissons fyrir mism. hnitakerfi og jaðarskilyrði

Fourierummyndun

„Þartil hæfileg" skalarog vigurföll má fourierummynda milli (staðarogtímarúms) og (bylgjuvigurs og tíðni rúms):

$$(\bar r, t) \longleftrightarrow (\bar k, \omega)$$

$$\bar F(\bar r, t) = \frac{1}{(2\pi)^4}\int_{\mathbb{R}^4} d\bar k\, d\omega\, e^{i\bar k\cdot\bar r - i\omega t}\,\bar F(\bar k, \omega)$$

$$\bar F(\bar k, \omega) = \int_{\mathbb{R}^4} d\bar r\, dt\, e^{-i\bar k\cdot\bar r + i\omega t}\,\bar F(\bar r, t) \qquad (3)$$

Einfalt er að sannreyna að

$$\bar\nabla\phi(\bar r t) \longleftrightarrow i\bar k\,\phi(\bar k\omega)$$

$$\bar\nabla\cdot\bar E(\bar r, t) \longleftrightarrow i\bar k\cdot\bar E(\bar k\omega) \qquad (4)$$

$$\bar\nabla\times\bar E(\bar r, t) \longleftrightarrow i\bar k\times\bar E(\bar k\omega)$$

Þægilegt er að innfæra Dirac δ-fallið

$$\delta(\bar k - \bar q) = \frac{1}{(2\pi)^3}\int d\bar r\, e^{-i\bar r\cdot(\bar k - \bar q)} \qquad (5)$$

með

$$f(\bar k) = \int d\bar q\, \delta(\bar q - \bar k)\, f(\bar q)$$

{ Fourierummyndun einsleits falls (fasta) gefur „óendanlegan topp í einum punkti" }

Með því má leiða út földunarsetninguna:

$$\boxed{\text{Ef}\quad \Phi(\bar r) = \int d\bar r'\, g(\bar r - \bar r')\, f(\bar r') \quad \text{þá er} \quad \Phi(\bar k) = g(\bar k)\, f(\bar k)} \qquad (6)$$

Fourier Transforms. If $f(z)$ is such a function of z that $\int_{-\infty}^{\infty} |f(z)|^2\,dz$ is finite and if the function

$$F(k) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(z) e^{ikz}\,dz$$

then $F(k)$ is called the *Fourier transform* of $f(z)$ and

$$f(z) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} F(k) e^{-ikz}\,dk; \quad \int_{-\infty}^{\infty} |F(k)|^2\,dk = \int_{-\infty}^{\infty} |f(z)|^2\,dz$$

Furthermore if the expansion for $f_+(z)$ in the neighborhood of $z = 0$ is

$$f_+(z) = \sum_{n=0}^{\infty} \left(\frac{z^n}{n!}\right) f^{(n)}(0); \quad z > 0; \quad f_+ = 0; \quad z < 0$$

then the asymptotic behavior of F for large k is

$$F_+(k) = -\frac{1}{\sqrt{2\pi}} \sum_{n=1}^{\infty} \left(\frac{i}{k}\right)^n f^{(n-1)}(0)$$

If $F(k)$ is the Fourier transform of $f(z)$ and $G(k)$ the Fourier transform of $g(z)$, then the Fourier transform of

$$\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(y) g(z - y)\,dy \quad \text{is} \quad F(k)G(k); \quad \text{faltung theorem}$$

and the Fourier transform of $f(z)g(z)$ is $\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} F(l)G(k - l)\,dl$. Also $\int_{-\infty}^{\infty} F(k)\bar{G}(k)\,dk = \int_{-\infty}^{\infty} f(z)\bar{g}(z)\,dz$.

If $F(k)$ is the Fourier transform of $f(z)$, then

$$\sum_{n=-\infty}^{\infty} f(\alpha n) = \frac{\sqrt{2\pi}}{\alpha} \sum_{m=-\infty}^{\infty} F\left(\frac{2\pi m}{\alpha}\right); \quad \text{Poisson sum formula}$$

Even if $\int_{-\infty}^{\infty} |f(z)|^2\,dz$ is not finite, if $\int_{-\infty}^{\infty} |f(z)|^2 e^{-2\tau_0 z}\,dz$ is finite and if $G(k)$ is the Fourier transform of $f(z)e^{-\tau_0 z}$, then the Fourier transform of $f(z)$ is $G(k - i\tau_0) = F(k)$, where

$$f(z) = \frac{1}{\sqrt{2\pi}} \int_{-\infty + i\tau_0}^{\infty + i\tau_0} F(k) e^{-ikz}\,dk$$

For other conditions of convergence see Eqs. (4.8.19) *et seq.*

Function $f(z)$	Fourier transform $F(k)$
$\lambda f(z)$	$\lambda F(k)$
$f(az)$	$(1/a)F(k/a)$
$izf(z)$	$\frac{d}{dk} F(k)$
$\frac{d}{dz} f(z)$	$-ikF(k)$
$e^{izk_0} f(z)$	$F(k + k_0)$
$f(z + z_0)$	$e^{-ikz_0} F(k)$
1	$\delta(k)$
$(z - iz_0)^{-1}$ $(\mathrm{Re}\, z_0 > 0)$	$i\sqrt{2\pi}\, e^{-kz_0}$ $(\mathrm{Re}\, k > 0)$
$[(z - iz_0)(z + iz_1)]^{-1}$ $(\mathrm{Re}\, z_0 \text{ and } \mathrm{Re}\, z_1 > 0)$	$\frac{\sqrt{2\pi}}{(z_0 + z_1)} \begin{cases} e^{-z_0 k} & (\mathrm{Re}\, k > 0) \\ e^{z_1 k} & (\mathrm{Re}\, k < 0) \end{cases}$
$\mathrm{sech}(k_0 z)$	$(1/k_0)\sqrt{\pi/2}\, \mathrm{sech}(\pi k/2k_0)$
$\tanh(k_0 z)$	$(i/k_0)\sqrt{\pi/2}\, \mathrm{csch}(\pi k/2k_0)$
$z^{-\alpha-1} e^{i/z}$	$i\sqrt{2\pi}\, e^{\frac{1}{2}\pi i\alpha} k^{\frac{1}{2}\alpha} J_\alpha(2\sqrt{k})$
$e^{-\frac{1}{2}z^2}$	$e^{-\frac{1}{2}k^2}$
$\sqrt{z}\, J_{-\frac{1}{4}}(\frac{1}{2}z^2)$	$\sqrt{k}\, J_{-\frac{1}{4}}(\frac{1}{2}k^2)$

Stórsætt líkan af rafsvörun

Til viðbótar við jöfnur Maxwells notum við samfelldnijöfnu efnis

$$\boxed{\frac{\partial}{\partial t}\rho^{(Ft)} + \bar{\nabla}\cdot J^{(Ft)} = 0} \quad (7)$$

{ tímabreyting á hleðslu orsakar straum og öfugt }

Einnig þurfum við hreyfijöfnu fyrir efni til þess að hafa fullkomna lýsingu

{ Við komum með hana síðar }

(Newton eða Schrödinger)

Lítum á efnisbút { athugum fyrst tímaóháð }

Óhlaðinn

$$\langle \rho^{(Ft)} \rangle = 0$$

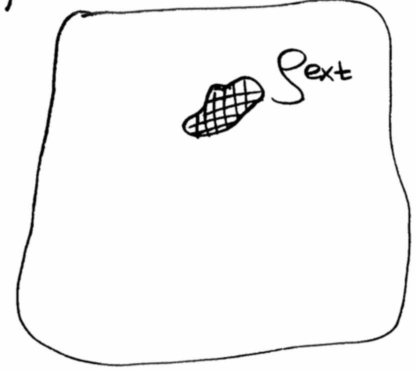
$$\bar{E} = 0$$

Stórsæja hleðsludreifingin

$\langle \rho^{(Ft)} \rangle = 0$ { eða bara $\rho^{(Ft)}$ }

Stórsæja rafsviðið $\bar{E} = 0$

Bætum við einni smá hleðslu $\rho_{ext}^{(Ft)}$



Búturinn er ekki lengur óhlaðinn

(8)

Stórsæja heildar rafsviðið

innan bútsins orsakast af tveimur þáttum

(1): Viðbótar hleðslunni $\rho_{ext}(\vec{r}t)$

(2): spönuðu hleðsludreifingunni $\langle\rho(\vec{r}t)\rangle \neq 0$ sem ρ_{ext} veldur

{frjálsar rafeindir í bútnum dragast eða hrindast frá viðbótar hleðslunni}

Því verður Maxwell's jafnan núna:

$$\boxed{\bar{\nabla}\cdot\bar{E} = \frac{1}{\epsilon_0}\left\{\langle\rho(\vec{r}t)\rangle + \rho_{ext}(\vec{r}t)\right\}} \quad (8)$$

(9)

fyrir heildar stórsæja rafsviðið inni í efnisbútnum

Viðbótar hleðslan (ytri hleðslan) veldur beint hinu svo kallaða færslu eða hliðrunarsviði D

$$\boxed{\bar{\nabla}\cdot\bar{D} = \frac{1}{\epsilon_0}\rho_{ext}(\vec{r}t)} \quad (9)$$

sem er ekki mælanlegt eitt sér!

Jöfnur (8) og (9) eru venjulega umskrifaðar sem Poissons jöfnur

$$\boxed{\begin{aligned} -\nabla^2\phi(\vec{r}t) &= \frac{1}{\epsilon_0}\left[\langle\rho(\vec{r}t)\rangle + \rho_{ext}(\vec{r}t)\right] \\ -\nabla^2\phi^{ext}(\vec{r}t) &= \frac{1}{\epsilon_0}\rho_{ext}(\vec{r}t) \end{aligned}} \quad (10)$$

með

$$\bar{E} = -\bar{\nabla}\phi \quad \text{og} \quad \bar{D} = -\bar{\nabla}\phi^{ext}$$

Athugasemdir !

Fyrir tímaháðsvið vitum við að

→ $\bar{E} = -\bar{\nabla}\phi$

nægir ekki því segulsvið getur líka valdið $\bar{E}$ ($\bar{E}$ er ekki geymið í þessu tilfelli)

þetta er því nálgun sem mun gilda við fremur lága tíðni og langa bylgjulengd þar sem ekki myndast sterkir straumar sem valda B. Hægt er að sýna fram á að leiðréttingar vegna B verða í okkar kerfum hér upp á 10^{-6}

Venjulega er ekki hægt að skilgreina mætti fyrir $\bar{D}$ þ.a.

$$\bar{D} = -\bar{\nabla}\phi^{ext}$$

En fyrir okkar kerfi má sýna að slíkt er skaðlaust (líka með fyrri athugasemd í huga).

Við ættum að nota þessar jöfnur t.þ.a. athuga hvað gerist þegar $\omega \to 0$

→ skýling

Síðar teygjum við okkur aðeins til að skoða lágtíðni fyrirbæri

→ hóphreyfingar

plasmabylgjur

(10)

Gert er ráð fyrir að þessi tvö mætti tengist á línulegan hátt {hér gætu aðrir mögul. komið til}

$$\phi^{ext}(\bar{r}) = \int d\bar{r}' \, \epsilon_r(\bar{r},\bar{r}')\, \phi(\bar{r}') \qquad (11)$$

{oft er notað $K(\bar{r},\bar{r}') = \epsilon_r(\bar{r},\bar{r}')$}

Í einsleitu rafeindagasi gildir

$$\epsilon_r(\bar{r},\bar{r}') = \epsilon_r(\bar{r}-\bar{r}') \qquad (12)$$

$$\rightarrow \phi^{ext}(\bar{r}) = \int d\bar{r}' \, \epsilon_r(\bar{r}-\bar{r}')\, \phi(\bar{r}') \qquad (13)$$

sem gefur með földunarsetningunni

$$\phi^{ext}(\bar{k}) = \underbrace{\epsilon_r(\bar{k})}_{\text{rafsvörunarfall}}\,\phi(\bar{k}) \qquad (14)$$

(11)

eða

$$\phi(\bar{k}) = \frac{1}{\epsilon_r(\bar{k})}\,\phi^{ext}(\bar{k}) \qquad (15)$$

Í einsleitu rafeindagasi er heildarmættið $\phi(\bar{k})$ jafn ytra mættinu deyfðu með $1/\epsilon_r(\bar{k})$.

{Ef rafeindakerfið var ekki einsleitt þá inniheldur $\phi(\bar{k})$ líka áhrif frá ϕ^{ext} með aðrabylgjuvigra $\bar{k}' \neq \bar{k}$.}

Flest líkön gefa ekki uppskrift fyrir reikningi á $\epsilon(\bar{k})$, heldur gefa rafviðtakið $\chi(\bar{k})$ í gegnum

$$\rho^{ind}(\bar{k}) = \chi(\bar{k})\,\phi(\bar{k}) \qquad (16)$$

!

(12)

þ.e. hvernig spanaði þéttleikinn ρ^{ind} tengist heildarmættinu ϕ

[t.d. truflanareikningur í skammtafr.]

- Athugum því fourier ummyndanir jöfnu (10)

$$k^2\phi(\bar{k}) = \frac{1}{\epsilon_0}\rho(\bar{k}) \quad (17)$$

$$k^2\phi^{ext}(\bar{k}) = \frac{1}{\epsilon_0}\rho^{ext}(\bar{k})$$

- og viðhöfðum

$$\rho = \rho^{ext} + \rho^{ind} \quad (18)$$

því fæst

$$k^2\left(\phi(\bar{k}) - \phi^{ext}(\bar{k})\right) = \frac{1}{\epsilon_0}\rho^{ind}(k) \quad (19)$$

$$= \frac{1}{\epsilon_0}\chi(\bar{k})\phi(\bar{k})$$

(13)

svo

$$\phi(\bar{k})\left\{k^2 - \frac{1}{\epsilon_0}\chi(\bar{k})\right\} = k^2\phi^{ext}(\bar{k})$$

eða

$$\phi(\bar{k}) = \frac{1}{\left\{1 - \frac{1}{\epsilon_0 k^2}\chi(\bar{k})\right\}}\phi^{ext}(\bar{k})$$

- sem gefur

$$\epsilon_r(\bar{k}) = \left\{1 - \frac{1}{\epsilon_0 k^2}\chi(\bar{k})\right\} \quad (20)$$

- Athugum einfallt líkan sem gefur gefið $\chi(\bar{k})$

<u>Thomas-Fermi líkan skýlingar</u>

<u>Ef</u> $\phi(r)$ breytist hægt með r

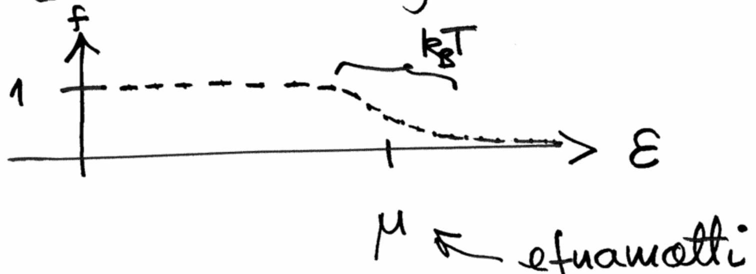
(Langbylgjunálgun)

Þá er heildarorka rafeindar

$$\varepsilon(\bar{k}) = \frac{(\hbar\bar{k})^2}{2m} - e\phi(\bar{r}), \qquad (21)$$

þ.s. $\bar{p} = \hbar\bar{k}$ er skriðþungi hennar.

Í þessu kerfi okkar er rafeindunum dreift á ástöndin merkt með $\bar{k}$ samkvæmt einsetulögmáli Paulis með Fermi dreifingu



Í einsleitu kerfi án truflunar er þéttleiki rafeindanna

$$n_0(\mu) = 2\int\frac{d\bar{k}}{(2\pi)^3} f(\varepsilon(\bar{k}) - \mu) \qquad (22)$$

spuni

með

$$f(x) = \frac{1}{\exp\left\{\frac{x}{k_BT}\right\}+1} \qquad (23)$$

Með ytri truflun verður nú

$$n(\bar{r}) = 2\int\frac{d\bar{k}}{(2\pi)^3} \frac{1}{\exp\left[\frac{1}{k_BT}\left(\frac{\hbar^2k^2}{2m} - e\phi(\bar{r}) - \mu\right)\right]+1}$$

$$= n_0(\mu + e\phi(\bar{r})) \qquad (24)$$

Berum saman við (18) til þess að fá

$$\rho^{ind}(\bar{r}) = -e\left\{n_0(\mu + e\phi(\bar{r})) - n_0(\mu)\right\} \qquad (25)$$

Sem er aðaljafna ólínulega Th-F-líkansins

$$\rho^{ind}(\bar{r}) = \underbrace{-e^2\left(\frac{\partial n_0}{\partial\mu}\right)\phi(\bar{r})}_{\text{línulegur liður}} + O(\phi^2) \qquad (26)$$

Berum saman við (16) sem gefur

$$\chi(\vec{k}) = -e^2 \frac{\partial n_0}{\partial \mu} \qquad (27)$$

og því

$$\boxed{\epsilon(\vec{k}) = 1 + \frac{e^2}{\epsilon_0 k^2} \frac{\partial n_0}{\partial \mu}} \qquad (28)$$

Venjulega er skilgreindur Th-F-bylgjuvigur

$$\boxed{k_0^2 = \frac{e^2}{\epsilon_0} \frac{\partial n_0}{\partial \mu}} \qquad (29)$$

$$\rightarrow \quad \boxed{\epsilon(\vec{k}) = 1 + \frac{k_0^2}{k^2}} \qquad (30)$$

$\frac{\partial n_0}{\partial \mu}$ er „varmafræðilegi ástanda-þéttleikinn" (TDOS)

$\boxed{\text{Hærri TDOS} \rightarrow \text{betri skýling}}$

Til dæmis

$$\phi^{ext}(\vec{r}) = \frac{Q}{4\pi\epsilon_0 r} \qquad (31)$$

Coulombóhreinindi í málmi

$$\rightarrow \quad \phi^{ext}(\vec{k}) = \frac{Q}{\epsilon_0 k^2} \qquad (32)$$

og

$$\phi(\vec{k}) = \frac{1}{\epsilon(\vec{k})} \phi^{ext}(\vec{k})$$

$$= \frac{Q}{\epsilon_0 (k^2 + k_0^2)} \qquad (33)$$

og því

$$\phi(\vec{r}) = \int \frac{d\vec{k}}{(2\pi)^3} e^{i\vec{k}\cdot\vec{r}} \frac{Q}{\epsilon_0 (k^2 + k_0^2)}$$

$$= \frac{Q}{4\pi\epsilon_0 r} e^{-k_0 r} \qquad (34)$$

Sterk skýling

tímaóháð

(Yukawa mætti) langbylgjunálgun $k \rightarrow 0$

Hvað gerist tímahάð?

Rafeindir færast til vegna ytri truflunar, þær fara yfir jafnvægis-stöðu sína

↳ mögulegar sveiflur

- Rafgasbylgjur í 3-víddarafeinda-gasi með föstum jákvætt hlöðnum bakgrunni (jónum).

- Kerfið er óhlaðið í heild $n_0 = n_b$

Vegna ytri tímaháð truflun

↳ $n(\bar{r}t) = n_0 + \delta n(\bar{r}t)$ (35)

jafnvægis þéttleiki

Notum samfelldnijöfnuna (7)

$$\frac{\partial}{\partial t}\rho(\bar{r}t) + \bar{\nabla}\cdot\bar{J}(\bar{r}t) = 0$$

og

$$\bar{J}(\bar{r}t) = -en\bar{v}\ ,\quad \rho = -en$$

- sem gefa

$$\frac{\partial}{\partial t}n(\bar{r}t) + \bar{\nabla}\cdot\left(n(\bar{r}t)\bar{v}\right) = 0$$

$$\frac{\partial}{\partial t}\left\{\delta n(\bar{r}t)\right\} + n_0\bar{\nabla}\cdot\bar{v} \simeq 0 \quad (36)$$

- Athugum hreyfijöfnu (Newtons)

$$m\frac{d}{dt}(n\bar{v}) = m\left\{\frac{\partial(n\bar{v})}{\partial t} + (\bar{v}\cdot\bar{\nabla})(n\bar{v})\right\}$$

$$= -en\bar{E} \quad (37)$$

Krafturinn

$$\boxed{mn_0\frac{\partial}{\partial t}\bar{v} \simeq -en_0\bar{E}}$$

Athugum afleiðuna (heildar af...)

$$\frac{d}{dt}\bar{v}$$

samferða afleiða (comoving....)

$\bar{v}(x,y,z,t)$ er hraði eindar vökva í punktinum (x,y,z) á tímanum t

Hraði sömu eindar á tíma $t+\Delta t$

er $v(x+\Delta x, y+\Delta y, z+\Delta z, t+\Delta t)$

með $\Delta x = v_x \Delta t$, ...

Í línulegri nálgun

$$v(x+v_x\Delta t,\ y+v_y\Delta t,\ z+v_z\Delta t,\ t+\Delta t)$$

$$\simeq v(x,y,z,t) + \frac{\partial v}{\partial x} v_x \Delta t + \frac{\partial v}{\partial y} v_y \Delta t + \frac{\partial v}{\partial z} v_z \Delta t + \frac{\partial v}{\partial t} \Delta t$$

Hröðunin

$$\frac{\Delta \bar{v}}{\Delta t} \simeq v_x \frac{\partial \bar{v}}{\partial x} + v_y \frac{\partial \bar{v}}{\partial y} + v_z \frac{\partial \bar{v}}{\partial z} + \frac{\partial}{\partial t}\bar{v}$$

$$= (\bar{v}\cdot\bar{\nabla})\bar{v} + \frac{\partial}{\partial t}\bar{v} \qquad (38)$$

$\frac{\partial}{\partial t}\bar{v}$ (hlutafleiðan)

er afleiða $\bar{v}$ m.t.t. t

í punktinum (x,y,z) (föstum)

$\frac{\partial}{\partial t}$ (36): $$\frac{\partial^2}{\partial t^2}\left\{\delta n(\bar{r}t)\right\} = -n_0 \frac{\partial}{\partial t} \bar{\nabla}\cdot\bar{v}(\bar{r}t)$$

$\bar{\nabla}\cdot$(37): $$m n_0 \frac{\partial}{\partial t} \bar{\nabla}\cdot\bar{v} = -e n_0 \bar{\nabla}\cdot\bar{E}$$

$$\frac{\partial^2}{\partial t^2}\left\{\delta n(\bar{r}t)\right\} = \frac{e n_0}{m} \bar{\nabla}\cdot\bar{E}$$

notum nú Maxwells jöfnuna

$$\bar{\nabla}\cdot\bar{E} = -\frac{e}{\epsilon_0}\left(n(\bar{r}t) - n_0\right) = -\frac{e}{\epsilon_0}\delta n(\bar{r}t)$$

til þess að fá hreyfijöfnuna

$$\boxed{\frac{\partial^2}{\partial t^2}\left\{\delta n(\bar{r}t)\right\} + \frac{n_0 e^2}{\epsilon_0 m}\delta n(\bar{r}t) = 0} \quad (39)$$

Hreintóna sveiflur með tíðni

$$\Omega_{pl}^2 = \frac{n_0 e^2}{\epsilon_0 m} \quad (40)$$

þrýstibylgjur $\Leftrightarrow$ langsbylgjur

fundnar með mjög einfaldaðri vökvaaflfræði, klassískri

Skammtafræðileg svörunarlíkön nota

$$\phi(\bar{k}\omega) = \frac{1}{\left\{1 - \frac{1}{\epsilon_0 k^2}\chi(\bar{k}\omega)\right\}}\phi^{ext}(\bar{k}\omega)$$

$$= \frac{1}{\epsilon_r(\bar{k}\omega)}\phi^{ext}(\bar{k}\omega)$$

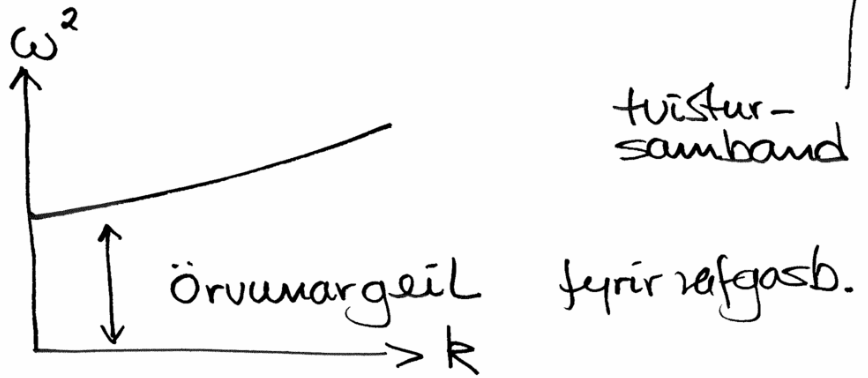
núllstöð í $\epsilon_r(\bar{k}\omega)$ gefur til kynna að ytrasvið (hverfandi) geti valdið miklu leiðarsviði

↳ sveiflur geta borist um kerfið

þá fæst

$$\boxed{\omega^2 = \Omega_{pe}^2 + \text{fasti} \cdot k^2 + \dots} \quad (41)$$

fyrir langs rafgasbylgjur í einsleitu rafeindagasi án segulsviðs



Við getum einsleitt út bylgjujöfnuna fyrir rafsegulsvið í efni og fundið þvers rafgasbylgju (ekki þrýstibylgju) með tvistursamband

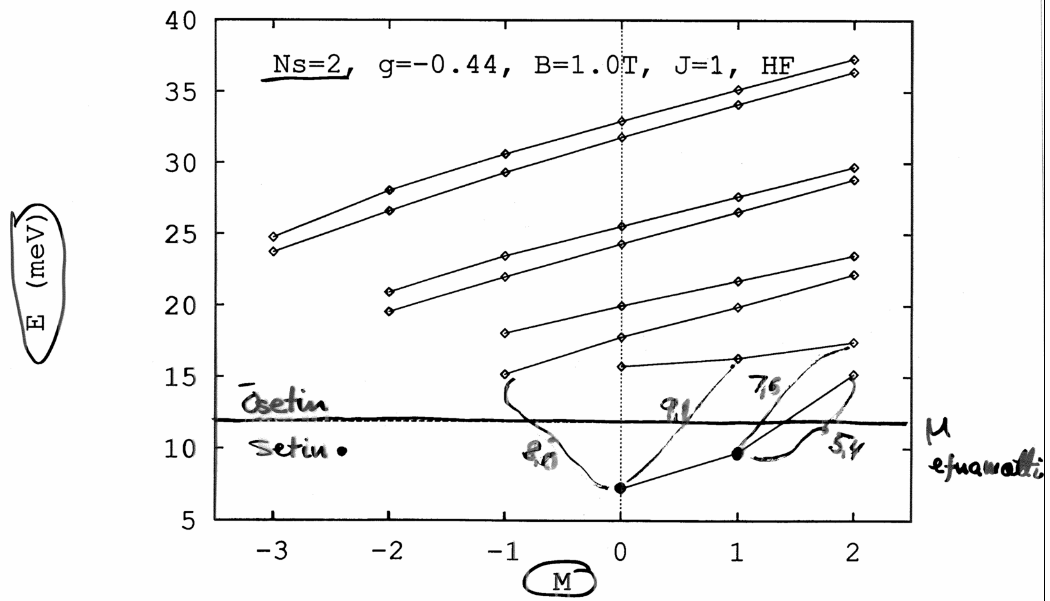
$$\boxed{\omega^2 = \underset{\uparrow\ \text{ljóshraði}}{c}k^2 + \Omega_{pe}^2 + \text{fasti} \cdot k^2 + \dots} \quad (42)$$

- möguleg rafsegulbylgja í efni án sterkrar dofnunar

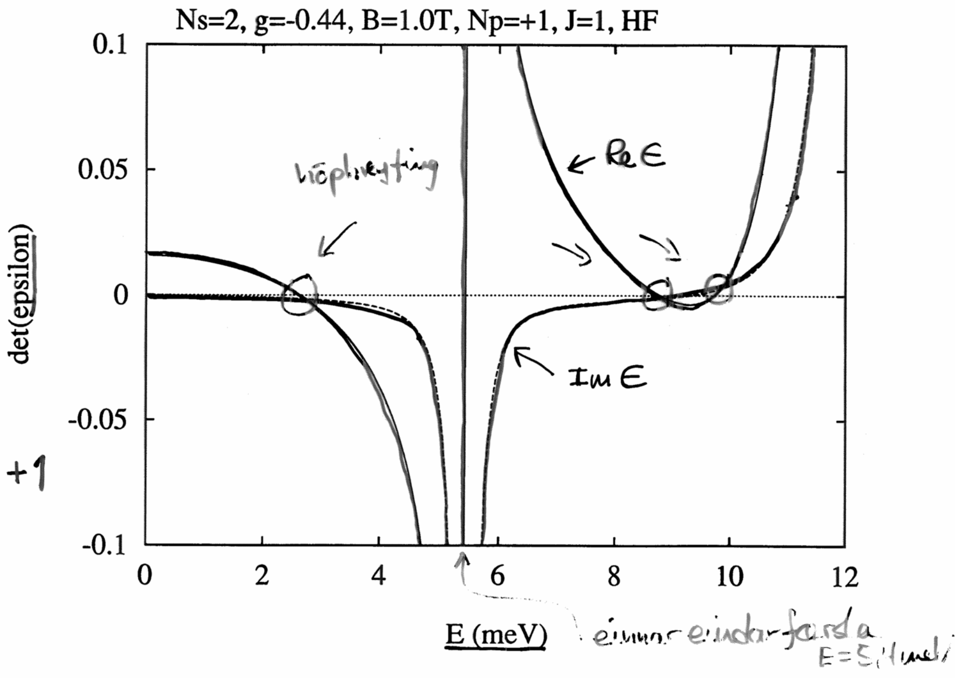
$\Updownarrow$

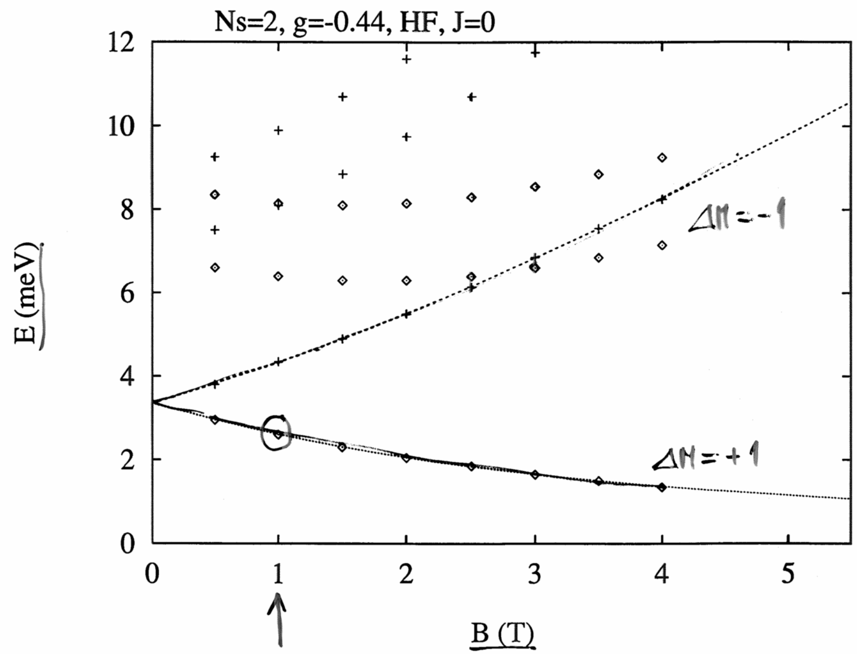
þvers rafgasbylgja

Örvunargeil

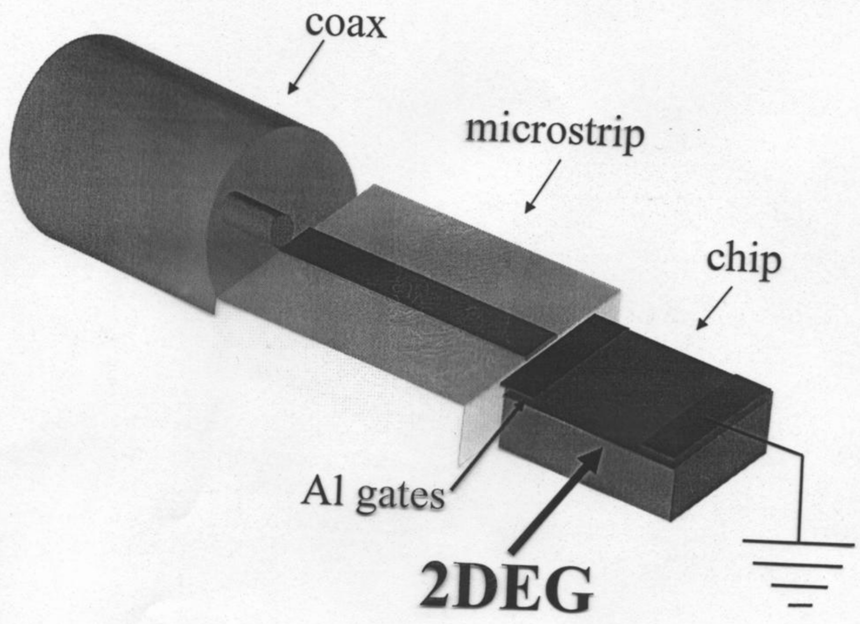

Ns=2, g=-0.44, B=1.0T, J=1, HF
E (meV)
M
Setin

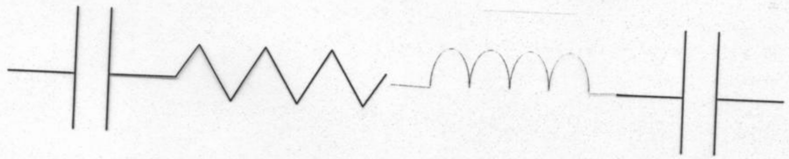
$\Delta M = +1$


Ns=2, g=-0.44, B=1.0T, Np=+1, J=1, HF
det(epsilon)
E (meV)
Re ∈
Im ∈



Measurement setup





$$\frac{V_{\text{reflected}}(\omega)}{V_{\text{incident}}(\omega)} = \frac{Z_{load}(\omega) - 50\,\Omega}{Z_{load}(\omega) + 50\,\Omega}$$

Peter John Burke

Ofurleiðni

Aðallega rafseguleiginleikar

Lýsing ofurleiðni

(1) Sum efni tapa öllu viðnámi fyrir neðan eitthvert T_c (markhitastig hvað efni)

Ef Ómslögmálið

$$\bar{J} = \sigma \bar{E}$$

gildir enn hér drögum við þá ályktun að $\bar{E} = 0$ innan ofurleiðara því segir (Maxwells jafnan)

$$\frac{\partial}{\partial t}\bar{B} = -\bar{\nabla}\times E$$

$\longrightarrow \partial_t \bar{B} = 0$ í ofurleiðara

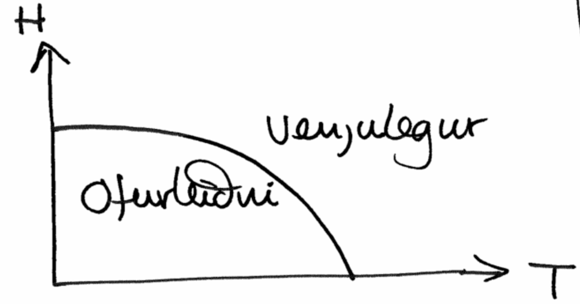
(2) Meissner-hrif

Þegar ofurleiðari er kældur niður fyrir T_c fara allar segulsviðslínur út fyrir hann

$\rightarrow$ í ofurleiðara er $\bar{B} = 0$

(3) Til er marksegulsvið H_c þ.a. ~~efni~~ ofurleiðari verður venjul. leiðari fyrir ofan það

$H = \frac{1}{\mu}B$ samsvarar D

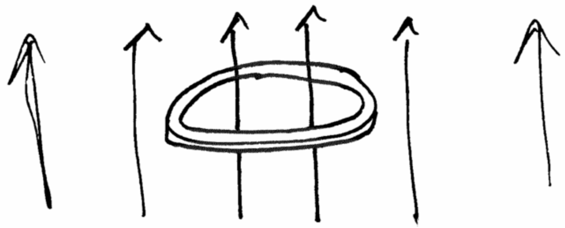


(4) Sístæðir ofurstraumar og flæðisskömmtun

Tökum hring í segulsviði

Lokkum T niður fyrir T_c

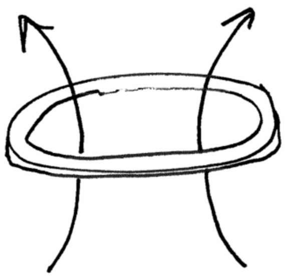
→ ofurleiðandi hringur



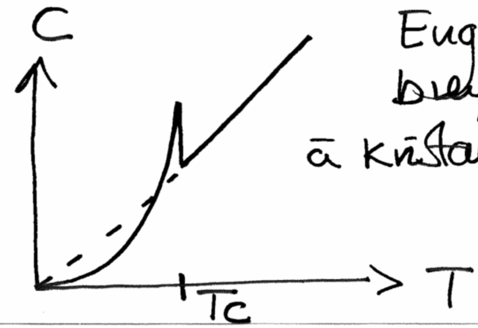
- slökkvum á segulsviðinu

Innan hringsins verður fangað flæði stammtað í flæðiseiningu

$$\phi_0 = \frac{h}{2e} \qquad (43)$$



(5) Varmarýmd



Engin breyting á kristallgrind

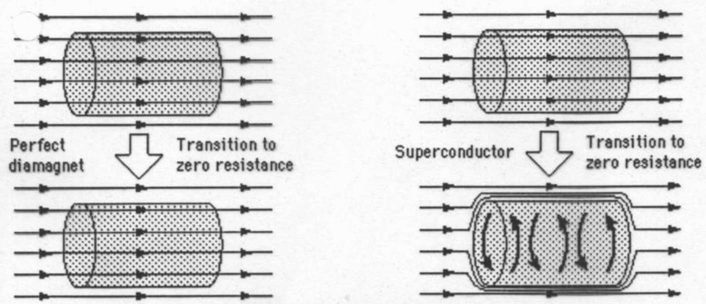
The Meissner Effect

Perfect Diamagnet

If a conductor already had a steady magnetic field through it and was then cooled through the transition to a zero resistance state, becoming a perfect diamagnet, the magnetic field would be expected to stay the same.

Superconductor

Remarkably, the magnetic behavior of a superconductor is distinct from perfect diamagnetism. It will actively exclude any magnetic field present when it makes the phase change to the superconducting state.



Magnetic levitation | Further discussion

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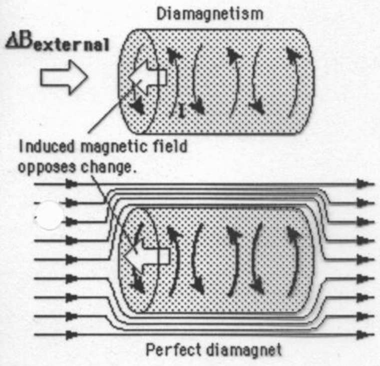
Perfect Diamagnetism



A conductor will oppose any change in externally applied magnetic field. Circulating currents will be induced to oppose the buildup of magnetic field in the conductor (Lenz's law). In a solid material, this is called diamagnetism, and a perfect conductor would be a perfect diamagnet. That is, induced currents in it would meet no resistance, so they would persist in whatever magnitude necessary to perfectly cancel the external field change. A superconductor is a perfect diamagnet, but there is more than this involved in the Meissner effect.

Illustrate mixed state

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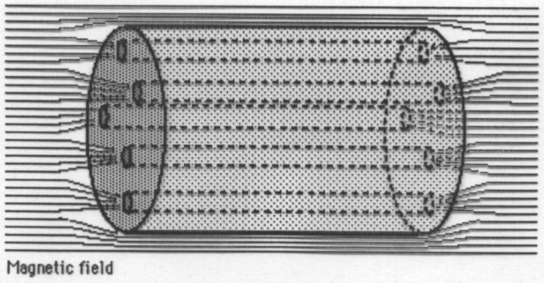
Mixed-State Meissner Effect

In Type II superconductors the magnetic field is not excluded completely, but is constrained in filaments within the material. These filaments are in the normal state, surrounded by supercurrents in what is called a vortex state. Such materials can be subjected to much higher external magnetic fields and remain superconducting.



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Spin Alignment vs Electron Pairs

The makers of superconducting magnets face a basic difficulty which Lindenfeld has put succinctly "magnetism and superconductivity are natural enemies". Macroscopic magnetization depends upon aligning the electron spins parallel to one another, while superconductivity depends upon pairs of electrons with their spins antiparallel. The Cooper pairs of electrons in the BCS theory have a very small binding energy, and external magnetic fields exert torques on the electron spins which tend to break up these pairs.

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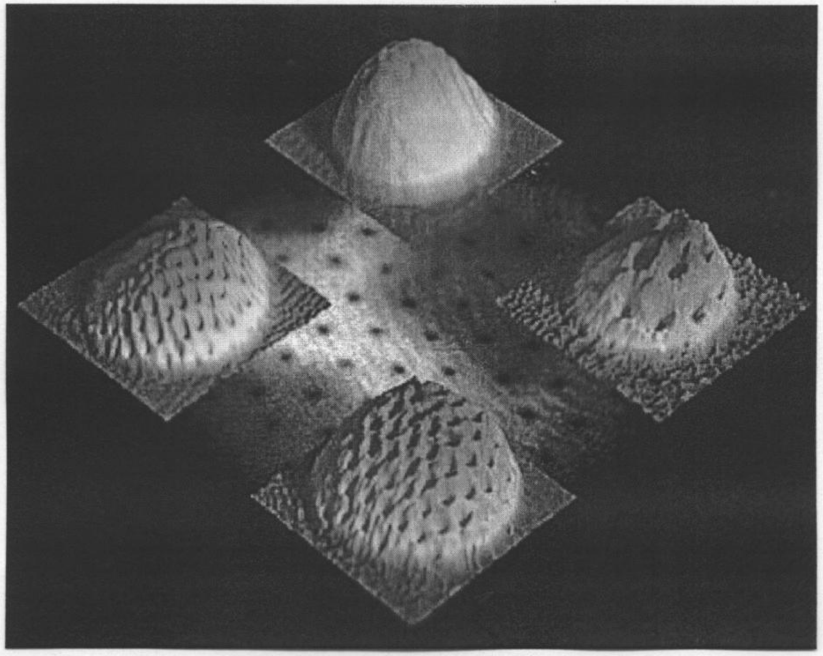
Bose-Einstein Vortices



The images show quantum vortices in a rotating condensate of sodium atoms. A condensate 60 micrometer in diameter and 250 micrometer in length was set in rotation by rotating laser beams. It then formed a regular lattice of vortices. The condensate was then allowed to ballistically expand which resulted in a twenty times magnification. The images represent two-dimensional cuts through the density distribution and show the density minima due to the vortex cores. The examples shown contain 0, 16, 70 and 130 vortices. The diameter of the cloud was about 1 mm.)

Figure courtesy of Todd Gustavson at MIT

Back to Physics News Graphics

Critical Temperature for Superconductors

The critical temperature for superconductors is the temperature at which the electrical resistivity of a metal drops to zero. The transition is so sudden and complete that it appears to be a transition to a different phase of matter; this superconducting phase is described by the BCS theory. Several materials exhibit superconducting phase transitions at low temperatures. The highest critical temperature was about 23 K until the discovery in 1986 of some high temperature superconductors.

Materials with critical temperatures in the range 120 K have received a great deal of attention because they can be maintained in the superconducting state with liquid nitrogen (77 K).

Material	T-Critical
Gallium	1.1 K
Aluminum	1.2 K
Indium	3.4 K
Tin	3.7 K
Mercury	4.2 K
Lead	7.2 K
Niobium	9.3 K
Niobium-Tin	17.9 K
La-Ba-Cu-oxide	30 K
Y-Ba-Cu-oxide	92 K
Tl-Ba-Cu-oxide	125 K

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Type I superconductors | Type II superconductors

Type I Superconductors

The thirty pure metals listed at right are called Type I superconductors. The identifying characteristics are zero electrical resistivity below a critical temperature, zero internal magnetic field (Meissner effect), and a critical magnetic field above which superconductivity ceases.

The superconductivity in Type I superconductors is modeled well by the BCS theory which relies upon electron pairs coupled by lattice vibration interactions. Remarkably, the best conductors at room temperature (gold, silver, and copper) do not become superconducting at all. They have the smallest lattice vibrations, so their behavior correlates well with the BCS Theory.

While instructive for understanding superconductivity, the Type I superconductors have been of limited practical usefulness because the critical magnetic fields are so small and the superconducting state disappears suddenly at that temperature. Type I superconductors are sometimes called "soft" superconductors while the Type II are "hard", maintaining the superconducting state to higher temperatures and magnetic fields.

Mat.	Tc
Be	0
Rh	0
W	0.015
Ir	0.1
Lu	0.1
Hf	0.1
Ru	0.5
Os	0.7
Mo	0.92
Zr	0.546
Cd	0.56
U	0.2
Ti	0.39
Zn	0.85
Ga	1.083

Mat.	Tc
Gd	1.1
Al	1.2
Pa	1.4
Th	1.4
Re	1.4
Tl	2.39
In	3.408
Sn	3.722
Hg	4.153
Ta	4.47
V	5.38
La	6.00
Pb	7.193
Tc	7.77
Nb	9.46

Type I superconductors on periodic table

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Superconductivity concepts

Type II Superconductors

Superconductors made from alloys are called Type II superconductors. Besides being mechanically harder than Type I superconductors, they exhibit much higher critical magnetic fields. Type II superconductors such as niobium-titanium (NbTi) are used in the construction of high field superconducting magnets.

Type-II superconductors usually exist in a mixed state of normal and superconducting regions. This is sometimes called a vortex state, because vortices of superconducting currents surround filaments or cores of normal material.

Material	Transition Temp (K)	Critical Field (T)
NbTi	10	15
PbMoS	14.4	6.0
V_3Ga	14.8	2.1
NbN	15.7	1.5
V_3Si	16.9	2.35
Nb_3Sn	18.0	24.5
Nb_3Al	18.7	32.4
$Nb_3(AlGe)$	20.7	44
Nb_3Ge	23.2	38

From Blatt, Modern Physics

New superconductor: magnesium diboride

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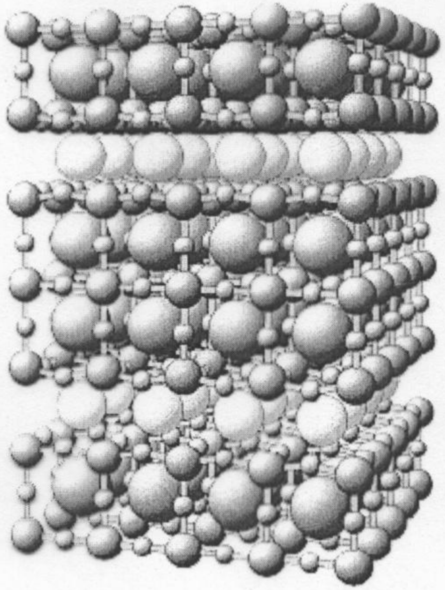
Superconductivity concepts

7004
2002)

27 February 2002

ctors Show Their Stripes

as, high-temperature superconductors appear
, distinguished by their stripes. Some
at electricity runs without resistance along
electric charge in these materials. That
gets a big boost now that a team has finally
es in the most widely studied of the so-called
ctor s reported in the 4 March print issue
hers' large crystal sample enabled them to
n neutron scattering data that support the
t separate a cuprate superconductor from its



K. Hermann/Fritz Haber Institute

Supercrystal. One theory claims that "stripes" of electric charges allow the planes of copper (green) and oxygen (blue) in $YBa_2Cu_3O_7$ to carry current without resistance at high temperatures. Now the stripes have been observed in this material.

nductors are something like multi-tiered dance
rges. They consist of parallel planes of copper
Within each plane the copper atoms arrange
are grid, with an oxygen atom sitting between
ring coppers. Between the planes lie atoms
nd some of these absorb electrons from the
ng positively charged "holes" behind.
hat these holes pair up to waltz without
planes. However, they aren't sure how the
nanage to cling together.

he h s first form long stripes in which it is
ugh the copper-oxygen terrain. A hole masks
e copper atom on which it sits. So as an
from one copper atom to the next, it appears
agnetism is jumping in the opposite direction.
quires lots of energy because it disrupts the
lown-up-down pattern of magnetic fields.
oles settle on a long stripe of several
, they form a runway along which there is no magnetism and no field pattern to disrupt. The holes still
t they don't wander from the stripe because it costs less energy to stay together and even to form
de along the runway without losing energy.

Electrical Resistivity Anisotropy from Self-Organized One Dimensionality in High-Temperature Superconductors

Yoichi Ando, Kouji Segawa, Seiki Komiya, and A. N. Lavrov
Central Research Institute of Electric Power Industry, Komae, Tokyo 201-8511, Japan
(Received 31 July 2001; published 19 March 2002)

We investigate the manifestation of stripes in the in-plane resistivity anisotropy in untwinned single crystals of $La_{2-x}Sr_xCuO_4$ ($x = 0.02-0.04$) and $YBa_2Cu_3O_y$ ($y = 6.35-7.0$). It is found that both systems show strongly temperature-dependent in-plane anisotropy in the lightly hole-doped region and that the anisotropy in $YBa_2Cu_3O_y$ *grows* with decreasing y below $\sim$6.60 despite the decreasing orthorhombicity, which gives most direct evidence that electrons self-organize into a macroscopically anisotropic state. The transport is found to be easier along the direction of the spin stripes already reported, demonstrating that the stripes are intrinsically conducting in cuprates.

DOI: 10.1103/PhysRevLett.88.137005 PACS numbers: 74.25.Fy, 74.25.Dw, 74.20.Mn, 74.72.Bk

Th echanism of the high-temperature superconduc-
vity is still not settled 15 years after its discovery, mostly
ecause it is unclear how best to describe the strongly cor-

conducting cuprates, evidence [2–7] is reasonably str
for *spin* stripes, but not at all conclusive for *charge* stri
Therefore, to really establish the charge stripe as a r

VOLUME 88, NUMBER 13 PHYSICAL REV

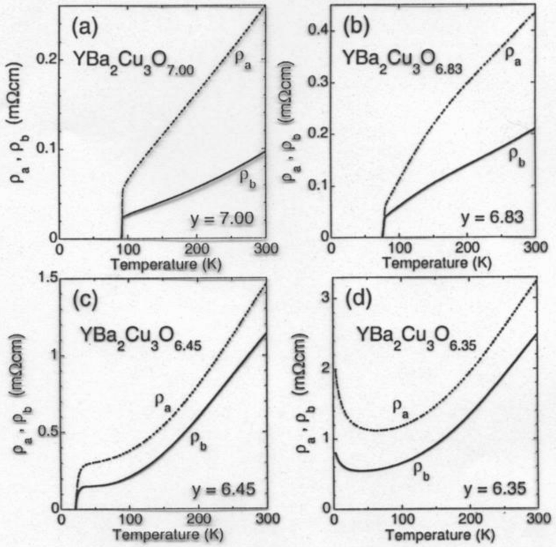


FIG. 3. Representative data sets of $\rho_a(T)$ and $\rho_b(T)$ for YBCO at selected y. The y values shown are 7.00 (a), 6.83 (b), 6.45 (c), and 6.35 (d). In nonsuperconducting samples at $y = 6.35$ (d), the anisotropy does not disappear even though the CuO chains are destroyed.

6 Samsætuhrif

skipt um samsætu í grind

$\rightarrow T_c \propto M^{-1/2}$

Jafna Londons $\Leftrightarrow$ ~~[illegible]~~ Smug dýpt

Ofurleiðari, gerum ráð fyrir

$$\bar{J}(\bar{r}t) = -n_s e \bar{v}(\bar{r}t)$$

samfelldnijafnan

$\rightarrow \quad \bar{\nabla}\cdot\bar{J} \Rightarrow \bar{\nabla}\cdot\bar{v} = 0$

Hreyfijafna

$$m\frac{d}{dt}\bar{v} = -e\left(\bar{E} + \bar{v}\times\bar{h}\right) \qquad (44)$$

meðalgildi segulsviðs á skala

$a_0 < L < \lambda_L \leftarrow$ kemur í ljós

umskrifum $\frac{d\bar{v}}{dt}$ (notum 38)

$$\frac{d\bar{v}}{dt} = \frac{\partial\bar{v}}{\partial t} + (\bar{v}\cdot\bar{\nabla})\bar{v} \qquad (45)$$

$$= \frac{\partial\bar{v}}{\partial t} + \bar{\nabla}\left(\frac{1}{2}v^2\right) - \bar{v}\times(\bar{\nabla}\times\bar{v})$$

---

44 og 45 gefa

$$\frac{\partial}{\partial t}\bar{v} + \frac{e\bar{E}}{m} + \bar{\nabla}\left(\frac{1}{2}v^2\right) = \bar{v}\times\left(\bar{\nabla}\times v - \frac{e\bar{h}}{m}\right) \qquad (46)$$

nú er notað

$$\bar{\nabla}\times\bar{E} = -\frac{\partial\bar{B}}{\partial t}$$

til þess að umskrifa $\bar{\nabla}\times$ (46) sem

$$\boxed{\frac{\partial\bar{Q}}{\partial t} = \bar{\nabla}\times\left(\bar{v}\times\bar{Q}\right)} \qquad (47)$$

með

$$\bar{Q} \equiv \left(\bar{\nabla}\times\bar{v} - \frac{e\bar{h}}{m}\right) \qquad (48)$$

Skoðum ofurleiðara í engu segulsviði, með $\bar{Q} = 0$

Af (47) má þá ráða að $\bar{Q}(t) = 0$ (jafnvel þó sviðar sé kveikt á sviði)

- Tilgátan

$$\boxed{\bar{Q} \equiv \left(\bar{\nabla}\times\bar{v} - \frac{e\bar{h}}{m}\right) = 0} \tag{49}$$

og því

$$\boxed{\frac{\partial \bar{v}}{\partial t} + \bar{\nabla}\left(\tfrac{1}{2}v^2\right) = -\frac{e\bar{E}}{m}}$$

hafa verið staðfestar sem réttar lýsingar á ofurleiðurum

$\boxed{\text{Jöfnur Londons}}$

Endurritum (49a) sem

$$\bar{h} = -\frac{m}{n_s e^2}\bar{\nabla}\times\bar{J}$$

saman með Maxwells jöfnunni

$$\bar{\nabla}\times\bar{B} = \mu_0\left(\bar{J} + \epsilon_0\frac{\partial \bar{E}}{\partial t}\right)$$

gefur hún

$$\bar{h} = -\frac{m}{n_s e^2}\bar{\nabla}\times\bar{J} = -\frac{m}{\mu_0 n_s e^2}\bar{\nabla}\times\bar{\nabla}\times\bar{h}$$

$$= \frac{m}{\mu_0 n_s e^2}\nabla^2 h \tag{50}$$

með lausn $h(z) = H_0 e^{-z/\lambda_L}$

og $\lambda_L = \left(\dfrac{m}{\mu_0 n_s e^2}\right)^{1/2}$

við slétt yfirborð ofurleiðara

λ_L : hversu langt segulsviðið smýgur inn í ofurleiðara

London gat einnig með jöfnu sinni sannað að

- <u>segulflæði í gegnum ofurleiðandi hring</u>

$\boxed{\text{sé fasti óháður tíma!}}$

- Með skammtafræði var síðan hægt að sýna að gildi flæðisins var skammtað

Hér er sagan rétt að byrja og verða spennandi, en hér hættum við