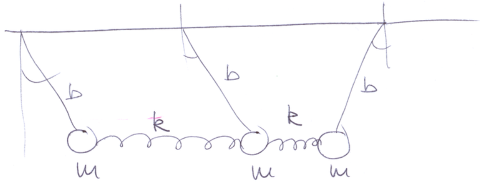
12-26




$k = 0{,}2\ \frac{N}{m}$

$m = 0{,}25\ \mathrm{kg}$

$b = 0{,}47\ m$

$$T = \frac{m}{2}\sum_{i=1}^{3}(b\dot{\theta}_i)^2$$

$$U = mgb\left\{(1-\cos\theta_1)+(1-\cos\theta_2)+(1-\cos\theta_3)\right\} + \frac{kb^2}{2}\left[(\sin\theta_2-\sin\theta_1)^2+(\sin\theta_3-\sin\theta_2)^2\right]$$

$$\simeq \frac{mgb}{2}\left\{\theta_1^2+\theta_2^2+\theta_3^2\right\} + \frac{kb^2}{2}\left\{\theta_1^2+\theta_3^2+2\theta_2^2-2\theta_1\theta_2-2\theta_2\theta_3\right\}$$

$$M = \frac{mb^2}{2}\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}$$

$$A = \frac{1}{2}\begin{pmatrix} mgb+kb^2 & -kb^2 & 0\\ -kb^2 & mgb+2kb^2 & -kb^2 \\ 0 & -kb^2 & mgb+kb^2\end{pmatrix}$$

Þurfum að leysa eigingildisverkefnið

$$A\bar{a} = \omega^2 M\bar{a}$$

Hægt er að láta þættina $\frac{1}{2}$ í M og A falla niður

M er á hornalínuformi, því er eigingildisverkefnið venjulegt

$$A = \frac{1}{2}mgb\begin{pmatrix}1+\epsilon & -\epsilon & 0\\ -\epsilon & 1+2\epsilon & -\epsilon\\ 0 & -\epsilon & 1+\epsilon\end{pmatrix}, \quad \epsilon = \frac{kb^2}{mgb} = \frac{kb}{mg}$$

$$\rightarrow \omega_i^2 = \begin{cases}\frac{mgb}{mb^2}\{1+3\epsilon\} & i=1\\ \frac{mgb}{mb^2}\{1\} & i=2\\ \frac{mgb}{mb^2}\{1+\epsilon\} & i=3\end{cases}$$

$$\rightarrow \omega_1 = \sqrt{\frac{g}{b}+\frac{3k}{m}} \qquad \omega_3 = \sqrt{\frac{g}{b}+\frac{k}{m}}$$

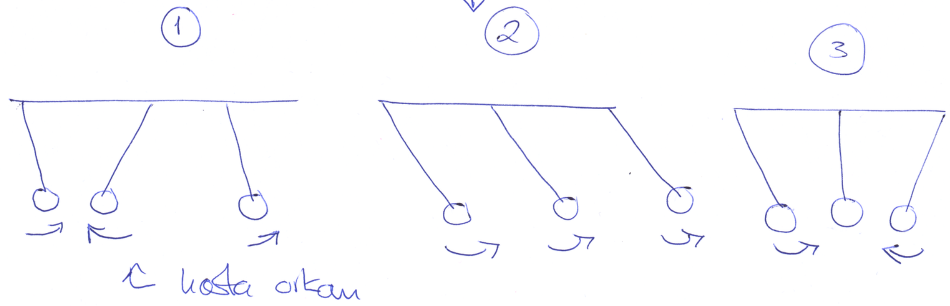
$$\omega_2 = \sqrt{\frac{g}{b}}$$

Eiginvigrar eru

$$\bar{a} = \begin{cases}\frac{1}{\sqrt{6}}(\theta_1, -2\theta_2, \theta_3) & i=1\\ \frac{1}{\sqrt{3}}(\theta_1, \theta_2, \theta_3) & i=2\\ \frac{1}{\sqrt{2}}(\theta_1, 0, -\theta_3) & i=3\end{cases}$$

með sveifluhætti

lægst orkan, minnst víxlverkun



12-19 Þrír pendúlar með

$$U = \frac{1}{2}\left\{ \theta_1^2 + \theta_2^2 + \theta_3^2 - 2\epsilon_{12}\theta_1\theta_2 - 2\epsilon_{13}\theta_1\theta_3 - 2\epsilon_{23}\theta_2\theta_3 \right\}$$

Öll $\epsilon_{ij}$ mismunandi

Ekki bara nágrannar víxlverkast eins í dæminu að undan

Setjum

$$T = \frac{1}{2}\left\{ \dot{\theta}_1^2 + \dot{\theta}_2^2 + \dot{\theta}_3^2 \right\} \Rightarrow M = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

og

$$A = \begin{Bmatrix} 1 & -\epsilon_{12} & -\epsilon_{13} \\ -\epsilon_{12} & 1 & -\epsilon_{23} \\ -\epsilon_{13} & -\epsilon_{23} & 1 \end{Bmatrix}$$

Sýna að margfeldni verði aðeins ef öll $\epsilon_{ij}$ eru þau sömu



Eigingildin má finna með wxmaxima, en þau eru flókin. Reynum því

$$\left\{ A - \omega^2 M \right\} \bar{a} = 0$$

línulegar óháðar jöfnur $\rightarrow \det\left\{ A - \omega^2 M \right\} = 0$

$$\rightarrow \begin{vmatrix} 1-\omega^2 & -\epsilon_{12} & -\epsilon_{13} \\ -\epsilon_{12} & 1-\omega^2 & -\epsilon_{23} \\ -\epsilon_{13} & -\epsilon_{23} & 1-\omega^2 \end{vmatrix} = 0$$

$$\rightarrow (1-\omega^2)^3 - (1-\omega^2)(\epsilon_{12}^2 + \epsilon_{13}^2 + \epsilon_{23}^2) - 2\epsilon_{12}\epsilon_{13}\epsilon_{23} = 0$$

Setjum $1-\omega^2 = X$



$$X^3 - X \cdot B - C = 0$$

$$\Delta = -4B^3 - 27C^2 = 4(\epsilon_{12}^2 + \epsilon_{13}^2 + \epsilon_{23}^2)^3 - 108(\epsilon_{12}\epsilon_{13}\epsilon_{23})^2$$

Ef $\Delta > 0$ þá eru þrjár mismunandi rætur

$$(\epsilon_{12}^2 + \epsilon_{13}^2 + \epsilon_{23}^2)^3 - 27(\epsilon_{12}\epsilon_{13}\epsilon_{23})^2 > 0$$

Ef $\Delta = 0$ þá er ein rót tvöföld

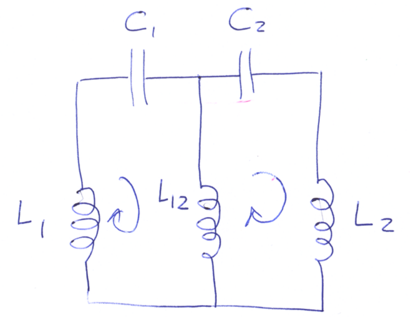
$\rightarrow$ ef

$$(\epsilon_{12}^2 + \epsilon_{13}^2 + \epsilon_{23}^2)^3 = 27(\epsilon_{12}\epsilon_{13}\epsilon_{23})^2$$

gerist aðeins ~~ef~~ $\epsilon_{12} = \epsilon_{13} = \epsilon_{23}$



12-13




Lögmál Faradays

$$\frac{q}{C} = -L\frac{di}{dt}$$

(ekki Kirchhoffs)

$$L_1\frac{di_1}{dt} + \frac{q_1}{C_1} + L_{12}\left(\frac{di_1}{dt} - \frac{di_2}{dt}\right) = 0$$

$$L_2\frac{di_2}{dt} + \frac{q_2}{C_2} + L_{12}\left(\frac{di_2}{dt} - \frac{di_1}{dt}\right) = 0$$

Tvær rásir með einfaldri hreintóna sveiflu tengdar saman

Notum $\dot q = i$ $\left(\frac{dq}{dt} = i\right)$ og tökum tíma afleiðu

$$L_1 \frac{d^2 i_1}{dt^2} + \frac{i_1}{C_1} + L_{12}\left(\frac{d^2 i_1}{dt^2} - \frac{d^2 i_2}{dt^2}\right) = 0$$

$$L_2 \frac{d^2 i_2}{dt^2} + \frac{i_2}{C_2} + L_{12}\left(\frac{d^2 i_2}{dt^2} - \frac{d^2 i_1}{dt^2}\right) = 0$$

umritum

$$\{L_1 + L_{12}\}\frac{d^2 i_1}{dt^2} + \frac{i_1}{C_1} - L_{12}\frac{d^2 i_2}{dt^2} = 0$$

$$\{L_2 + L_{12}\}\frac{d^2 i_2}{dt^2} + \frac{i_2}{C_2} - L_{12}\frac{d^2 i_1}{dt^2} = 0$$

Gerum ráð fyrir lausnum

$$i_1(t) = Ae^{i\omega t}, \quad i_2(t) = Be^{i\omega t}$$



Þá fæst

$$\left[\omega^2\{L_1 + L_{12}\} - \frac{1}{C_1}\right]A - L_{12}\omega^2 B = 0$$

$$\left[\omega^2\{L_2 + L_{12}\} - \frac{1}{C_2}\right]B - L_{12}\omega^2 A = 0$$

$$\begin{pmatrix} \omega^2\{L_1+L_{12}\} - \frac{1}{C_1} & -L_{12}\omega^2 \\ -L_{12}\omega^2 & \omega^2\{L_2+L_{12}\} - \frac{1}{C_2} \end{pmatrix}\begin{pmatrix} A \\ B \end{pmatrix} = 0$$

óhliðruð línuleg jafna → ákveða fylkisins verður að hverfa



$$\left\{\omega^2[L_1+L_{12}] - \frac{1}{C_1}\right\}\left\{\omega^2[L_2+L_{12}] - \frac{1}{C_2}\right\} - L_{12}^2\omega^4 = 0$$

$$\to \omega^2 = \frac{(L_1+L_{12})C_1 + (L_2+L_{12})C_2 \pm \sqrt{\left[(L_1+L_{12})C_1 - (L_2-L_{12})C_2\right]^2 - 4L_{12}^2 C_1 C_2}}{2C_1C_2\left[(L_1+L_{12})(L_2+L_{12}) - L_{12}^2\right]}$$

ef $L_{12} \to 0$, og $L_1 = L_2 = L$, $C_1 = C_2 = C$

$$\to \omega^2 = \frac{1}{LC}$$

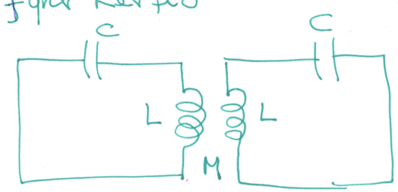


12-12 Höfum

$$L\ddot I_1 + \frac{I_1}{C} + M\ddot I_2 = 0$$

$$L\ddot I_2 + \frac{I_2}{C} + M\ddot I_1 = 0$$

fyrir kerfið




notum þessa jöfnu og eins . . . .

$$L\ddot I_1 + \frac{I_1}{C} + M\left\{-\frac{I_2}{LC} - \frac{M}{L}\ddot I_1\right\} = 0$$

$$\to \left\{L - \frac{M^2}{L}\right\}\ddot I_1 + \frac{I_1}{C} - \frac{M}{LC}I_2 = 0$$

einsfæst

$$\left\{L - \frac{M^2}{L}\right\}\ddot I_2 + \frac{I_2}{C} - \frac{M}{LC}I_1 = 0$$

tengingin er núna ekki um lið með $\ddot I_i$

(13)

Berum saman við (12.1) og (12.8) fyrir 2 massa tengda gorm


Setjum

$$m = L - \frac{M^2}{L}, \quad k_{12} = \frac{M}{LC}, \quad k = \frac{1}{C}\left(1 - \frac{M}{L}\right)$$

þá fæst

$$m\ddot{I}_1 + (k+k_{12})I_1 - k_{12}I_2 = 0$$
$$m\ddot{I}_2 + (k+k_{12})I_2 - k_{12}I_1 = 0$$

og því

$$\omega_1 = \sqrt{\frac{k+2k_{12}}{m}} = \sqrt{\frac{1+\frac{M}{L}}{C\left(L-\frac{M^2}{L}\right)}} = \sqrt{\frac{1}{C(L-M)}}$$

$$\omega_2 = \sqrt{\frac{k}{m}} = \sqrt{\frac{1-\frac{M}{L}}{C\left(L-\frac{M^2}{L}\right)}} = \sqrt{\frac{1}{C(L+M)}}$$

(14)

12-21 þrír sveiflar tengdir þ.a.

$$U = \frac{1}{2}\left\{K_1(x_1^2 + x_3^2) + K_2x_2^2 + K_3(x_1x_2 + x_2x_3)\right\}$$

$$K_3 = \sqrt{2K_1K_2}$$

$$M = \frac{1}{2}\begin{pmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & m \end{pmatrix} \qquad A = \frac{1}{2}\begin{pmatrix} K_1 & \frac{K_3}{2} & 0 \\ \frac{K_3}{2} & K_2 & \frac{K_3}{2} \\ 0 & \frac{K_3}{2} & K_1 \end{pmatrix}$$

$$\omega_1^2 = -\frac{\sqrt{2(2K_1K_2) + K_2^2 - 2K_1K_2 + K_1^2} - K_1 - K_2}{2m} = 0$$

$$\omega_2^2 = \frac{\sqrt{2(2K_1K_2) + K_2^2 - 2K_1K_2 + K_1^2} + K_2 + K_1}{2m} = \frac{K_2 + K_1}{m}$$

$$\omega_3^2 = \frac{K_1}{m}$$

(15)

Við fáum því eigintíðnirnar og normalsveifluháttina

$$\omega_1 = 0 \quad \longleftrightarrow \quad \bar{\eta}_1 \sim \left(1, -\tfrac{1}{2}\sqrt{\tfrac{K_1}{K_2}}, 1\right)$$

$$\omega_2 = \sqrt{\frac{K_2+K_1}{m}} \quad \longleftrightarrow \quad \bar{\eta}_2 \sim \left(1, +\tfrac{1}{2}\sqrt{\tfrac{K_2}{K_1}}, 1\right)$$

$$\omega_3 = \sqrt{\frac{K_1}{m}} \quad \longleftrightarrow \quad \bar{\eta}_3 \sim (1, 0, -1)$$

$\omega_1 \to \ddot{\bar{\eta}}_1 = 0$ með lausn $\bar{\eta}_1(t) = at + b$

því $\ddot{\bar{\eta}}_i + \omega_i \bar{\eta} = 0$ eða öfugt

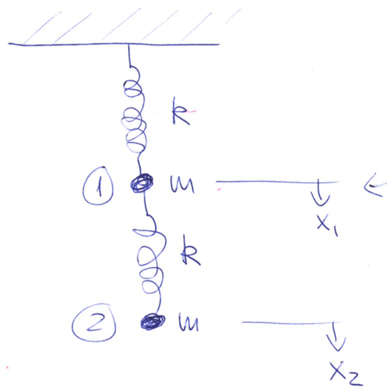
þessu fylgir líka $\bar{\nabla} U = \left(K_1x_1 + K_3x_3,\ K_2x_2 + K_3(x_1+x_3),\ K_1x_3 + K_3x_2\right)$

og $\bar{\nabla} U\Big|_{(x,y,z) = \alpha\cdot\bar{\eta}_1} = (0,0,0)$ fyrir öll $\alpha \in \mathbb{R}$

leiðir til sömu útkomu sem fannst til að finna eigingildin

(16)

12-07




þyngdarkrafturinn er inni í jafnvægis-stöðu massanna

$$m\ddot{x}_1 = -kx_1 + k(x_2 - x_1) = -2kx_1 + kx_2$$
$$m\ddot{x}_2 = -k(x_2 - x_1) = -kx_2 + kx_1$$

Reynum lausn $x_1(t) = Ae^{i\omega t}$, $x_2(t) = Be^{i\omega t}$

(17)

$$\rightarrow \quad -m\omega^2 A + 2kA - kB = 0$$
$$-m\omega^2 B + kB - kA = 0$$

$$\begin{pmatrix} -m\omega^2 + 2k & -k \\ -k & -m\omega^2 + k \end{pmatrix} \begin{pmatrix} A \\ B \end{pmatrix} = 0$$

$$\rightarrow \quad (-m\omega^2 + 2k)(-m\omega^2 + k) - k^2 = 0$$

$$m^2\omega^4 - 2km\omega^2 - km\omega^2 + 2k^2 - k^2 = 0$$

$$m^2\omega^4 - 3km\omega^2 + k^2 = 0$$

(18)

$$\omega^4 - \frac{3k}{m}\omega^2 + \left(\frac{k}{m}\right)^2 = 0$$

$$\omega_{12}^2 = \left\{ \frac{3}{2} \pm \frac{\sqrt{5}}{2} \right\} \frac{k}{m}$$

$$\omega_1 = \sqrt{\left(\frac{3}{2} - \frac{\sqrt{5}}{2}\right)\frac{k}{m}}, \quad \omega_2 = \sqrt{\left(\frac{3}{2} + \frac{\sqrt{5}}{2}\right)\frac{k}{m}}$$

$$a_1 \sim \left(1, \frac{1+\sqrt{5}}{2}\right)$$

$$a_2 \sim \left(1, \frac{1-\sqrt{5}}{2}\right)$$

Ef $x_2$ er fast, $x_2 = 0$

$$\rightarrow \quad m\ddot{x}_1 = -2kx_1$$

$$\rightarrow \quad \omega_1^0 = \sqrt{\frac{2k}{m}}$$

(19)

Ef $x_1$ er fast, $x_1 = 0$

$$\rightarrow \quad m\ddot{x}_2 = -kx_2$$

$$\rightarrow \quad \omega_2^0 = \sqrt{\frac{k}{m}}$$

Berum saman

gullinsnið....

$$\omega_1 = \sqrt{\left(\frac{3}{2} - \frac{\sqrt{5}}{2}\right)\frac{k}{m}} \simeq 0{,}618 \cdot \sqrt{\frac{k}{m}} \qquad \omega_1^0 \simeq 1{,}414 \cdot \sqrt{\frac{k}{m}}$$

$$\omega_2 = \sqrt{\left(\frac{3}{2} + \frac{\sqrt{5}}{2}\right)\frac{k}{m}} \simeq 1{,}618 \cdot \sqrt{\frac{k}{m}} \qquad \omega_2^0 = \sqrt{\frac{k}{m}}$$

Tíðnir óvíxlverkandi kerfisins finnast ekki í víxlverkandi kerfinu, víxlverkun hliðrar eigintíðnum

(20)

Eigin vigrar víxlverkandi kerfis

$a_1 \sim (1, 1{,}618)$ – samhverfur sveifluháttur

$a_2 \sim (1, -0{,}618)$ andsamhverfur sveifluháttur

hærri orka vegna $\rightarrow$
hreyfingar miðju gorms