(10-12) Á breiddargráðu λ reikna frávik lóðlínu frá „lóðréttu" skilgreindu án snúnings (1)

Sýna að

$$\epsilon = \frac{R\omega^2 \sin\lambda \cos\lambda}{g_0 - R\omega^2 \cos^2\lambda}$$

Notum (10.32)

$$\bar F_{eff} = \bar S + m\bar g_0 - m\bar\omega \times (\bar\omega \times \bar r) - 2m\bar\omega \times \bar v_r$$

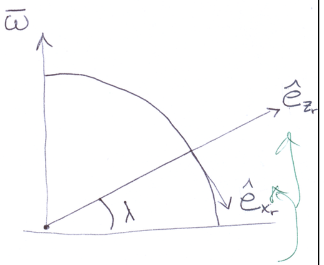
Setjum

$\bar S = 0$ aðrir kraftar

$\bar v_r = 0$ enginn hraði lóðs

$\bar r = (0,0,R)$ geisli jarðar

$\bar\omega = (-\omega\cos\lambda, 0, \omega\sin\lambda) \leftarrow \bar\omega$ í local hnitunum r



(2)

$\bar g_0 = (0,0,-g_0)$

því fæst að

$$\bar\omega \times \bar r = (-\omega\cos\lambda, 0, \omega\sin\lambda) \times (0,0,R) = \omega R\cos\lambda \cdot \hat e_{y_r}$$

$$-m\bar\omega \times (\bar\omega \times \bar r) = -m(-\omega\cos\lambda, 0, \omega\sin\lambda) \times (0,1,0)\,\omega R\cos\lambda = m\omega^2 R\left\{\sin\lambda\cos\lambda \cdot \hat e_{x_r} + \cos^2\lambda \cdot \hat e_{z_r}\right\}$$

og því

$$\bar F_{eff} = -mg_0\hat e_{z_r} + m\omega^2 R\left\{\sin\lambda\cos\lambda\,\hat e_{x_r} + \cos^2\lambda\,\hat e_{z_r}\right\}$$

Sem hefur bæði $\hat e_{x_r}$ og $\hat e_{z_r}$ þátt

(3)

því gildir um frávikið að

$$\tan\epsilon = \frac{|(F_{eff})_{x_r}|}{|(F_{eff})_{z_r}|} = \frac{\omega^2 R\sin\lambda\cos\lambda}{g_0 - \omega^2 R\cos^2\lambda}$$

$$= \frac{\sin\lambda\cos\lambda}{\frac{g_0}{\omega^2 R} - \cos^2\lambda}$$

$$\frac{g_0}{\omega^2 R} = \frac{9.81\ m/s^2}{(7.3\cdot 10^{-5}\,\frac{1}{s})^2\ 6.4\cdot 10^5 m} \sim 2.9\cdot 10^3$$

$$\to \epsilon \approx \frac{\sin\lambda\cos\lambda}{\frac{g_0}{\omega^2 R} - \cos^2\lambda} \approx \frac{\sin\lambda\cos\lambda}{\frac{g_0}{\omega^2 R}} = \frac{\omega^2 R}{g_0\, 2}\sin(2\lambda)$$

(10-03) μ_s , $\bar r' = \bar R + \bar r$ (4)

upphaf snúningskerfis

snúnings kerfi

fast kerfi

í fasta kerfinu

Jafna (10.25) gefur

$$\bar F_{eff} = \bar F - m\ddot{\bar R}_f - m\dot{\bar\omega}\times\bar r - m\bar\omega\times(\bar\omega\times\bar r) - 2m\bar\omega\times\bar v_r$$

↑ í snúningskerfinu

Einu kraftarnir eru miðsóknar og viðnáms

Hversu langt frá miðju getur pökkurinn legið án þess að renna til

$$\to \mu_s mg = m\omega^2 r \quad \to \quad r = \frac{\mu_s g}{\omega^2}$$

10-05

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Eins og nevnt er í Ex. 10.2 í bókini

$$\bar{F}_{eff} = m\bar{\omega}\times(\bar{\omega}\times\bar{r}) - 2m\bar{\omega}\times\bar{v}_r$$

$$\bar{a}_{eff} = -\bar{\omega}\times(\bar{\omega}\times\bar{r}) - 2\bar{\omega}\times\bar{v}_r$$

$$\bar{\omega} = \omega\hat{e}_z \qquad \bar{\omega} = (0,0,\omega)$$

$$\bar{r} = (x,y,0)$$

$$\bar{v}_r = (\dot{x},\dot{y},\dot{z})$$

$$\rightarrow \quad \bar{a}_{eff} = (\ddot{x},\ddot{y},0) = \omega^2(x,y,0) - 2\omega(\dot{x},-\dot{y},0)$$

$$\rightarrow \quad \boxed{\begin{aligned}\ddot{x} &= \omega^2 x + 2\omega\dot{y}\\ \ddot{y} &= \omega^2 y - 2\omega\dot{x}\end{aligned}}$$

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Umskrivum sum

$$y_1 = x \rightarrow \dot{y}_1 = \dot{x} = y_2$$

$$y_2 = \dot{x} \rightarrow \dot{y}_2 = \ddot{x} = \omega^2 x + 2\omega\dot{y} = \omega^2 y_1 + 2\omega y_4$$

$$y_3 = y \rightarrow \dot{y}_3 = \dot{y} = y_4$$

$$y_4 = \dot{y} \rightarrow \dot{y}_4 = \ddot{y} = \omega^2 y - 2\omega\dot{x} = \omega^2 y_3 - 2\omega y_2$$

Hneppið er tí

$$\boxed{\begin{aligned}\dot{y}_1 &= y_2\\ \dot{y}_2 &= \omega^2 y_1 + 2\omega y_4\\ \dot{y}_3 &= y_4\\ \dot{y}_4 &= \omega^2 y_3 - 2\omega y_2\end{aligned}}$$

upphafsskilyrði

$$y_1 = -0{,}5\,m$$

$$y_3 = 0\ m$$

$$y_2 = \frac{v_0}{\sqrt{2}}\ \frac{m}{s}$$

$$y_4 = \frac{v_0}{\sqrt{2}}\ \frac{m}{s}$$

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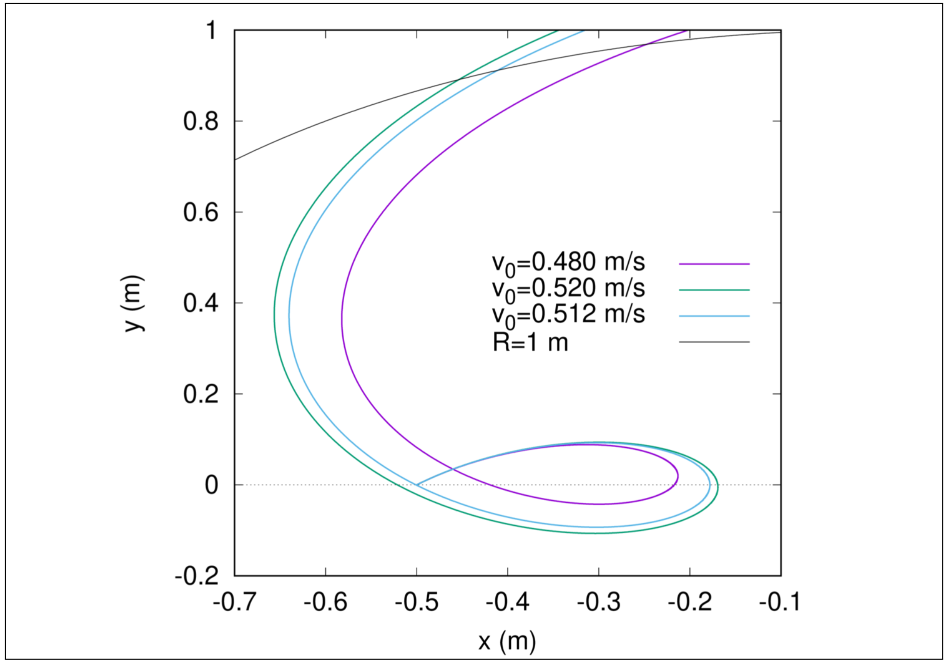
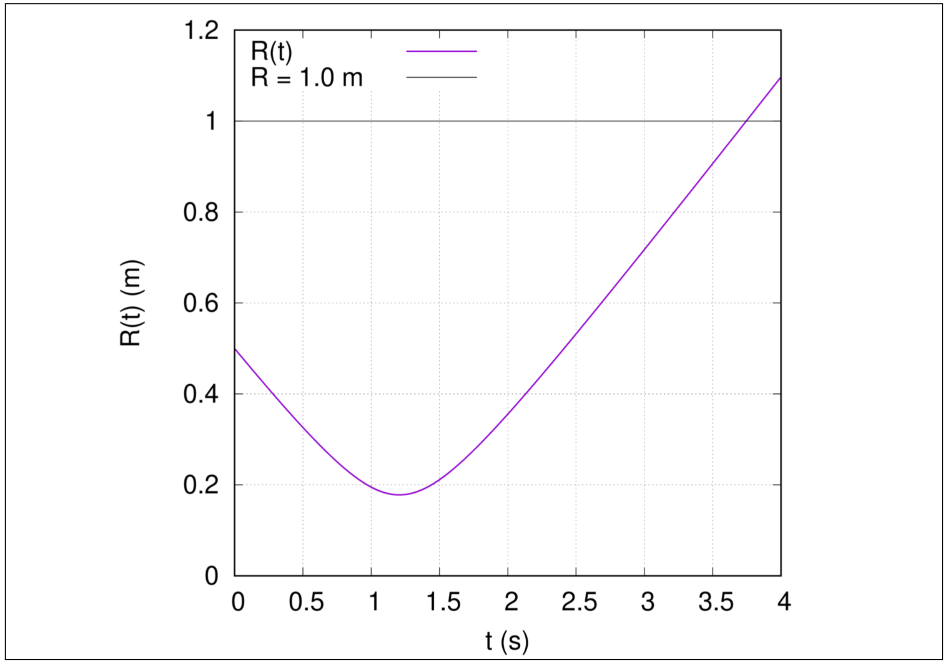
samkvæmt Ex 10.2 þarf at reyna v_0 á bilið

$$0{,}47\ \frac{m}{s} < v_0 < 0{,}53\ \frac{m}{s}, \quad \omega = 1\ \frac{rad}{s}, \quad R = 1\,m$$

Viljum finna v_0 sum fær pøkkin til at koma aftur í gjøgnum upphafspunktið, og tíman þar til hann fer út af

Sjá graf á tveimur næstu síðum





10-15 10

Höfum fasta kerfið með hnit x_f, og snúningskerfið með x_r

Gerum ráð fyrir að upphafspunktar kerfanna falli saman

$$\longrightarrow \quad \bar{v}_f = \bar{v}_r + \bar{\omega}\times\bar{r}_r$$

og í fasta kerfinu er

$$L = \frac{m}{2} v_f^2 - U(r_f)$$

með breyttum hnitum fæst

$$L = \frac{m}{2}\left\{ (\bar{v}_r + \bar{\omega}\times\bar{r}_r)^2 \right\} - U(r_r)$$

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$$\longrightarrow \quad L = \frac{m}{2}\left\{ v_r^2 + 2\,\bar{v}_r\cdot(\bar{\omega}\times\bar{r}_r) + (\bar{\omega}\times\bar{r}_r)^2 \right\} - U(r_r)$$

$\bar{p}_r$ verðum við að skilgreina sem

$$\bar{p}_r = \frac{\partial L}{\partial \bar{v}_r} = m\bar{v}_r + m(\bar{\omega}\times\bar{r}_r)$$

og fall Hamiltons verður

$$H = \bar{v}_r\cdot\bar{p}_r - L = \frac{m}{2}v_r^2 - \frac{m}{2}(\bar{\omega}\times\bar{r}_r)^2 - U(r_r)$$

H er ekki fall af t, ekki heldur L

$$\longrightarrow \quad \frac{\partial H}{\partial t} = 0 \qquad \frac{\partial L}{\partial t} = 0$$

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En hnitin $\bar{r}_f$ og $\bar{r}_r$ tengjast á tímaháðan hátt

$$\longrightarrow \quad H \neq E$$

Skoðum liðinn

$$U_c = -\frac{m}{2}(\bar{\omega}\times\bar{r}_r)^2$$

$$-\nabla_r U_c = \frac{m}{2}\nabla_r\left\{ \omega^2 r_r^2 - (\bar{\omega}\cdot\bar{r}_r)^2 \right\}$$

$$= m\left\{ \omega^2 \bar{r}_r - (\bar{\omega}\cdot\bar{r}_r)\bar{\omega} \right\} =$$

$$= \underbrace{-m\bar{\omega}\times(\bar{\omega}\times\bar{r}_r)}_{\text{miðsóknarkrafturinn}}$$

$(a\times b)\cdot(c\times d)$
$= (a\cdot c)(b\cdot d) - (a\cdot d)(b\cdot c)$

$(a\times b)\cdot(a\times b)$
$= (a^2 b^2 - (a\cdot b)(a\cdot b))$

$\bar{\nabla}(\bar{a}\cdot\bar{b})$
$= (\bar{a}\cdot\bar{\nabla})\bar{b} + (\bar{b}\cdot\bar{\nabla})a$
$+ \bar{a}\times(\bar{\nabla}\times\bar{b})$
$+ \bar{b}\times(\bar{\nabla}\times\bar{a})$

$\bar{\nabla}\times\bar{r} = 0$

$\bar{a}\times(\bar{b}\times\bar{c})$
$= (\bar{a}\cdot\bar{c})b - (\bar{a}\cdot\bar{b})\bar{c}$

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10-22 Við $\lambda = 42^\circ N$ fellur blý $h = 27m$

reikna frávik frá lóðréttu ....

Coriolis hröðun $\bar{a}_c = 2\bar{v} \times \bar{\omega}$

$\to$ frávik í austur með $|\bar{a}_c| = a_c = 2v\omega\cos\lambda$

$$\to v_c(t) = \int_0^t a_c \, dt' = \int_0^t dt' \, 2v(t')\omega\cos\lambda$$

$$= 2\omega\cos\lambda \int_0^t dt' \, (gt) = \omega\cos\lambda \cdot gt^2$$

$$\to x_c(t) = \int_0^t dt' \, v_c(t') = \omega\cos\lambda \, \frac{gt^3}{3}$$

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Falltími blýsins er $t = \sqrt{\frac{2h}{g}}$

$$\to x_c(t) = \omega\cos\lambda \, \frac{g}{3}\left(\frac{2h}{g}\right)^{3/2}$$

$$= \omega\cos\lambda \cdot \frac{1}{3}\sqrt{\frac{8h^3}{g}}$$

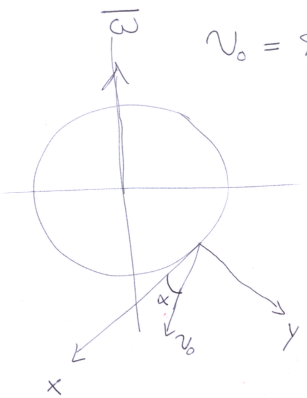
$$= \left(7.3 \cdot 10^{-5}\frac{1}{s}\right)\cos(42^\circ)\sqrt{\frac{8(27m)^3}{9{,}81\, m/s^2}}$$

$$\approx 0{,}0023 \text{ m}$$

15

10-18 $\lambda = 50^\circ S$ skotið suður $\alpha = 37^\circ$

$v_0 = 800$ m/s hve stór er Coriolis geigun



$$v = \begin{pmatrix} v_x \\ v_y \\ 0 \end{pmatrix} = \begin{pmatrix} v_0\cos\alpha \\ v_0\sin\alpha - gt \\ 0 \end{pmatrix}$$

$$\bar{\omega} = \begin{pmatrix} -\omega\cos\lambda \\ -\omega\sin\lambda \\ 0 \end{pmatrix}$$

$$\bar{a}_c = 2\bar{v} \times \bar{\omega}$$

$$= \hat{e}_z\left\{-2v_0\omega\cos\alpha\sin\lambda + 2(v_0\sin\alpha - gt)\omega\cos\lambda\right\}$$

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$$v_c = \int_0^t a_c(t')dt' = \hat{e}_z\left\{2v_0\omega t(\sin\alpha\cos\lambda - \cos\alpha\sin\lambda) - \omega\cos\lambda \cdot gt^2\right\}$$

flugtími er $t = \frac{2v_0\sin\alpha}{g}$

Geigunin er

$$z_c = \int_0^t v_c \, dt' = v_0\omega t^2(\sin\alpha\cos\lambda - \cos\alpha\sin\lambda) - \omega\cos\lambda\frac{gt^3}{3}$$

$\to z_c = -272$ m

inn í blaðið til austurs