04-06 Einföldur pendúll með massa m og lengd l

Stöðuorka, mætti: $U = mgl\{1-\cos\theta\}$

Hreyfiorka: $T = \frac{1}{2}mv^2 = \frac{1}{2}ml^2\dot{\theta}^2$

Mættið $U(\theta)$ er lotubundið eins og sést á mynd á síðu 3.

Könnum feril í fasarúminu $\{\theta, \dot{\theta}\}$ fyrir orku $2mgl$ og sitt hvoru megin við. þ.e. þegar hreyfingin er að verða hringur í $\{t, \theta\}$ sléttunni

$$E = T + U, \text{ ekkert tap}$$
$$= \frac{1}{2}ml^2\dot{\theta}^2 + mgl\{1-\cos\theta\}$$

$$\rightarrow \frac{1}{2}ml^2\dot{\theta}^2 = E - mgl\{1-\cos\theta\}$$

$$\rightarrow \dot{\theta}^2 = \frac{2}{ml^2}\left[E - mgl\{1-\cos\theta\}\right]$$

$$\rightarrow \dot{\theta} = \sqrt{\frac{2}{ml^2}}\left[E - mgl\{1-\cos\theta\}\right]^{1/2}$$

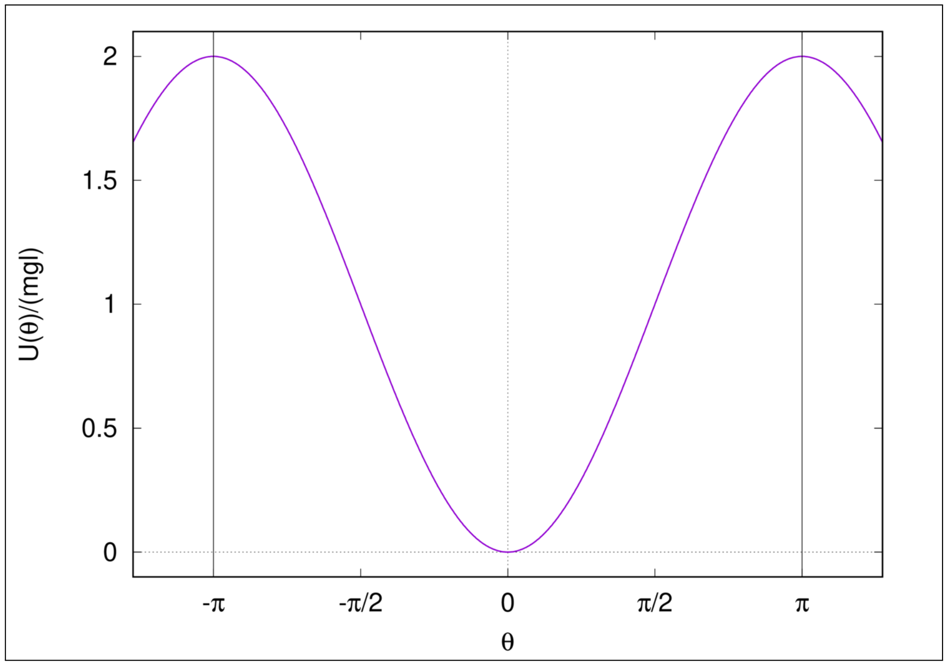
munum eftir tveimur rótum hér

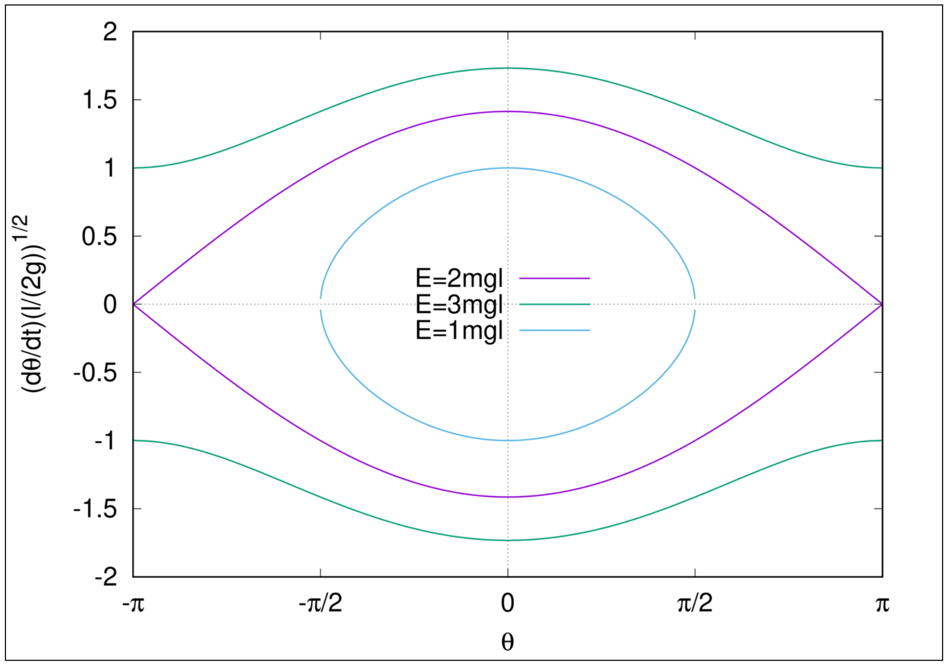
Fyrir graf:
$$\dot{\theta} = \sqrt{\frac{2g}{l}}\left[\frac{E}{mgl} - (1-\cos\theta)\right]^{1/2}$$

sjá mynd á síðu 4

$$\rightarrow \dot{\theta}\sqrt{\frac{l}{2g}} = \left[\frac{E}{mgl} - (1-\cos\theta)\right]^{1/2}$$

Eðlilegar víddar-lausar breytur fyrir graf





04-09

Taplaus hreyfing í krafti

$$F(x) = \begin{cases} -kx & |x|<a \\ -(k+\delta)x + \delta a & |x|>a \end{cases}$$

Kraftinum má lýsa með mættinu $\left(F = -\frac{d}{dx} U \right)$

$$U(x) = \begin{cases} \frac{kx^2}{2} & |x|<a \\ \frac{(k+\delta)x^2}{2} - \delta a x & |x|>a \end{cases}$$

skölun fyrir graf

$$= \begin{cases} \frac{ka^2}{2}\left(\frac{x}{a}\right)^2 & \left|\frac{x}{a}\right|<1 \\ \frac{(k+\delta)a^2}{2}\left(\frac{x}{a}\right)^2 - \delta a^2\left(\frac{x}{a}\right) & \left|\frac{x}{a}\right|>1 \end{cases}$$

$$E = T + U, \quad T = \frac{1}{2}mv^2 = \frac{1}{2}m\dot{x}^2$$

$$T = E - U \longrightarrow \frac{1}{2}mv^2 = E - U(x)$$

$$v = \sqrt{\frac{2}{m}}\left(E - U(x)\right)^{1/2} = \sqrt{\frac{2}{m}}\left(E - \frac{ka^2}{2}V(x)\right)^{1/2}$$

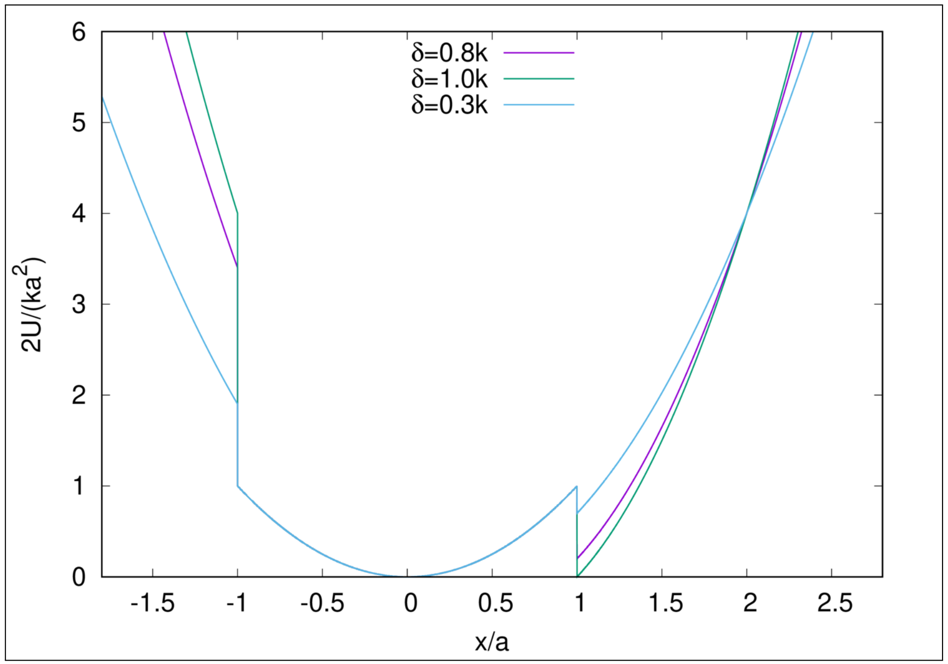
V er víddarlaust

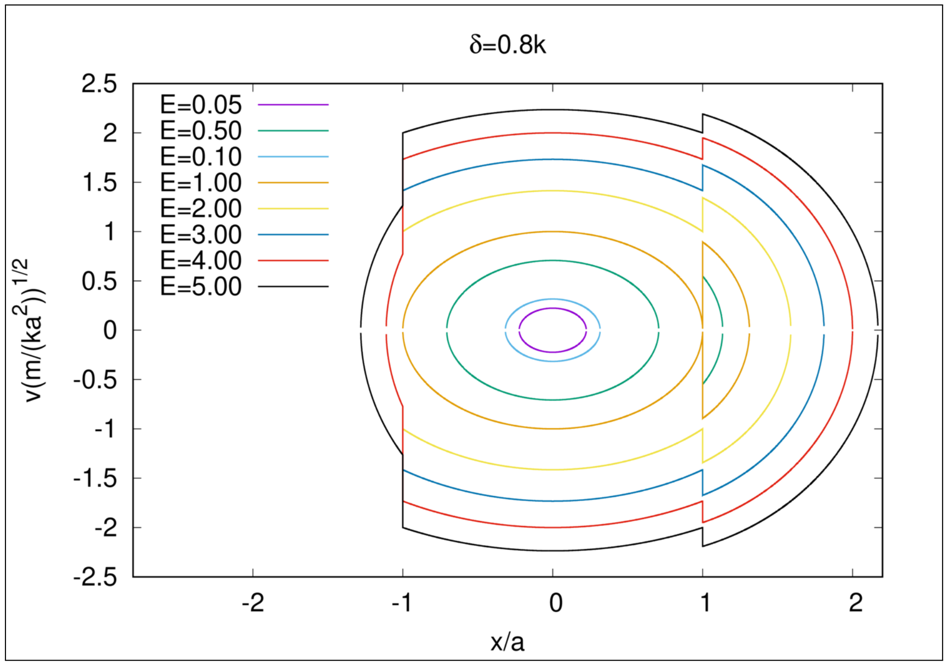
$$v = \sqrt{\frac{2}{m}\frac{ka^2}{2}}\left(\frac{2E}{ka^2} - V(x)\right)^{1/2}$$

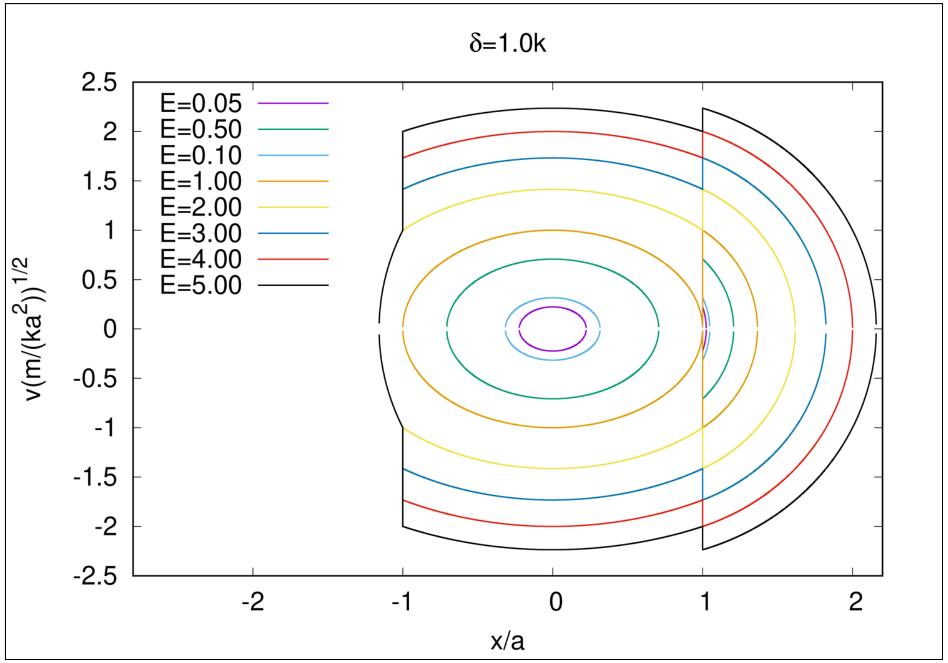
$$= \sqrt{\frac{ka^2}{m}}\left(\frac{2E}{ka^2} - V(x)\right)^{1/2}$$

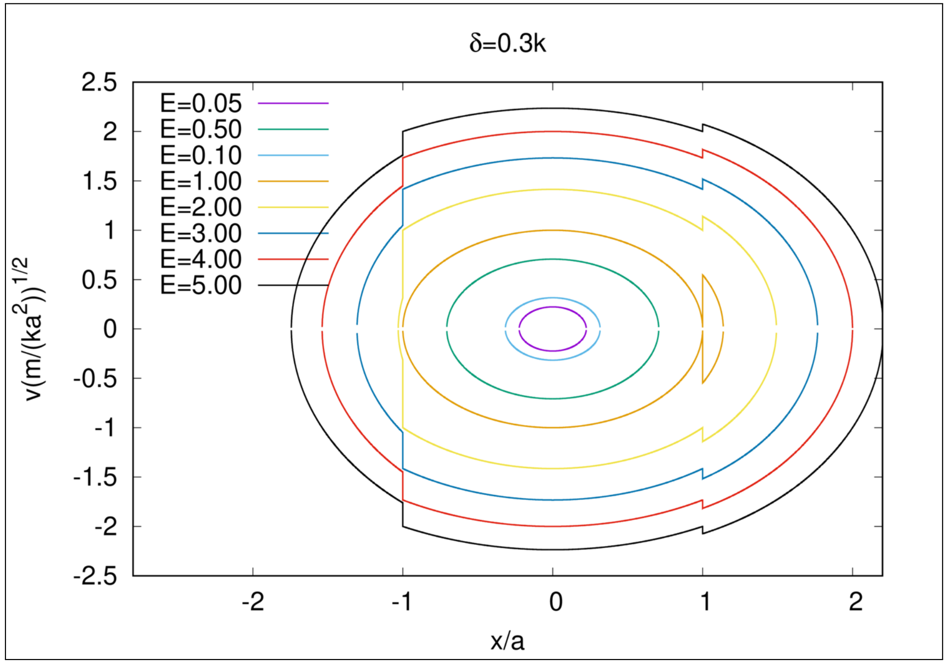
skölun orku notuð á gröfunum í fasarúminu

Gröfin eru á næstu 3 síðum









04-03

$$U(x) = -\left(\frac{\lambda}{3}\right)x^3$$

Eins og í dæmina á undan er

$$v = \sqrt{\frac{2}{m}}\left(E - U(x)\right)^{1/2}$$

$$= \sqrt{\frac{2}{m}}\left(E + \frac{\lambda}{3}x^3\right)^{1/2}$$

skölum

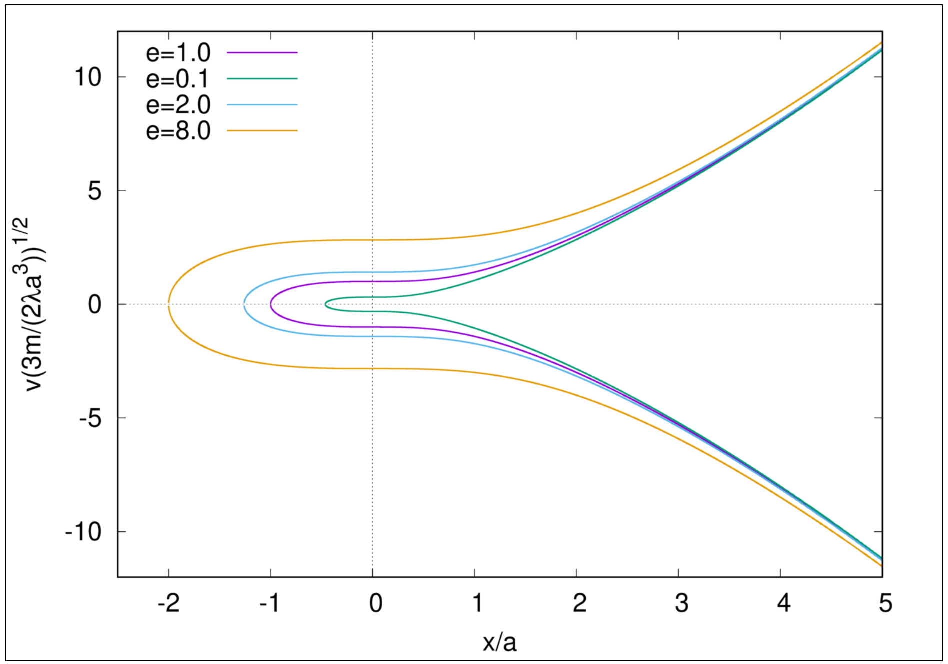
$$v = \sqrt{\frac{2}{m}}\left(E + \frac{\lambda a^3}{3}\left(\frac{x}{a}\right)^3\right)^{1/2}$$

$$= \sqrt{\frac{2\lambda a^3}{3m}}\left(\frac{3E}{\lambda a^3} + \left(\frac{x}{a}\right)^3\right)^{1/2}$$

$$v\sqrt{\frac{3m}{2\lambda a^3}} = \left(\frac{3E}{\lambda a^3} + \left(\frac{x}{a}\right)^3\right)^{1/2}$$

"e" á grafi

Sjá næstu síðu



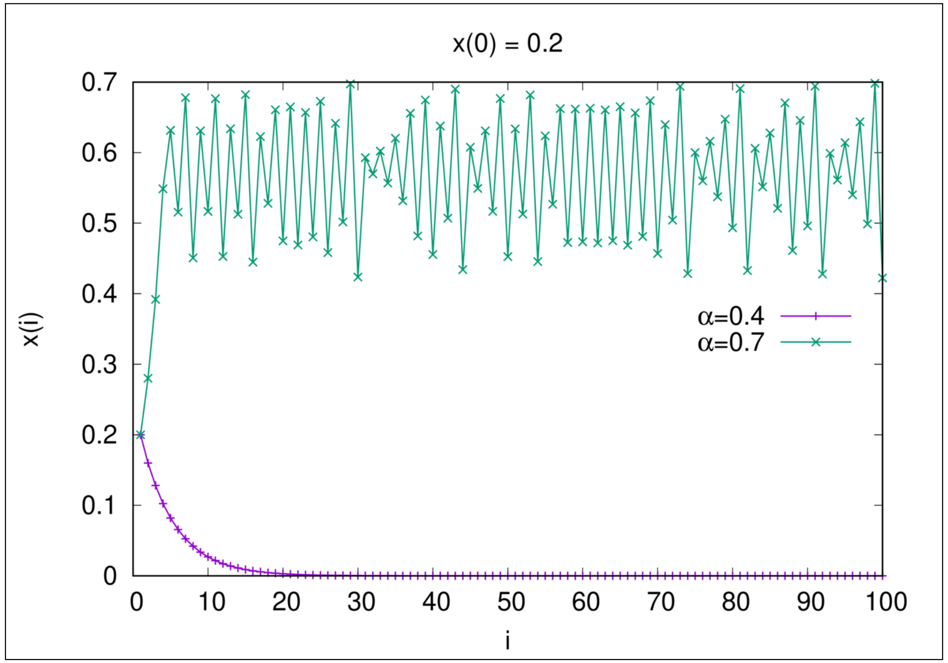
04-17 Tjald-vörpunin

$$x_{n+1} = 2\alpha x_n \qquad 0 < x < 1/2$$

$$x_{n+1} = 2\alpha(1 - x_n) \qquad \tfrac{1}{2} < x < 1 \qquad\qquad 0 < \alpha < 1$$

Kanna fyrir $\alpha = 0,4$ og $0,7$ með $x_1 = 0,2$

Sjá graf á næstu síðu



04-04 Jafna Rayleigh er

$$\ddot{x} - (a - b\dot{x}^2)\dot{x} + \omega_0^2 x = 0 \qquad (*)$$

Sýna að breytuskiptin $y = y_0\sqrt{3\frac{b}{a}}\,\dot{x}$ leiði til jöfnu van der Pol

$$\ddot{y} - \frac{a}{y_0^2}(y_0^2 - y^2)\dot{y} + \omega_0^2 y = 0$$

Breytuskiptin benda til þess að best sé að diffra $(*)$

$$\dddot{x} + 2b\dot{x}\ddot{x}\dot{x} - (a - b\dot{x}^2)\ddot{x} + \omega_0^2\dot{x} = 0$$

$$\to \quad \dddot{x} - \{a - 3b\dot{x}^2\}\ddot{x} + \omega_0^2\dot{x} = 0$$

$$y = y_0\sqrt{\frac{3b}{a}}\,\dot{x} \quad \to \quad \dot{x} = \sqrt{\frac{a}{3b}}\,\frac{y}{y_0}$$

$$\to \quad \ddot{x} = \sqrt{\frac{a}{3b}}\,\frac{\dot{y}}{y_0}$$

$$\dddot{x} = \sqrt{\frac{a}{3b}}\,\frac{\ddot{y}}{y_0}$$

Innsetning

$$\sqrt{\frac{a}{3b}}\left\{\frac{\ddot{y}}{y_0} - \left[a - 3b\frac{a}{3b}\frac{y^2}{y_0^2}\right]\frac{\dot{y}}{y_0} + \omega_0^2\frac{y}{y_0}\right\} = 0$$

$$\to \quad \ddot{y} - \frac{a}{y_0^2}\{y_0^2 - y^2\}\dot{y} + \omega_0^2 y = 0$$

Jafna van der Pols

04-22

$$\frac{dx}{dt} = y$$

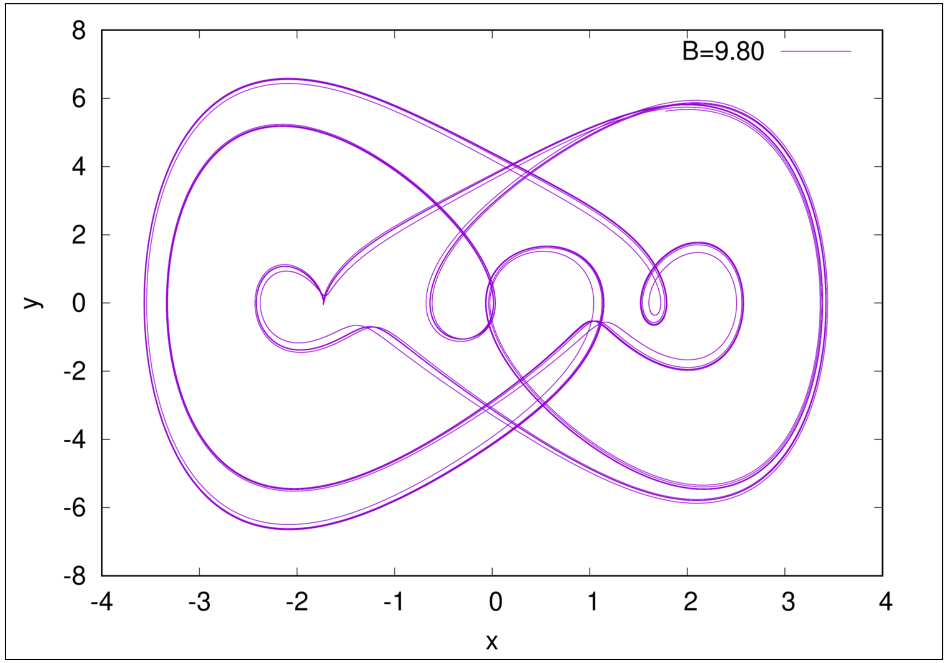
$$\frac{dy}{dt} = -ky - x^3 + B\cos t$$

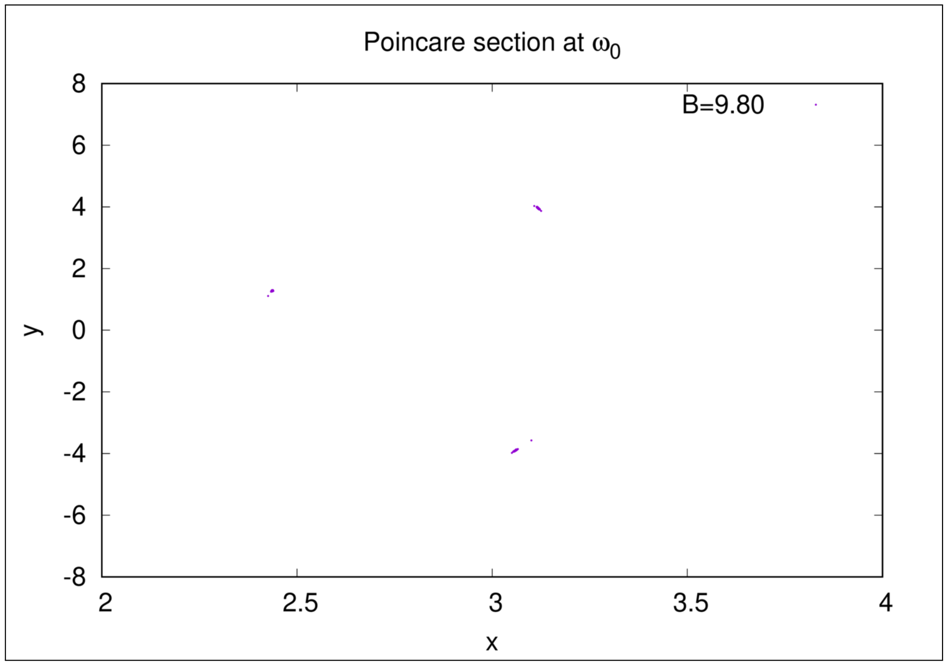
Poincaré snið : $k=0.1$ $\quad 9.8 \le B \le 13.4$

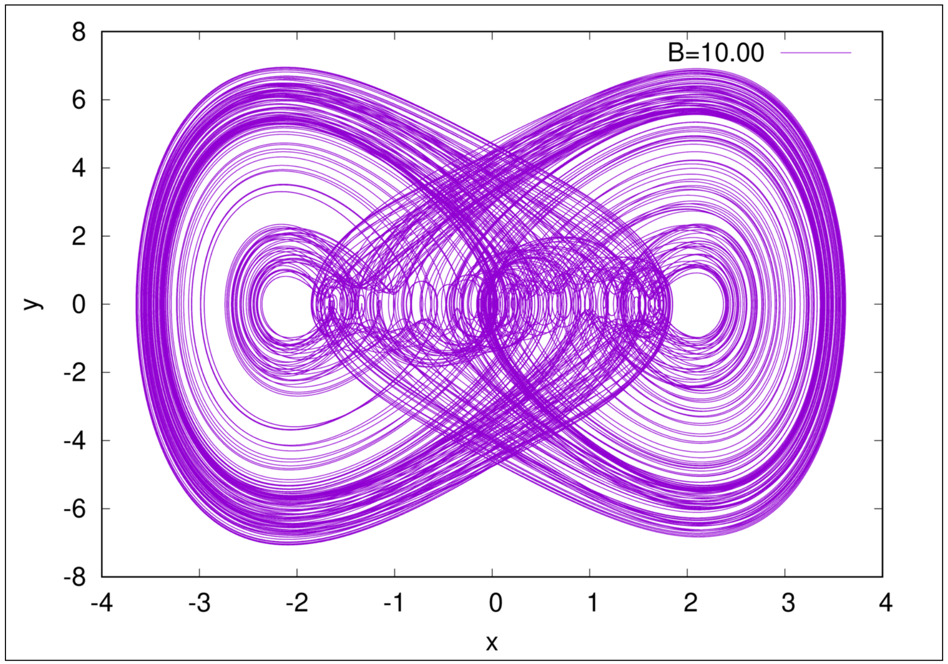
Eg bý til fasarit fyrir 4 virði á B, en kanna ikki alt sniðið. Eg nýta 1000000 punkta í tíma og sleppi 100000 fyrstu punktunum. Í fasaritinum er $\Delta t = 0{,}001$, en í sniðinum nýta eg $t_n = 2\pi n$ sum samsvarar $\omega_0 = 1$ her.

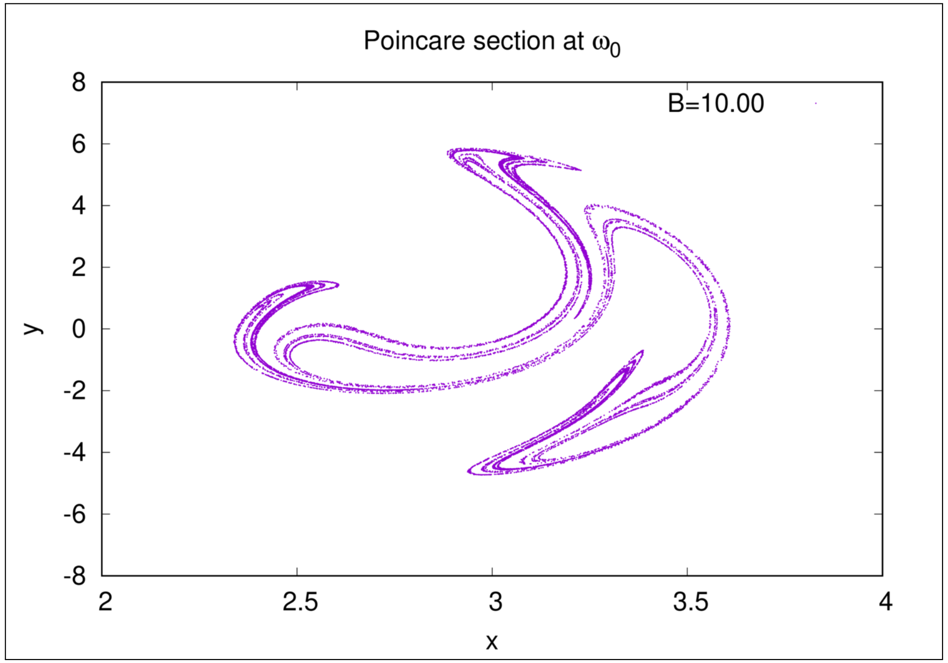
Hvør koyrsla í forritinum tekur ~ 2s.

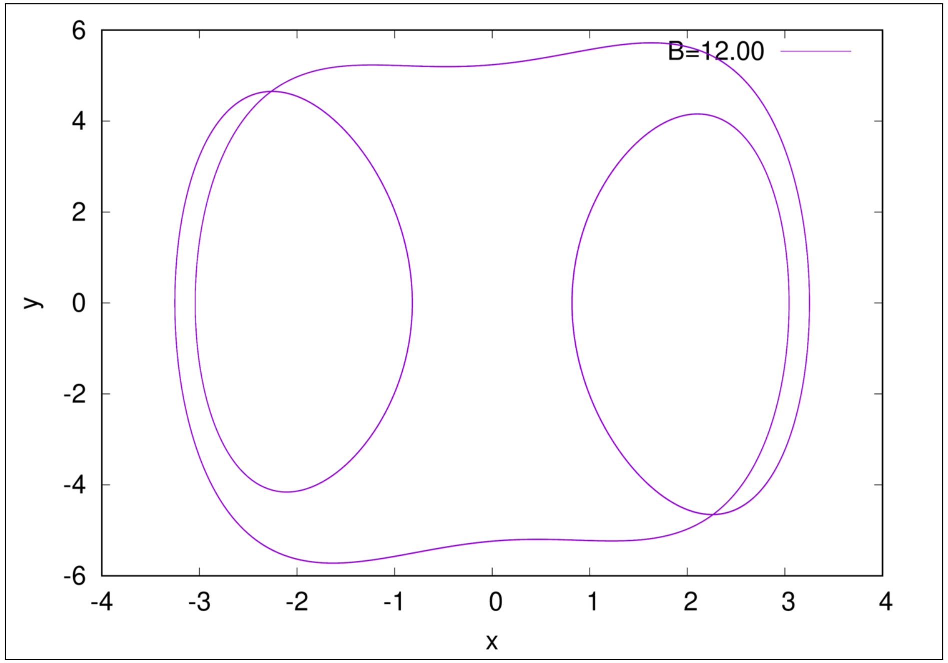
Upphafspunktur $(x,y) = (0,0)$

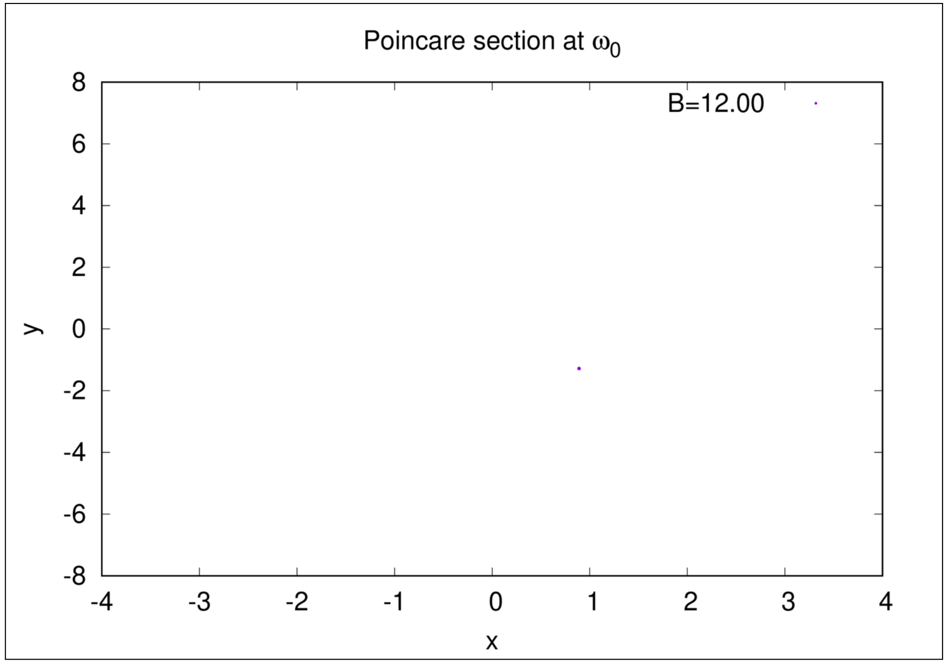


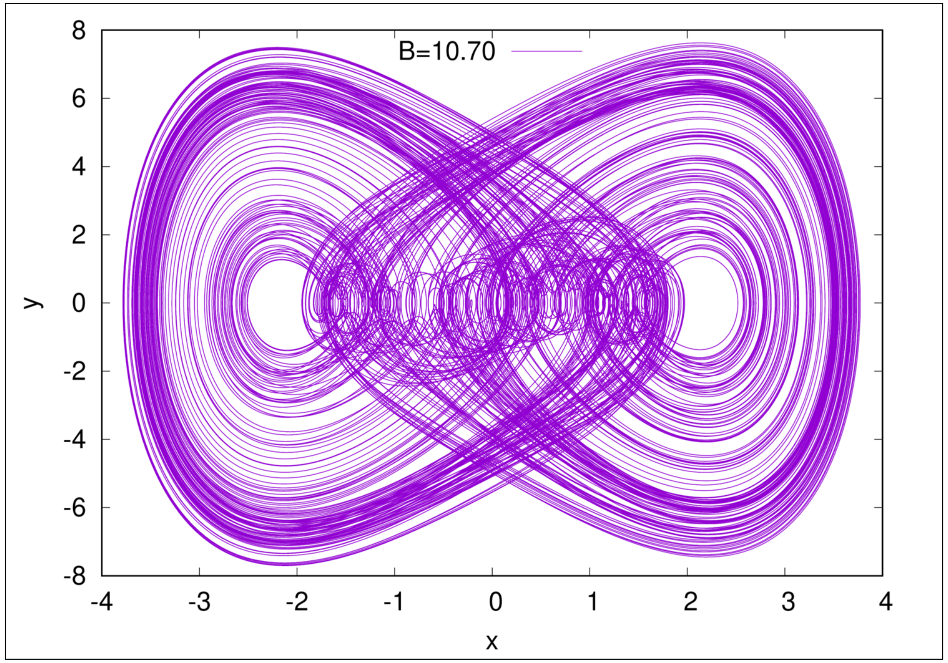


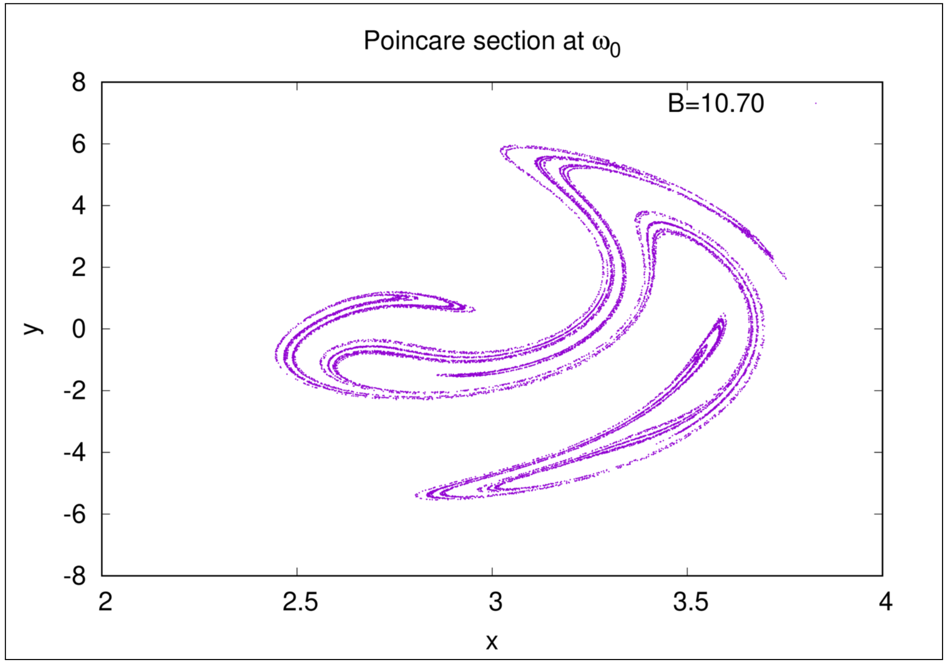



Poincare section at ω_0
B=10.00
8
6
4
2
0
-2
-4
-6
-8
y
2
2.5
3
3.5
4
x


B=12.00
6
4
2
0
-2
-4
-6
y
-4
-3
-2
-1
0
1
2
3
4
x


Poincare section at ω_0
B=12.00
8
6
4
2
0
-2
-4
-6
-8
y
-4
-3
-2
-1
0
1
2
3
4
x


B=10.70
8
6
4
2
0
-2
-4
-6
-8
y
-4
-3
-2
-1
0
1
2
3
4
x


Poincare section at ω_0
B=10.70
y
x
8
6
4
2
0
-2
-4
-6
-8
2
2.5
3
3.5
4