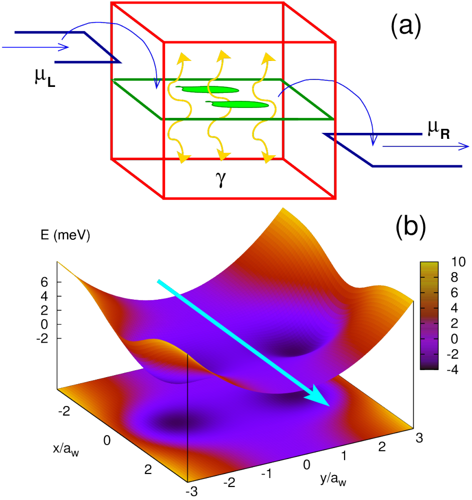
Time-dependent transport of electrons through nanosystems in a photon cavity

Viðar Guðmundsson

Science Institute, University of Iceland

vidar@hi.is

Transport of electrons through dots in a photon cavity



Exactly interacting electrons and photons, geometry

Time-dependent transport

$\Downarrow$

Time scales

$\Downarrow$

Transient – intermediate – long time – steady state

$\Uparrow$

Density operator

$\Uparrow$

Open Systems

Equation of motion

Liouville-von Neumann

$$\partial_t W = \mathcal{L}W, \quad \mathcal{L}\rho = -\frac{i}{\hbar}[H, W]$$

$$H = H_\mathrm{S} + H_\mathrm{LR} + H_\mathrm{T}(t), \quad H_\mathrm{S} = H_\mathrm{e} + H_\mathrm{EM}$$

$$H_\mathrm{S} = \int d^2r \psi^\dagger(\mathbf{r}) \left\{ \frac{\pi^2}{2m^*} + V(\mathbf{r}) \right\} \psi(\mathbf{r}) + H_\mathrm{Coul} + \hbar\omega a^\dagger a$$
$$-\frac{1}{c}\int d^2r\, \mathbf{j}(\mathbf{r}) \cdot \mathbf{A}_\gamma - \frac{e}{2m^* c^2}\int d^2r\, \rho(\mathbf{r}) A_\gamma^2$$

$$\boldsymbol{\pi} = \left(\mathbf{p} + \frac{e}{c}\mathbf{A}_\mathrm{ext}\right), \quad \rho = -e\psi^\dagger\psi, \quad \mathbf{j} = -\frac{e}{2m^*}\left\{\psi^\dagger\left(\boldsymbol{\pi}\psi\right) + \left(\boldsymbol{\pi}^*\psi^\dagger\right)\psi\right\}$$

$$\mathbf{A}(\mathbf{r}) = \begin{pmatrix} \hat{\mathbf{e}}_x \\ \hat{\mathbf{e}}_y \end{pmatrix} \mathcal{A} \left\{ a + a^\dagger \right\} \begin{pmatrix} \cos\left(\frac{\pi y}{a_c}\right) \\ \cos\left(\frac{\pi x}{a_c}\right) \end{pmatrix} \cos\left(\frac{\pi z}{d_c}\right), \qquad \begin{matrix} \mathrm{TE}_{011}, & x\text{-pol.} \\ \mathrm{TE}_{101}, & y\text{-pol.} \end{matrix}$$

Projection on the central system

Reduced density operator

$$\rho_{\rm S}(t) = \mathcal{P}W(t) = \rho_{\rm LR}(0){\rm Tr}_{\rm LR}W(t)$$

Liouville-von Neumann $\Rightarrow$ Nakajima-Zwanzig equation (to 2nd order in $H_{\rm T}$)

$$\partial_t\rho_{\rm S}(t) = \mathcal{L}_{\rm S}\rho_{\rm S}(t) + \int_0^t dt' K[t,t-t';\rho_{\rm S}(t')]$$

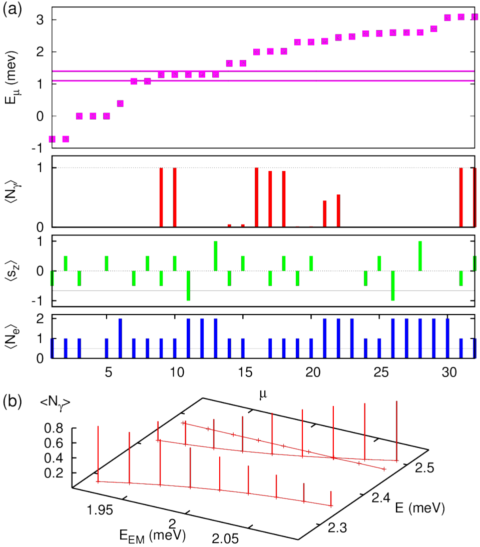
with

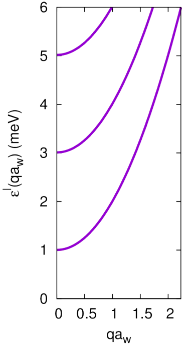
$$K[t,s;\rho_{\rm S}(t')] = {\rm Tr}_{\rm LR}\left\{\left[H_{\rm T}(t),\left[U(s)H_{\rm T}(t')U^+(s),\right.\right.\right. \\ \left.\left.\left. U_{\rm S}(s)\rho_{\rm S}(t')U_{\rm S}^+(s)\rho_{\rm L}\rho_{\rm R}\right]\right]\right\} \quad (1)$$

and

$$H_{\rm T}(t) = \sum_{i,l}\chi(t)\int dq\, \left\{T^l_{qi}c^\dagger_{ql}d_i + (T^l_{qi})^* d^\dagger_i c_{ql}\right\} \quad (2)$$

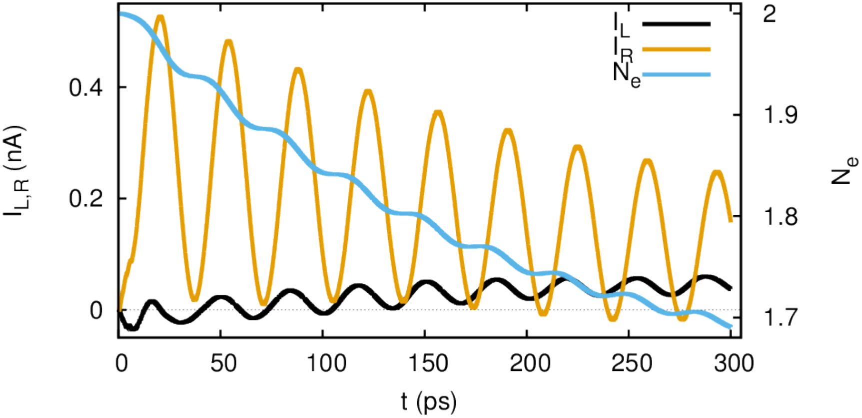
Spectrum of closed systems, y-polarized cavity photons





$\hbar\omega$	$=$	2.0	meV
g_{EM}	$=$	0.05	meV
B	$=$	0.1	T
a_w	$=$	23.8	nm
V_g	$=$	0.1	mV

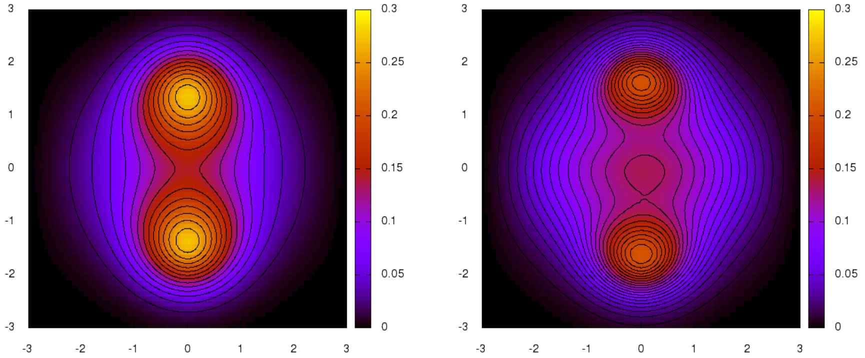
Rabi-oscillations seen in transport current



Initial state: fully entangled 2-e Rabi-split $|21)$ and $|22)$

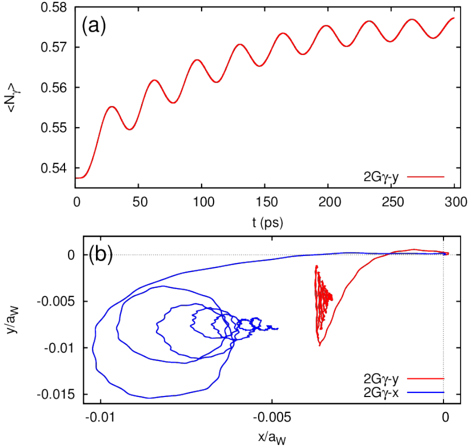
Charge density oscillations

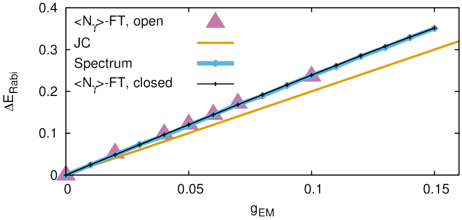
$t = 10$ ps $\qquad t = 60$ ps



Variable probability in contact area $\rightarrow$ variable current

Consequences of geometry





Rabi-splitting

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Dynamical Hall effect

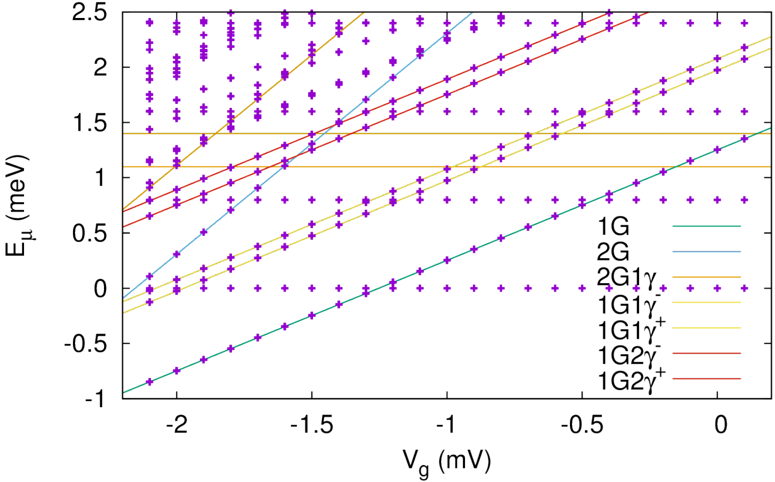
Questions

- What happens beyond 300 ps?
- How long time is needed to get 2 electrons into the system?
- Steady state?
- Are there different time-regimes?

$$\Updownarrow$$

- Time-integration not feasible
- Consider Markovian instead of non-Markovian system
- Continue with no rotating wave approximation
- Start with short quantum wire without embedded dots

Spectrum of closed system vs. plunger gate voltage V_g



x-polarization, $\hbar\omega = 0.8$ meV, $g_{\mathbf{EM}} = 0.05$ meV, $\hbar\Omega = 2.0$ meV, $B = 0.1$ T

Nakajima-Zwanzig

$$\partial_t \rho = -\frac{i}{\hbar}\left[H_{\mathrm{S}}, \rho\right] - \sum_l \Lambda(\Omega_{ql}, \tau_{ql}, \chi_l, t)$$

with

$$\Lambda(\Omega_{ql}, \tau_{ql}, \chi_l, t) = \frac{1}{\hbar^2}\int dq\ \chi_l(t)\left\{\left[\tau_{ql}, \Omega_{ql}(t)\right] + h.c.\right\}$$

where,

$$\Omega_{ql}(t) = \int_0^t ds\ \chi(s) U(t-s)\Big\{\tau_{ql}^\dagger \rho(s)(1-f_{ql}) \\ - \rho(s)\tau_{ql}^\dagger f_{ql}\Big\} U^\dagger(t-s) e^{i(s-t)\omega_{ql}}$$

Change of variable $t - s \to s'$, set $\rho(t-s) \to \rho(t)$

use

$$\int_0^t ds \exp\left[is(E_\nu - E_\mu - \epsilon_{ql})\right] \to \pi\delta(E_\nu - E_\mu - \epsilon_{ql})$$

and

$$\begin{aligned}\int dq A(q)\delta(E_\alpha - E_\beta - \epsilon_{ql}) &= \int d\epsilon (dq/d\epsilon) A(\epsilon)\delta(E_\alpha - E_\beta - \epsilon)\\ &= A^{\alpha\beta}D^{\alpha\beta}\end{aligned}$$

$$\chi_l(t) \to \theta(t)$$

Leads to

$$\Omega_{\alpha\beta} = \left\{ \mathcal{R}[\rho]_{\alpha\beta} - \mathcal{S}[\rho]_{\alpha\beta} \right\} \delta^{\beta\alpha}$$
$$\mathcal{R}[\rho] = \rho\pi f \tau^\dagger, \quad \mathcal{S}[\rho] = \pi(1-f)\tau^\dagger\rho$$

Introduce

$$\Delta_{\alpha\beta} = \delta^{\alpha\beta} = \delta(E_\alpha - E_\beta - \epsilon)$$

to obtain

$$\mathcal{Z}_{\alpha\beta} = \int DA_{\alpha\lambda}\Omega_{\lambda\sigma}B_{\sigma\beta}d\delta^{\sigma\lambda}$$
$$\Downarrow$$
$$\mathcal{Z} = \int DA \left\{ (\mathcal{R}[\rho] - \mathcal{S}[\rho]) \odot d\Delta^T \right\} B.$$

Fock $\rightarrow$ Liouville space

Use vectorization and Kronecker tensor product

$$\mathrm{vec}(\mathbf{AXB}) = \left\{\mathbf{B}^T \otimes \mathbf{A}\right\} \mathrm{vec}(\mathbf{X})$$

$\dim(N)$ Fock-state-space $\rightarrow$ $\dim(N^2)$ Liouville-transition-space
Markovian equation of motion

$$\partial_t \rho_{\mathrm{S}}^{\mathrm{vec}} = \mathcal{L} \rho_{\mathrm{S}}^{\mathrm{vec}}$$

where

$$\mathcal{L} = \left\{ -\frac{i}{\hbar}(I \otimes H - H^T \otimes I) + \sum_{X=R,S} (\mathfrak{Z}_{X_1} \mathfrak{Z}_{X_2}) \right\}$$

and

$$\mathfrak{Z}_{X_1} = \int \left(B^T \otimes DA\right) \mathrm{Diag}(\Delta^T), \qquad X = R, S$$

$$\mathfrak{Z}_{R_2} = \int \mathrm{Diag}(\Delta^T)(I \otimes R)$$

$$\mathfrak{Z}_{S_2} = -\int \mathrm{Diag}(\Delta^T) \left(S^T \otimes I\right)$$

with solution

$$\rho_{\rm S}^{\rm vec}(t) = \left[\mathcal{U}\exp\left(\mathcal{L}_{\rm diag}t\right)\mathcal{V}\right]\rho_{\rm S}^{\rm vec}(0)$$

where

$$\mathcal{L}\mathcal{V} = \mathcal{V}\mathcal{L}_{\rm diag}, \quad \mathcal{U}\mathcal{L} = \mathcal{L}_{\rm diag}\mathcal{U}, \quad \mathcal{U}\mathcal{V} = \mathcal{V}\mathcal{U} = \mathcal{I}$$

Steady state can be found as the eigenvalue 0 of

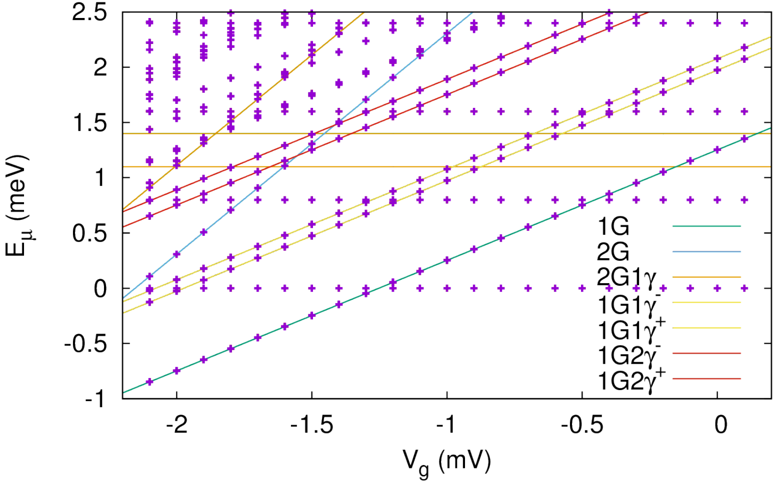
$$0 = \mathcal{L}\rho_{\rm S}^{\rm vec}$$

but we use

$$\lim_{t\to\infty}\left[\mathcal{U}\exp\left(\mathcal{L}_{\rm diag}t\right)\mathcal{V}\right]\rho_{\rm S}^{\rm vec}(0)$$

Here, $N = 120$, $V_g = -1.6$ mV

Spectrum of closed system vs. plunger gate voltage V_g



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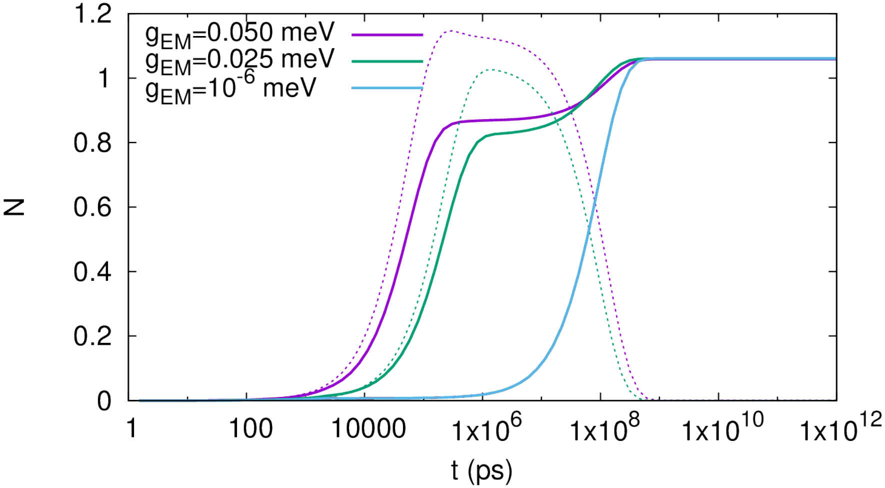
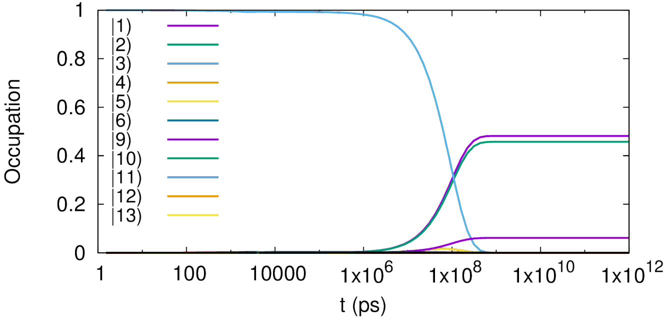
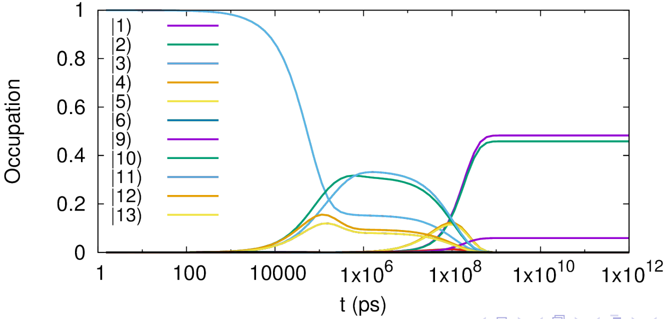
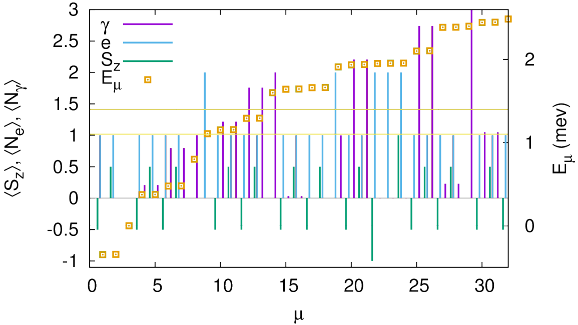
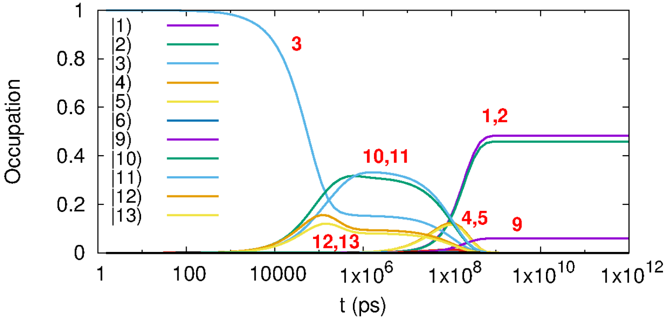


1.2
1
0.8
0.6
0.4
0.2
0
N
g_{EM}=0.050 meV
g_{EM}=0.025 meV
g_{EM}=10^{-6} meV
1
100
10000
1x10^{6}
1x10^{8}
1x10^{10}
1x10^{12}
t (ps)


Occupation
1
0.8
0.6
0.4
0.2
0
|1)
|2)
|3)
|4)
|5)
|6)
|9)
|10)
|11)
|12)
|13)
1
100
10000
1x10^6
1x10^8
1x10^10
1x10^12
t (ps)


Occupation
1
0.8
0.6
0.4
0.2
0
|1)
|2)
|3)
|4)
|5)
|6)
|9)
|10)
|11)
|12)
|13)
1
100
10000
1x10^6
1x10^8
1x10^10
1x10^12
t (ps)


γ
e
S_z
E_μ
$\langle S_z \rangle$, $\langle N_e \rangle$, $\langle N_\gamma \rangle$
E_μ (mev)
μ


|1)
|2)
|3)
|4)
|5)
|6)
|9)
|10)
|11)
|12)
|13)
3
1,2
10,11
4,5
9
12,13
Occupation
t (ps)

Conclusions

- We can identify regimes of different types of transitions, electromagnetic, non-electromagnetic
- We can analyze steady state and long-time behavior of complex cavity systems
- http://arxiv.org/abs/1605.08248

Collaboration and support

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Spectrum of the Liouvillian