með

$$H_1(t) = e^{iH_0t/\hbar} H_1 e^{-iH_0t/\hbar}$$

þegar $t \to -\infty$ þá er $|\Phi_0\rangle$

tíma óháð ástand H_0

$$H_0|\Phi_0\rangle = E_0|\Phi_0\rangle$$

nærri eigin ástand H

$$|\Psi_H\rangle = |\Psi_I(0)\rangle = U_\epsilon(0,-\infty)|\Phi_0\rangle$$

Gell-Mann Low setn

Ef eftirfarandi er til samkv. truflanareikn.

$$\lim_{\epsilon\to 0} \frac{U_\epsilon(0,-\infty)|\Phi_0\rangle}{\langle\Phi_0|U_\epsilon(0,-\infty)|\Phi_0\rangle} \equiv \frac{|\Psi_0\rangle}{\langle\Phi_0|\Psi_0\rangle}$$

þá er það eiginástand H.

[þó $|\Phi_0\rangle$ sé grunnástand H_0 þá er ekki alveg öruggt að $|\Psi_0\rangle$ sé grunnástand H]

$$\frac{\langle\Psi_0|\hat{O}_H(t)|\Psi_0\rangle}{\langle\Psi_0|\Psi_0\rangle}$$

$$= \frac{\langle\Phi_0|U_\epsilon(\infty,t)\,\hat{O}_I(t)\,U_\epsilon(t,-\infty)|\Phi_0\rangle}{\langle\Phi_0|\underbrace{U_\epsilon(\infty,-\infty)}_{S}|\Phi_0\rangle}$$

þessvegna fæst:

$$\langle\Psi_0|T(\psi(\bar{x}t)\psi^{\dagger}(\bar{x}'t'))|\Psi_0\rangle = iG(\bar{x}t,\bar{x}'t')$$

$$= \sum_{n=0}^{\infty}\left(-\frac{i}{\hbar}\right)\frac{1}{n!}\int_{-\infty}^{\infty}dt_1 \cdots dt_n \frac{\langle\Phi_0|T(H_1(t_1)\cdots H_1(t_n)\psi(\bar{x}t)\psi^{\dagger}(\bar{x}'t')|\Phi_0\rangle}{\langle\Phi_0|U_{\epsilon}(\infty,-\infty)|\Phi_0\rangle}$$

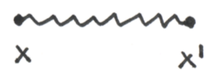
Nú væri hægt að nota víxlreglur fyrir sviðsvirkjana til þess að reikna

$$\frac{\langle\Phi_0|T(\quad)|\Phi_0\rangle}{\langle\Phi_0|U..|\Phi_0\rangle}$$

fyrir allar tímaraðanir.
Aðferðirnar hafa verið hafa verið teknar saman í Wicks setningunni sem síðan er notuð til þess að leiða út reglur Feynmanns

Reglur Feynmanns fyrir n. stigs Greensfallið í staðarrúminu þegar spuna er sleppt. Fermí eindir

1) $G^0(\bar{x}t,\bar{x}'t') = G^0(x,x') \rightarrow$ 

$V(x,x') = V(\bar{x},\bar{x}')\delta(t-t') \rightarrow$ 

2) Dragið allar „tópológísk" mismunandi tengdar myndir með n víxlvertunum og $2n+1$ beinum Greensföllum

$\langle\Phi_0|U|\Phi_0\rangle$ liðurinn í nefnara styttir út allar ótengdar myndir

3) merkið alla hnútapunkta með fjórhniti $x = xt$

4) Heildið allar innri rúm og tíma-breytur

5) Fyrir hverja lokaða Fermi lykkju kemur stuðull (-1)

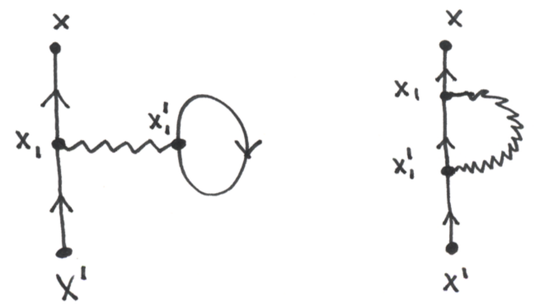
6) Stuðull $(-i)\left(-\frac{i}{\hbar}\right)^n (i)^{2n+1} = \left(\frac{i}{\hbar}\right)^n$

7) Greens fall með $t=t'$ verður að skiljast sem

$$G^0(\bar{x}t, \bar{x}'t^+)$$

$\left\{ \text{Reglurnar má einnig finna í skriðþunga rúminu} \right\}$

1. stigs myndirnar eru

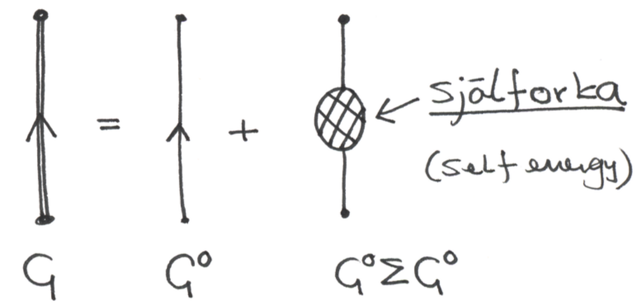


$$G^{(1)}(x,x') = \frac{i}{\hbar}\int dx_1 dx_1' \Big\{(-1)G^0(x,x_1)V(x_1,x_1')G^0(x_1',x') \cdot G^0(x_1',x_1') + G^0(x,x_1)V(x_1,x_1')G^0(x_1,x_1')G^0(x_1',x')\Big\}$$

fyrir G^2 eru 10-myndir!

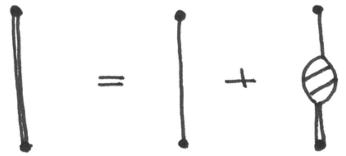
Síðan má einnig sýna

Eðlilega



og

þannig að:



jafna Dysons

$$G(x,y) = G^0(x,y) + \int dx_1\, dx_2\, G^0(x,x_1)\, \Sigma^*(x_1,x_2)\, G(x_2,y)$$

Ef kerfið er óbreytt við hliðrun

þ.a. $G^0(x,y) = G^0(x-y) \quad \Sigma^*(x,y) = \Sigma(x-y) \ldots$

þá fæst í Fourier rúmi (földun)

$$G(k) = G^0(k)\, \Sigma^*(k)\, G(k) \qquad k = \bar{k}, \omega$$

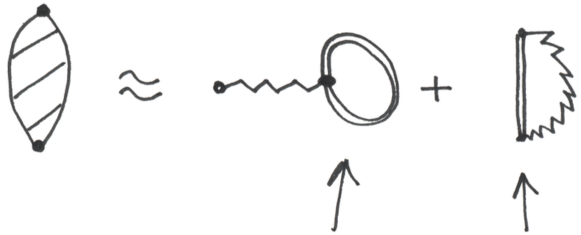
$$\rightarrow \quad G(k) = \frac{1}{\{G^0(k)\}^{-1} - \Sigma^*(k)}$$

$$= \frac{1}{\omega - \omega_k^0 - \Sigma^*(\bar{k}, \omega)}$$

eiginleg sjálforka

stýrir nálgunin

t.d. má skrifa Hartree Fock nálgunina sem



beinn liður

skiptivíxlverkun

1

Rafsegulsvið

Sígilt rafsegulsvið í skammtafræði

Víxlverkun við efnisvið

Kvarðasamhverfa

Landau stig

spunahermа

tímubg svörun

Skömmtun Rafsegulsviðs

sjálfgeislun

cgs-einingar

2

Víxlverkun við efnisvið

Einnar agnar skammtafr.

Hamiltonvirkinn fyrir ögn með hleðslu e í rafsegulsviði er

$$H = \frac{\left(p - \frac{e}{c}A(\bar{x}t)\right)^2}{2m} + e\phi(\bar{x}t) + V(\bar{x}t)$$

tímaháða Schrödingerjafnan

$$i\hbar\partial_t\psi(\bar{x}t) = \left\{\frac{1}{2m}\left(\frac{\hbar}{i}\nabla - \frac{e}{c}A(\bar{x}t)\right)^2 + e\phi(\bar{x}t) + V(\bar{x}t)\right\}\psi(\bar{x}t)$$

er óbreytt við kvarðaskiptin

$$\bar{A}(\bar{x}t) \longrightarrow \bar{A}(\bar{x}t) + \bar{\nabla}\chi(\bar{x}t)$$

$$\phi(\bar{x}t) \longrightarrow \phi(\bar{x}t) - \frac{1}{c}\partial_t\chi(\bar{x}t)$$

ef

$$\psi(\bar{x}t) \longrightarrow e^{\frac{ie}{\hbar c}\chi(\bar{x}t)}\psi(\bar{x}t)$$

Rafsviðið

$$\bar{E} = -\bar{\nabla}\phi - \frac{1}{c}\partial_t \bar{A}$$

og segulsviðið

$$\bar{B} = \bar{\nabla} \times \bar{A}$$

eru óbreytt við kvarðaskiptin. Schrödinger jafnan inniheldur mættin $\bar{A}$ og ϕ

$\longrightarrow$ breytist bylgjufallið

Allar mælistærðir verða að vera óbreyttar

$\longrightarrow$ bylgjufallið fær staðbundin fasastuðul

hér er ekki átt við virkjana eina sér, heldur virkja og fallagrunn

$\longrightarrow$ væntingargildi - - - | Öðruvísi en við aðra
Schrödinger jöfnu | skömmtun

t.d. er skriðþunginn $\bar{p}$ ekki kvarðaóháður því

$$\langle \psi_1 | p | \psi_2 \rangle = -i\hbar \int d\bar{x}\, \psi_1(\bar{x}t) \vec{\nabla} \psi_2(\bar{x}t)$$

er greinilega ekki óháð kvarða

En fylkisstök af

$$\bar{p} - \frac{e}{c}\bar{A}(\bar{x}t)$$

eru kvarðaóháð og því verð þessi breyting í H

Heisenberg jöfnurnar sýna einnig að

$$m d_t \bar{x}(t) = \bar{p}(t) - \frac{e}{c}\bar{A}(\bar{x}(t), t)$$

Nú getur verið þægilegt að greina

$$H = H_0 + H_I$$

þetta er gert á mismunandi hátt eftir þörfum í hvert skipti

Einnig eru mismunandi kvarðar valdir eftir þörfum!

T.d. er oft þegar áhrif rafsegulgeislunar (veikrar) á kerfi eru athuguð notað

$$H = H_0 + H_I$$

með

$$H_0 = \frac{p^2}{2m} + V(\vec{r},t)$$

$$H_I = -\frac{e}{2mc}\left\{\vec{p}\cdot\vec{A} + \vec{A}\cdot\vec{p}\right\} + \frac{e^2}{2mc^2}\vec{A}^2 + e\phi$$

Hér mun verða litið á rafsegulsviðið sem truflun á kerfinu

> Sjálfsamkvæma meðhöndlun á rafsegulsviðinu verður litið á undir „<u>línulegri svörun</u>" þar sem tekið er tillit til spansviðs sem kerfið getur framkallað sem svörun við ytra sviðinu

Skammtað fjöleindakerfi + sígilt rafsegulsvið

Almennt eru hreyfijöfnurnar <u>cgs-einingar</u>

$$\left\{ i\hbar\partial_t + \frac{\hbar^2}{2m}\left(\vec{\nabla} - i\frac{e}{\hbar c}\vec{A}(\vec{x}t)\right)^2 \right\}\psi(\vec{x}t) = e\phi(\vec{x}t)\psi(\vec{x}t)$$

$$\left(\nabla^2 - \frac{1}{c^2}\partial_t^2\right)\vec{A}(\vec{x}t) = -\frac{4\pi}{c}\vec{J}(\vec{x}t) + \frac{1}{c}\vec{\nabla}\phi(\vec{x}t)$$

þar sem $\vec{A}$ er heildar vektor mættið, bæði utanaðkomandi svið og svið eindanna sjálfra

$$\vec{J}(\vec{x}t) = -i\frac{e\hbar}{2m}\left\{\psi^+(\vec{x}t)\left(\vec{\nabla} - \frac{ie}{\hbar c}\vec{A}(\vec{x}t)\right)\psi(\vec{x}t) - \left(\vec{\nabla} + \frac{ie}{\hbar c}\vec{A}(\vec{x}t)\right)\psi^+(\vec{x}t)\cdot\psi(\vec{x}t)\right\} + \vec{J}_{ext}$$

Á sama hátt er $e\phi$ heildar mættið, bæði utanaðkomandi og innra mætti

$$\phi_{in}(\bar{x}t) = \int d\bar{x}' \frac{\rho(\bar{x}'t)}{|\bar{x}-\bar{x}'|}$$

með

$$\rho(\bar{x}t) = e\psi^{+}(\bar{x}t)\,\psi(\bar{x}t)$$

hér hefur verið notast við coulomb kvarðann

$$\bar{\nabla}\cdot\bar{A} = 0$$

Hamiltonvirkjarnir sem leiða til hreyfingarjafnanna eru

$$H_e = \frac{\hbar^2}{2m}\int d\bar{x}\; \bar{\nabla}\psi^{+}(\bar{x}t)\,\bar{\nabla}\psi(\bar{x}t)$$

$$H_{em} = \frac{1}{8\pi}\int d\bar{x}\left\{\underbrace{\frac{1}{c^2}\left(\partial_t\bar{A}(\bar{x}t)\right)^2}_{E^2} + \underbrace{\left(\bar{\nabla}\times\bar{A}\right)^2}_{B^2}\right\}$$

$$H_I = \int d\bar{x}\left\{-\frac{1}{c}\,\bar{j}(\bar{x}t)\cdot\bar{A}(\bar{x}t) + \frac{e^2}{2mc^2}\rho(\bar{x}t)A^2(\bar{x}t) + e\rho(\bar{x}t)\phi(\bar{x}t)\right\}$$

$$+\frac{e^2}{2}\int d\bar{x}d\bar{x}'\,\frac{\psi^{+}(\bar{x}t)\,\psi^{+}(\bar{x}'t)\,\psi(\bar{x}'t)\,\psi(\bar{x}t)}{|\bar{x}-\bar{x}'|}$$

þar sem

$$\bar{j}(\bar{x}t) = -i\frac{e\hbar}{2m}\left\{\psi^{+}(\bar{x}t)\vec{\nabla}\psi(\bar{x}t) - \bar{\nabla}\psi^{+}(\bar{x}t)\psi(\bar{x}t)\right\}$$

þannig að

$$\bar{J}(\bar{x}t) = \underbrace{\bar{j}(\bar{x}t)}_{\substack{\text{mótseglunar}\\ \text{straumur}\\ \text{para-}}} - \underbrace{\frac{e}{mc}\bar{A}(\bar{x}t)\rho(\bar{x}t)}_{\substack{\text{meðseglunar}\\ \text{straumur}\\ \text{dia-}}}$$

Ef rafsegulsviðinu er skipt í tvennt

$$\bar{A}(\bar{x}t) \longrightarrow \bar{A}(\bar{x}t) + \bar{A}_{ext}(\bar{x}t)$$

og truflanareikningi verður beitt með tilliti til $\bar{A}_{ext}$

þá er eðlilegt að nota sundurgreininguna

$$\boxed{H = \int d\bar{x}\, \psi^{+}(\bar{x}t)\left\{\frac{1}{2m}\left(-i\hbar\bar{\nabla} + \frac{e}{c}\bar{A}(\bar{x}t) + \frac{e}{c}\bar{A}_{ext}(\bar{x}t)\right)^{2}\right\}\psi(\bar{x}t)}$$

$$= \int d\bar{x}\, \psi^{+}(\bar{x}t)\left\{\frac{1}{2m}\left(-i\hbar\bar{\nabla} + \frac{e}{c}\bar{A}(\bar{x}t)\right)^{2}\right\}\psi(\bar{x}t)$$

$$- \frac{1}{c}\int d\bar{x}\, \bar{j}(\bar{x}t)\cdot\bar{A}_{ext}(\bar{x}t) - \frac{e^2}{2mc}\int d\bar{x}\, \rho(\bar{x}t)\,\bar{A}^{2}_{ext}(\bar{x}t)$$

þar sem

$$\bar{j}(\bar{x}t) = -\frac{ie\hbar}{2m}\left\{\psi^{+}(\bar{x}t)\left(\bar{\nabla} - \frac{ie}{\hbar c}\bar{A}(\bar{x}t)\right)\psi(\bar{x}t) - \left(\bar{\nabla} + \frac{ie}{\hbar c}\bar{A}(\bar{x}t)\right)\psi^{+}(\bar{x}t)\cdot\psi(\bar{x}t)\right\}$$

Þannig mætti t.d. fjalla um ferðlur atóms sem er í segulsviði $\bar{B} = \bar{\nabla}\times\bar{A}$ og verður fyrir truflunni $\bar{A}_{ext}$

Liðurinn $\bar{A}^2$ hefur þá oft sterk áhrif á atómið en liðinn $\bar{A}^2_{ext}\rho$ má oft sleppa m.t.t. $\bar{j}\cdot\bar{A}$ liðsins.

Dæmi þar sem truflanareikn verður ekki beitt m.t.t. rafsegulsviðs

Landau stig (Capri..., Landau Lifs)

Fjálsar rafeindir í segulsviði

fast einsleitt segulsvið $\bar{B} = B\hat{z}$

Einn möguleiki fyrir $\bar{A}$ er t.d.

$$\bar{A} = \left(-\tfrac{1}{2}By,\ \tfrac{1}{2}Bx,\ 0\right)$$

$$\longrightarrow\ \bar{B} = \bar{\nabla}\times\bar{A}$$

$$= \left(\left\{\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right\}, \left\{\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right\}, \left\{\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right\}\right)$$

$$= \left(0, 0, \frac{B+B}{2}\right) = B\hat{z}$$

Hér er talað um hringsamhverfa kvarðann eða hringkvarðann

Einnig má nota $\bar{A} = (-By, 0, 0)$

„Landau kvarði" ← innan þéttefnisedlisfr.

$$\longrightarrow \bar{B} = \bar{\nabla} \times \bar{A} = B\hat{z}$$

þessa tvo kvarða má tengja saman með

$$\bar{A}_h(\bar{x}) = \bar{A}_L(\bar{x}) + \bar{\nabla}\chi(\bar{x})$$

þ.s.

$$\bar{\nabla}\chi(\bar{x}) = \left(\tfrac{1}{2}By, \tfrac{1}{2}Bx, 0\right)$$

$\longrightarrow$ t.d. $\quad \chi(\bar{x}) = \frac{1}{2}Bxy$

síðan er sá kvarðinn valinn sem á betur við hnitakerfið sem Schrödingerjafnan er leyst í

Landau kvarði $\qquad \bar{A} = (-By, 0, 0)$

$$H = \frac{\hbar^2}{2m}\left(-i\bar{\nabla} + \frac{e}{c\hbar}\bar{A}(\bar{x})\right)^2$$

$$= -\frac{\hbar^2}{2m}\left\{\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} - \frac{2i}{\ell^2}y\partial_x - \frac{y^2}{\ell^4}\right\} - \frac{\hbar^2}{2m}\frac{\partial^2}{\partial z^2}$$

þar sem $\quad \ell^2 = \frac{c\hbar}{eB}$: segullengd

Í H eru engin x og z

$$\longrightarrow [H, p_x] = [H, p_z] = 0$$

svo lausnin á Schrödingerjöfnunni verður

$$\psi \sim e^{ik_x x + ik_z z}\phi(y)$$

hvar er nú sígilda hugmyndin um hringsnúning?

Schrödinger jafnan verður þá

$$-\frac{\hbar^2}{2m}\left\{\partial_y^2 - k_x^2 + \frac{2k_x}{\ell^2}y - \frac{y^2}{\ell^4} - k_z^2\right\}\phi(y) = E_{k_xk_z}\phi$$

hreyfingin í z-áttina er frjáls með orkuna $\frac{k_z^2\hbar^2}{2m}$

Látum því nægja að fjalla um hreyfingar í x-y planinu $\quad k_x = k$

$$\left\{-\frac{\hbar^2}{2m}\partial_y^2 + \frac{1}{2}m\omega_c^2(y-y_0)^2\right\}\phi_{ny_0}(y) = E_{ny_0}\phi_{ny_0}$$

með

$y_0 = k\ell^2$: miðjuhnit

$\omega_c = \frac{eB}{mc}$: hringhermutíðni

x-y-hreyfingar tengjast þ.a.

$\boxed{\text{miðjuhnit } y_0 \text{ hreintóna sveiflunnar í y-átt gefur skriðþunga } \frac{y_0}{\ell^2} \text{ planbylgju í x-átt}}$

Ef $\phi_{ny_0}(\pm\infty) = 0$ og ef $L_x \to \infty$ þá fæst

$$E_{ny_0} = \hbar\omega_c\left(n+\frac{1}{2}\right) \qquad n = 0,1,2\ldots$$

$$\phi_{ny_0}(x,y) = \frac{1}{\sqrt{2\pi}}\left(\frac{1}{\pi\ell^2}\right)^{1/4}\left(\frac{1}{n!\,2^n}\right)^{1/2} \cdot \exp\left\{i\frac{y_0}{\ell^2}x - \frac{1}{2\ell^2}(y-y_0)^2\right\}H_n\left(\frac{y-y_0}{\ell}\right)$$

Hermite fl.

<u>n: númer Landau stigs</u>

Landaustigin eru margföld m.t.t. y_0

Ef Normað á L_x þá fast

$$\tilde{\phi}_{Nk} = \frac{\sqrt{2\pi}}{\sqrt{L_x}}\,\phi_{Nk}(x,y)$$

lotubundið

$$\tilde{\phi}_{Nk}(x,y) = \tilde{\phi}_{Nk}(x+L_x, y)$$

$$\rightarrow e^{i\frac{y_0 L_x}{\ell^2}} = 1 \quad \rightarrow \frac{i y_0 L_x}{\ell^2} = i 2\pi N$$

$$\rightarrow k L_x = 2\pi N$$

$$\rightarrow k = \frac{2\pi N}{L_x}$$

$$\Delta k = \frac{2\pi}{L_x}$$

$$n_N(\bar{x}) = \sum_k \left|\tilde{\phi}_{Nk}\right|^2$$

$$= \frac{2\pi}{L_x}\sum_k \left|\phi_{Nk}(\bar{x})\right|^2 = \sum_k \left|\phi_{Nk}(\bar{x})\right|^2 \Delta k$$

$$= \int_{-\infty}^{\infty} dk \left|\phi_{Nk}(\bar{x})\right|^2$$

margfeldnina er best að kanna með seinni skömmtun

þéttleiki ástanda í fylltu Landaustigi N er

$$n_N(\bar{x}) = \langle \Phi_N | \psi^+(\bar{x})\psi(\bar{x}) | \Phi_N \rangle$$

þar sem

$$\psi(\bar{x}) = \sum_{n=0}^{\infty}\int_{-\infty}^{\infty} dk\, C_{nk}(t)\,\phi_{nk}(\bar{x})$$

$$k = \frac{y_0}{\ell^2}$$

með

$$\left\{C_{nk}(t), C^+_{n'k'}(t)\right\} = \delta_{nn'}\delta(k-k')$$

$$\rightarrow n_N(\bar{x}) = \sum_{nn'}\int_{-\infty}^{\infty} dk\,dp\,\phi^*_{nk}(\bar{x})\,\phi_{n'p}(\bar{x})\,\delta(k-p)\,\delta_{n,N}\delta_{n'N}$$

$$= \int_{-\infty}^{\infty} dk\,\phi^*_{Nk}(\bar{x})\,\phi_{Nk}(\bar{x})$$

$$n_N(\bar{x}) = \frac{1}{\ell^2}\int_{-\infty}^{\infty} dy_0\, \phi^*_{Ny_0}(\bar{x})\,\phi_{Ny_0}(\bar{x})$$

$$= \frac{1}{2\pi\ell^2}\left(\frac{1}{\pi\ell^2}\right)^{1/2}\frac{1}{N!\,2^N}\int_{-\infty}^{\infty} dy_0 \exp\left\{-\frac{(y-y_0)^2}{\ell}\right\} H_N^2\left(\frac{y-y_0}{\ell}\right)$$

$$= \frac{1}{2\pi\ell^2} = \frac{eB}{hc}$$

þéttleiki ástanda í fylltu Landau stigi n_0 er því fasti óháður N

$$\longrightarrow \quad n_0 = \frac{1}{2\pi\ell^2}$$

í slíku kerfi í jafnvægi er $n(\bar{x})$ ávalt óháð $\bar{x}$: n_s

fyllitala Landau stiga kerfisins er því stiðgreind

$$\boxed{\nu = \frac{n_s}{n_0} = 2\pi\ell^2 n_s}$$

hve mörg Landau-stig eru setin!

samakerfi, hringkvarði

$$\bar{A} = \left(-\frac{1}{2}By, \frac{1}{2}Bx, 0\right)$$

þáttur Schrödingerjöfnunnar í z-átt er eins og áður, svo honum er sleppt hér

$$\longrightarrow$$

$$H = \frac{\hbar^2}{2m}\left(-i\tilde{\nabla} + \frac{e}{c\hbar}\tilde{A}(\bar{x})\right)^2$$

með

$$\tilde{\nabla} = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}\right), \quad \tilde{A} = (A_x, A_y)$$

$$H = \frac{\hbar^2}{2m}\left[\left(-i\partial_x - \frac{y}{2\ell^2}\right)^2 + \left(-i\partial_y + \frac{x}{2\ell^2}\right)^2\right]$$

$$= -\frac{\hbar^2}{2m}\left\{\partial_x^2 + \partial_y^2 + \frac{i}{\ell^2}(x\partial_y - y\partial_x)\right\} + \frac{m\omega_c^2}{8}(x^2+y^2)$$

$$= -\frac{\hbar^2}{2m}\left\{\tilde{\nabla}^2 + \frac{i}{\ell^2}\partial_\varphi\right\} + \frac{m\omega_c^2}{8}r^2$$

pólhnit

$$H = -\frac{\hbar^2}{2m}\left\{\partial_r^2 + \frac{1}{r}\partial_r + \frac{1}{r^2}\partial_\varphi^2 + \frac{i}{\ell^2}\partial_\varphi\right\} + \frac{m\omega_c^2}{8} r^2$$

Hreintónasveifill í 2-vídd fyrir utan liðinn

<u>Kerfið er hringsamhverft</u>

$$\longrightarrow \psi(r,\varphi) = \phi(r)\, e^{-iM\varphi} \qquad M \in \mathbb{Z}$$

$$\longrightarrow \left\{\partial_r^2 + \frac{1}{r}\partial_r - \frac{M^2}{r^2} - \frac{r^2}{4\ell^4} + \left(\frac{M}{\ell^2} + \epsilon\right)\right\}\phi(r) = 0$$

$$\epsilon = \frac{2m}{\hbar^2} E$$

bera saman við hreintónasveifil í 2-vídd
Flügge bl. 108

$$\longrightarrow \qquad E = \hbar\omega_c\left(n + \frac{1}{2}\right)$$

$$n = \frac{|M| - M}{2} + n_r \quad , n_r = 0, 1, 2, \dots$$

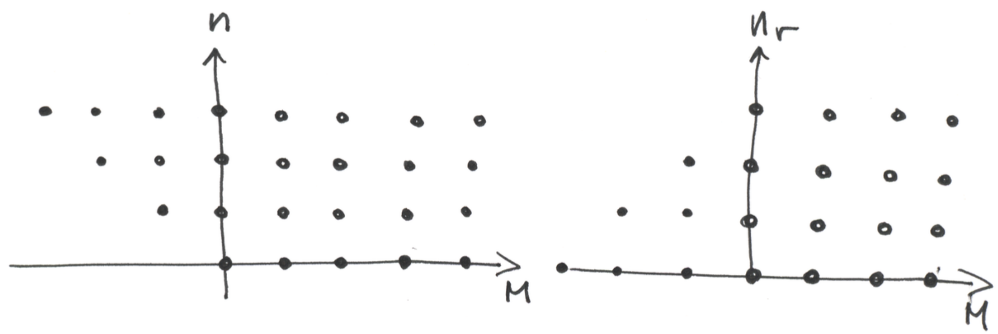
$$\phi_{n_r M}(r) = \left\{\pi(|M| + n_r)!\,(2\ell^2)^{|M|+1}(n_r!)^{-1}\right\}^{-1/2} r^{|M|} e^{-\frac{r^2}{4\ell^2}} L_{n_r}^{|M|}\left(\frac{r^2}{2\ell^2}\right) e^{-iM\varphi}$$

M: hverfiþungi ástandsins um $r=0$

$\longrightarrow$ líking við sígilda hringhreyfingu

greinilega $n = 0, 1, 2 \dots$ Landau stig

hvaða skammtatölur eru mögulegar



$-n \leq M$

Segulsviðið orsakar því stjál jafndreifð orkustig, Landau stig með $\Delta E = E_{n+1} - E_n = \hbar\omega_c$ sem hvert hefur margfeldnina

$$n_0 = \frac{1}{2\pi\ell^2}$$

Eins og búist var við, leiða báðir kvarðarnir til sömu orku

Einnig var áður sagt að

$$\boxed{\psi(\bar{x}) \longrightarrow e^{\frac{ie}{\hbar c}\chi}\, \psi(\bar{x})} \quad *$$

við kvarðaskiptin

Hér var $\chi = \frac{1}{2}Bxy$ og

$$\phi_{n_r M} \sim r^{|M|} e^{-\frac{r^2}{4l^2}} L_{n_r}^{|M|}\left(\frac{r^2}{2l^2}\right) e^{-iM\varphi}$$

$$\phi_{n y_0} \sim e^{+\frac{i y_0 x}{l^2}} e^{-\frac{1}{2l^2}(y-y_0)^2} H_n\left(\frac{y-y_0}{l}\right)$$

<u>Svo hvernig verður * uppfyllt?</u>

Landaustigs skammtatalan <u>n</u> í báðum kvörðum hefur sömu merkingu

en <u>M og k</u> hafa mismunandi merkingu því er hægt að uppfylla * á eftirfarandi hátt.

$$\phi_{nM}(\bar{x}) e^{\frac{ie}{\hbar c}\chi} = \int_{-\infty}^{\infty} dk\, C^n(k,M)\, \phi_{nk}(\bar{x})$$

$$\phi_{nk}(\bar{x}) e^{-\frac{ie}{\hbar c}\chi} = \sum_{M=-\infty}^{\infty} b^n(k,M)\, \phi_{nM}(\bar{x})$$

og $b^{n*}(k,M) = C(k,M)$

finnið $b^0(k,M)$ eða $C^0(k,M)$ fyrir dæmatíma

(22)

Eins og sást í dæminu með

hringkvarðann má einfaldlega bæta

innilokunar mættinu $\frac{1}{2} m \omega_0 r^2$ við

Schrödingerjöfnuna og fá nákvæmalausn.

$$E_n \to E_{nM} = \left(n + \frac{M}{2} + \frac{1}{2}\right)\hbar\left(\omega_c^2 + 4\omega_0^2\right)^{1/2} - \frac{M\hbar\omega_c}{2}$$

$$l \to a = \frac{l^2}{\sqrt{1+4\left(\frac{\omega_0}{\omega_c}\right)^2}} = \frac{\hbar}{m}\frac{1}{\sqrt{\omega_c^2 + 4\omega_0^2}}$$

heppilegur grunnur fyrir hringlaga endanleg tvívíð kerfi

$\longrightarrow$ skammtapunkta

Einfalt að bera saman við þekkt markgildi þegar

$\omega_c \gg \omega_0$ eða $\omega_c \ll \omega_0$

$\uparrow$ sterkt segulsvið $\qquad$ $\uparrow$ veikt segulsvið

(1)

Línuleg svörun

Ryogo Kubo: Journ. Phys. Soc. Japan **12** (1957)
"Statistical-Mechanical Theory of Irreversible Processes"

Fetter Walecka . . .

Víxlverkandi fjöleindakerfi

tíma óháður H, Schrödingermynd

$$\to \quad i\hbar\partial_t |\Psi_S(t)\rangle = H|\Psi_S(t)\rangle$$

með lausn

$$|\Psi_S(t)\rangle = e^{-iHt/\hbar}|\Psi_S(0)\rangle$$

$\boxed{\text{Klukkan } t = t_0 \text{ er kveikt á truflun } H_{ext}(t)}$

<u>nýja</u> schrödinger ástandið $|\overline{\Psi}_s(t)\rangle$

uppfyllir því fyrir $t > t_0$

$$i\hbar\partial_t|\overline{\Psi}_s(t)\rangle = (H + H_{ext}(t))\,|\overline{\Psi}_s(t)\rangle$$

Leitað verður að lausn

$$|\overline{\Psi}_s(t)\rangle = e^{-iHt/\hbar} A(t)\,|\Psi_s(0)\rangle$$

þar sem

$$A(t) = 1 \qquad t \leq t_0$$

Ef þessi lausn er sett inn í jöfnuna fæst fyrir A

$$i\hbar\partial_t A(t) = e^{iHt/\hbar} H_{ext}(t)\, e^{-iHt/\hbar} A(t)$$

$$= H_H^{ext}(t)\, A(t)$$

<u>þ.s. hér er að ræða Heisenb. mynd m.t.t. H</u>
en ekki $H + H_{ext}$

heilda $\longrightarrow$

$$A(t) = \overset{1}{\overset{\shortparallel}{A(t_0)}} - \frac{i}{\hbar}\int_{t_0}^{t} dt'\, H_H^{ext}(t')\, A(t')$$

ítra $\longrightarrow$

$$A(t) = 1 - \frac{i}{\hbar}\int_{t_0}^{t} dt'\, H_H^{ext}(t') + \cdots\cdots$$

$\rightarrow$ ástandið

$$|\overline{\Psi}_s(t)\rangle = e^{-iHt/\hbar}|\Psi_s(0)\rangle - \frac{i}{\hbar}\, e^{-iHt/\hbar}\int_{t_0}^{t} dt'\, H_H^{ext}(t')|\Psi_s(0)\rangle + \cdots$$

$$\rightarrow \langle O\rangle_{ext} \equiv \langle\overline{\Psi}_s(t)|\,O_s(t)\,|\overline{\Psi}_s(t)\rangle$$

$$= \langle\Psi_s(0)|\left\{1 + i\hbar^{-1}\int_{t_0}^{t} dt'\, H_H^{ext}(t') + \cdots\right\} e^{iHt/\hbar}\, O_s(t)\, e^{-iHt/\hbar}$$

$$\cdot \left\{1 - i\hbar^{-1}\int_{t_0}^{t} dt'\, H_H^{ext}(t') + \cdots\right\}|\Psi_s(0)\rangle$$

$$|\Psi_s(0)\rangle = |\Psi_H(0)\rangle$$

$$\langle O\rangle_{ext} = \langle \Psi_H^{(0)}| O_H(t) | \Psi_H^{(0)}\rangle + \frac{i}{\hbar}\int_{t_0}^{t} dt' \langle \Psi_H^{(0)}| [H_H^{ext}(t'), O_H(t)] |\Psi_H^{(0)}\rangle$$

$\boxed{\text{línulegt í } H^{ext}}$ $+ \cdots\cdots$

→ fyrsta stigs breyting á fylkisstaki

Vegna ytri áhrifa er tákníð við nákvæma Heisenberg virkja víxlverkandi en ótruflaða kerfisins

setjum $|\Psi_H(0)\rangle = |\Psi_0\rangle$ grunnástand

væntingargildi 0 í truflaða kerfinu

$$\langle O\rangle_{ext} = \langle O\rangle_0 + \frac{1}{\hbar}\int_{-\infty}^{\infty} dt'\, i\theta(t-t') \langle \Psi_0| [H_H^{ext}(t'), O(t)] |\Psi_0\rangle$$

$H_H^{ext} = 0$ fyrir $t < t_0$

heildi yfir svörunarfall

athugum með dæmum

ytra rafmætti

$$H_H^{ext}(t) = \int d\bar{x}\; n_H(\bar{x}t)\, e\phi^{ext}(\bar{x}t)$$

→

$$\langle n(\bar{x}t)\rangle_{ext} = \langle n(\bar{x}t)\rangle_0 + \frac{i}{\hbar}\int_{-\infty}^{\infty} dt' d\bar{x}'\, e\phi(\bar{x}'t')\,\theta(t-t') \cdot \langle \Psi_0| [n_H(\bar{x}'t'), n_H(\bar{x}t)] |\Psi_0\rangle$$

skilgreinum þéttni-þéttni svörunarfall

$$i D^R(x,x') \equiv \theta(t-t') \langle \Psi_0| [n_H(x), n_H(x')] |\Psi_0\rangle$$

→

$$\boxed{\langle n(\bar{x}t)\rangle_{ext} = \langle n(\bar{x}t)\rangle_0 + \frac{1}{\hbar}\int_{-\infty}^{\infty} dt' d\bar{x}'\, D^R(\bar{x}t,\bar{x}'t')\, e\phi^{ext}(\bar{x}'t')}$$

þar sem notuð var Heisenberg mynd fyrir H (en ekki H + Hext) þá tengist $D^R(x,x')$ svörunarfallið Greens fallinu fyrir H-kerfið (sjá síðar)

Ekki er hægt að beyta Feynmann-Dyson truflana reikn á seinkuð föll þ.s. Wick setn. gildir ekki

$\rightarrow$ tengja þarf D^R við D

Ef kerfi er einsleitt þ.a. $D^R(x-x')$ þá fæst með földun

$$\langle n(\bar{k}\omega)\rangle_{ext} = \langle n(\bar{k}\omega)\rangle_0 + \frac{1}{\hbar} D^R(\bar{k},\omega)\, e\phi(\bar{k}\omega)$$

og sýna má

$$\mathrm{Re}\, D(\bar{k}\omega) = \mathrm{Re}\, D^R(\bar{k}\omega)$$

$$\mathrm{Im}\, D(\bar{k}\omega)\,\mathrm{sgn}\,\omega = \mathrm{Im}\, D^R(\bar{k},\omega)$$

$$\mathrm{sgn}\,\omega = \frac{\omega}{|\omega|}$$

Af viðbrögðum kerfis við $\phi(\bar{x}t)$ má reikna „tíðni háða og statiska stýlingu" ásamt plasma bylgjum í gegnum D^R

ytra rafsvið

Coulomb kvarði

$$\bar{E}(\bar{x}t) = -\frac{1}{c}\partial_t \bar{A}(\bar{x}t)$$

Víxlverkun

$$H(t) = \frac{1}{c}\int d\bar{x}\, \bar{A}(\bar{x}t)\cdot\bar{j}(\bar{x}t)$$

$$\bar{J}(\bar{x}t) = \bar{j}(\bar{x}t) - \frac{e^2}{mc}\bar{A}(\bar{x}t)\psi^\dagger(\bar{x}t)\psi(\bar{x}t)$$

$\langle j\rangle_0 = 0$

$$\rightarrow \langle J_k(\bar{x}t)\rangle_{ext} = -\frac{e^2}{mc} n(\bar{x}t) A_k(\bar{x}t)$$

$$+\frac{i}{\hbar c}\int_{-\infty}^{\infty} d\bar{x}'dt' \langle\Psi_0|[j_k(\bar{x}'t'), j_l(\bar{x}t)]|\Psi_0\rangle \cdot A_l(\bar{x}'t')\,\theta(t-t')$$

línulegt í $\bar{A}$

Skilgreinum straum-straum svörunarfall

$\longrightarrow$

$$\langle J_k(\bar{x}t)\rangle_{ext} = -\frac{e^2}{mc}n(\bar{x}t)A_k(\bar{x}t) + \frac{-1}{\hbar c}\int_{-\infty}^{\infty} d\bar{x}'dt'\Lambda^R_{kl}(\bar{x}t,\bar{x}'t')A_l(\bar{x}'t')$$

Fyrir einsleitt kerfi er leiðni skilgreind sem

$$\langle J_k(\bar{q}\omega)\rangle_{ext} = \sigma_{kl}(\bar{q}\omega)\,E_l$$

$\longrightarrow$

$$\Lambda^R_{kl}(\bar{q}\omega) = e^2 D^R_{kl}(\bar{q}\omega)$$

$$\boxed{\sigma_{kl}(\bar{q}\omega) = \frac{ie^2}{\hbar}\frac{D^R_{kl}(\bar{q}\omega)}{\omega} - \frac{ie^2 n}{\omega m}\delta_{kl}}$$

leiðnin tengist einnig Greensföllum kerfisins H

Λ

Í $D^R_{kl}(\bar{q}\omega)$ eru bæði innbyrðis víxlverkanir rafeindanna og víxlverkanir þeirra við óreglur í kristallinum

Sjálfsamkvæm svörun

hleðsla $-e$

t.d. ytra rafmætti $\phi_{ext}(\bar{x}t)$

Heildarmættið í kerfinu:

spanmætti

$$\phi_{sc} = \phi_{ext} + \phi_{ind} \qquad (1)$$

Veldur þéttleikahnikum:

$$\delta n(\bar{x}t) = \langle n(\bar{x}t)\rangle_{ext} - \langle n(\bar{x}t)\rangle_0 \qquad (2)$$

$$= \frac{1}{\hbar}\int d\bar{x}'dt'\, D(\bar{x}t,\bar{x}'t')\left(-e\phi_{sc}(\bar{x}'t')\right)$$

sem leiðir til spanmættis:

$$\phi_{ind}(\bar{x}t) = -\frac{e}{\kappa}\int d\bar{x}'\frac{\delta n(\bar{x}'t)}{|\bar{x}-\bar{x}'|} \qquad (3)$$

$(2) \to (3) \to (1)$

$$\phi_{sc}(\bar{x}t) = \phi_{ext}(\bar{x}t) + \frac{1}{\hbar}\int_{-\infty}^{\infty} dt' d\bar{x}' d\bar{x}'' \frac{e^2 D^R(\bar{x}''t, \bar{x}'t')}{K|\bar{x}-\bar{x}''|}\phi_{sc}(\bar{x}'t')$$

heildisjafna fyrir ϕ_{sc} Heildarmættið

Ef $\underline{D^R(x,x') = D^R(\bar{x},\bar{x}', t-t')}$

og ytra mættið $\phi_{ext}(\bar{x}t) = \phi_{ext}(\bar{x})e^{-i\omega t}$

þá fæst í Fourierrúmi

$$\boxed{\phi_{sc}(\bar{q}\omega) = \phi_{ext}(\bar{q}\omega) + \frac{4\pi e^2}{\hbar \chi q}\int d\bar{k}\, D^R(\bar{q},\bar{k})^{\omega}\phi_{sc}(\bar{k}\omega)}$$

og ef $D^R(\bar{x},\bar{x}', t-t') = D^R(x'-x')$

þá fæst

$$\phi_{sc}(\bar{q}\omega) = \phi_{ext}(\bar{q}\omega) + \frac{4\pi e^2}{\hbar \chi q} D^R(\bar{q}\omega)\phi_{sc}(\bar{q}\omega)$$

$$\to \phi_{sc}(\bar{q}\omega) = \frac{\phi_{ext}(\bar{q}\omega)}{\left\{1 - \frac{4\pi e^2}{\hbar\kappa q}D^R(\bar{q}\omega)\right\}}$$

Samkvæmt rafsegulfræði

$$\phi_{sc}(\bar{q}\omega) = \epsilon^{-1}(\bar{q}\omega)\,\phi_{ext}(\bar{q}\omega)$$

$$\to \epsilon(\bar{q}\omega) = 1 - \frac{4\pi e^2}{\kappa q}\frac{1}{\hbar}D^R(\bar{q}\omega)$$

$$\equiv 1 - \frac{4\pi e^2}{\kappa q}\chi(\bar{q}\omega)$$

<u>þetta tengist</u>

ytra rafsvið $\bar{D}(\bar{q}\omega)$

Heildar -‖- $\bar{E}(\bar{q}\omega)$

spanaða -‖- $\bar{E}_{pol}(\bar{q}\omega)$

$$\bar{\nabla}\cdot\bar{D} = 4\pi z \rho_{ext}$$

$$\bar{\nabla}\cdot\bar{E} = 4\pi\left\{z\rho_{ext} + e\langle\rho_{pol}\rangle\right\}$$

fyrir A·J svörunina hefði mátt fá

$$\sigma_{kl} = \frac{ie^2}{\omega}\left\{\chi_{kl} + \frac{n}{m\kappa}\delta_{kl}\right\}$$

$$\chi_{kl} = \frac{1}{\hbar} D^R_{kl}$$

og

$$\epsilon_{kl} = 1 - \frac{\omega_p^2}{\omega^2}\delta_{kl} - \frac{4\pi e^2}{\omega^2}\chi_{kl}$$

með

$$\omega_p^2 = \frac{4\pi e^2 n}{\kappa m}$$

Almenn varðveislu lögmál (hleðsla - straumur) tengja síðan saman D^R_{kl} og D^R

(tengist Ward-Takahashi venslum)

Nálgun fyrir D^R

$$D(x,x') = -2i\, G^0(x,x')\, G^0(x',x) + \cdots$$

nákvæmt Greensfall fyrir H
með víxlverkunum

$\longrightarrow$

$$D^R(\bar{x},\bar{x}',\omega) = \hbar \sum_{\alpha\beta} \phi_\alpha(\bar{x})\, \phi^*_\beta(\bar{x})\, \phi^*_\alpha(\bar{x}')\, \phi_\beta(\bar{x}') \cdot \left\{ \frac{\theta(\mu-E_\alpha) - \theta(\mu-E_\beta)}{\hbar\omega + (E_\alpha - E_\beta) + i\eta} \right\}$$

$=$ 

$+ \cdots\cdots$

Skömmtun rafsegulsviðs

Hér þyrfti að skammta rafsegulsviðið „óháð kvarða" og „formlega Lorentz óbreytt"

En þar sem víxlverkun við Galilei-óbreytanlegt efnissvið verður athugað er venja að nota Coulomb-kvarða og ekki augsæilega Lorentz óbreyttan rithátt.

$$\boxed{\text{Jöfnur Maxwells verða uppfylltar af skömmtuðu sviðunum}}$$

$\overline{\nabla}\cdot\overline{A} = 0$ Coulomb kvarði
þver kvarði

Orka sígilds rafsegulsviðs í tómarúmi er

$$\frac{1}{8\pi}\int d\overline{x}\left\{ E^2 + B^2 \right\}$$

því er notaður Hamiltonvirkinn

$$H_{em} = \frac{1}{8\pi}\int d\overline{x}\left\{\frac{1}{c^2}\dot{A}_i(\overline{x},t)\dot{A}_i(\overline{x},t) + \left(\partial_i A_j(\overline{x}t)\partial_i A_j(\overline{x}t) - \partial_i A_j(\overline{x}t)\partial_j A_i(\overline{x}t)\right)\right\}$$

fyrir frjálst rafsegulsvið í tómarúmi.

(notað var

$$\overline{E}(\overline{x}t) = -\frac{1}{c}\partial_t\overline{A}(\overline{x}t)$$

$$\overline{B}(\overline{x}t) = \overline{\nabla}\times\overline{A}(\overline{x}t)$$
)

Körskömmtun er síðan framkvæmd þ.a. víxlreglur þarf að finna fyrir $\overline{A}$ þ.a. Heisenbergvenslin

$$i\hbar\dot{A}_i(\overline{x}t) = \left[\overline{A}_i(\overline{x}t), H\right]$$

geti hreyfingarjöfnuna

$$\left(\nabla^2 - \frac{1}{c^2}\partial_t^2\right) A_i(\bar{x}t) = -\frac{4\pi}{c} J_i(\bar{x}t)$$

þegar einnig er tekið t.t.

$$H_{int} = \int d\bar{x} \left\{ -\frac{e}{c} \bar{J}(\bar{x}t)\cdot \bar{A}(\bar{x}t) + \frac{e^2}{2mc^2} \rho(\bar{x}t) \bar{A}^2(\bar{x}t) \right\}$$

Sígilt rafsegulsvið í <u>holrúmi</u> má skrifa sem samantekt flatrabylgna

$$\bar{A}(\bar{x}t) = \sum_{l} \sum_{\tau=1}^{2} \sqrt{\frac{2\pi\hbar c^2}{\omega_l V}}\, \hat{e}_{l\tau} \left\{ a_{l\tau} e^{i(\bar{k}_l\cdot\bar{x} - \omega_l t)} + a^{+}_{l\tau} e^{-i(\bar{k}_l\cdot\bar{x} - \omega_l t)} \right\} \quad (*)$$

þar sem

$l = (n_x, n_y, n_z)$ $\quad \tau = 1, 2$: skautunarstefna

$$k_l^2 = \frac{\omega_l^2}{c^2} \qquad k_l = \frac{2\pi}{L}(n_x, n_y, n_z)$$

$$\bar{\nabla}\cdot\bar{A} = 0 \quad \rightarrow \quad \hat{e}_{l\tau}\cdot\bar{k}_l = 0$$

Ef $(*)$ er sett inn fyrir $\bar{A}$ í H_{em} fæst

$$H_{em} = \frac{1}{2}\sum_{l,\tau} \hbar\omega_l \left(a_{l\tau} a^{+}_{l\tau} + a^{+}_{l\tau} a_{l\tau} \right)$$

$a_{l\tau} \in \mathbb{C}$

því er frjálst rafsegulsvið sem safn hreintóna sveifla

Skömmtun rafsegulsviðsins fylgir því skömmtun hreintóna sveifils. Krefjumst

$$[a_{l\tau}, a^{+}_{l'\tau'}] = \delta_{ll'}\delta_{\tau\tau'}$$

$$[a_{l\tau}, a_{l'\tau'}] = 0$$

Myndað er eins og áður Fock-rúm

$$a^+_{\vec{e}\tau}|\cdots n_{\vec{e}\tau}\cdots\rangle = \sqrt{n_{\vec{e}\tau}+1}\,|\cdots n_{\vec{e}\tau}+1\cdots\rangle$$

$$a_{\vec{e}\tau}|\cdots n_{\vec{e}\tau}\cdots\rangle = \sqrt{n_{\vec{e}\tau}}\,|\cdots n_{\vec{e}\tau}-1\cdots\rangle$$

$$a_{\vec{e}\tau}|\cdots 0\cdots\rangle = 0$$

$$N_{\vec{e}\tau}|\cdots n_{\vec{e}\tau}\cdots\rangle = a^+_{\vec{e}\tau}a_{\vec{e}\tau}|\cdots n_{\vec{e}\tau}\cdots\rangle$$

$$= n_{\vec{e}\tau}|\cdots n_{\vec{e}\tau}\cdots\rangle$$

Skammta Hamiltonvirkinn er

$$H_{em} = \frac{1}{2}\sum_{\vec{e}\tau}\hbar\omega_e\left\{2a^+_{\vec{e}\tau}a_{\vec{e}\tau}+1\right\}$$

þ.s. víxlin hafa verið notuð

$$\rightarrow \boxed{H_{em} = \sum_{\vec{e}\tau}\hbar\omega_e\, a^+_{\vec{e}\tau}a_{\vec{e}\tau}}$$

ef núllpunkts orkunni er sleppt

Skammta flökt

$$\langle E_j\rangle = \langle\Psi(0)|E_j|\Psi(0)\rangle$$

tökum til einföldunar að $|\Psi(0)\rangle$ sé eigin ástand $N_{\vec{e}\tau}$ og orkunnar

t.d. $\quad |\Psi(0)\rangle = |n_1, n_2 \cdots\rangle$

$$\rightarrow \quad \langle E_j\rangle = \langle B_j\rangle = 0$$

{ til eru blönduð ástönd þ.a. $\langle E_j\rangle \neq 0 \ldots$ }

en greinilegt er að

$$\langle E_j^2(\vec{x}t)\rangle \neq 0$$

$$\langle E_j^2(\bar{x}t)\rangle = \sum_{l,\tau} \frac{\hbar\omega_l}{V}\left(n_{l\tau}+\tfrac{1}{2}\right)\left(\hat{e}_{l\tau}\right)_j^2$$

$$\rightarrow \quad \langle \bar{E}^2\rangle = \frac{1}{V}\sum_{l,\tau} \hbar\omega_l\left(n_{l\tau}+\tfrac{1}{2}\right)$$

því $\sum_j \left(\hat{e}_{l\tau}\right)_j^2 = 1$

$$\rightarrow \quad \langle \bar{E}^2\rangle_0 = \frac{1}{V}\sum_{l,\tau} \frac{\hbar\omega_l}{2}$$

$\uparrow$

fyrir tómarúmið

$\rightarrow$ $\boxed{\text{í tómarúminu flöktir rafsegulsviðið}}$

Síðan má finna víxlin fyrir vigurmættið og rafsviðið

$$\boxed{\left[A_i(\bar{x}t), \mathring{A}_j(\bar{x}'t)\right] = i4\pi\hbar c^2\, \delta_{ij}^T(\bar{x}-\bar{x}')}$$

ásamt

$$\left[A_i(\bar{x}t), A_j(\bar{x}'t)\right] = \left[\mathring{A}_i(\bar{x}t), \mathring{A}_j(\bar{x}'t)\right] = 0$$

hér er

$$\delta_{ij}^T(\bar{x}-\bar{x}') = \frac{1}{(2\pi)^3}\int d\bar{q}\, e^{-i\bar{q}\cdot(\bar{x}-\bar{x}')}\left(\delta_{ij} - \frac{q_i q_j}{q^2}\right)$$

þ.a. oft er ritað

$$\delta_{ij}^T(\bar{x}) = \left\{\delta_{ij} - \frac{\partial_i\partial_j}{\nabla^2}\right\}\delta(\bar{x})$$

Hægt er að skipta öllum vigrum
þannig í langs- og þverhluta

$$A_i = A_i^t + A_i^l$$

$$= \left\{ \delta_{ij} - \frac{\partial_i \partial_j}{\nabla^2} \right\} A_j + \frac{\partial_i \partial_j}{\nabla^2} A_j$$

því

$$\overline{\nabla} \cdot \overline{A}^t = 0 \quad \text{og} \quad \overline{\nabla} \times \overline{A}^l = 0$$

{ sést best í fourier rúminu }

---

t. þ. a. spara skriftir þá munum
við liða $\overline{A}$ á eftirfarandi hátt

$$\overline{A}(\overline{x}t) = \sum_{\overline{k}\tau} \left\{ A_{\overline{k}\tau}\, \hat{e}_{\overline{k}\tau} \frac{e^{i\overline{k}\cdot\overline{x} - i\omega t}}{\sqrt{V}} + h.c. \right\}$$

$$\longrightarrow \left[A_{\overline{k}\tau}, A^{\dagger}_{\overline{k}'\tau'} \right] = \frac{2\pi\hbar c^2}{\omega} \delta_{\overline{k},\overline{k}'}\, \hat{e}_{\overline{k}\tau} \cdot \hat{e}_{\overline{k}'\tau'}$$

Stór kassi með rúmmál $V \to \infty$ síðar
(Lotubundin jaðarskilyrði) $\omega = ck$

$$\boxed{\begin{array}{l}\text{Ísog og útgeislun sígilds} \\ \text{rafsegulsviðs} \\ \text{Fyrstastigsnálgun} \Leftrightarrow \text{Fermi gullnareglan}\end{array}}$$

{ síðan borið saman við
skammtaða rafsegulsviðið }

Eins og áður var kynnt þarf
hærri nálganir (dofnunarfræði)
til þess að ráða við línubreidd og . . . .
(Heitler, Wigner Weißkopf . . .)

notum

$$H_{int} = -\frac{e}{c} \int \overline{j}(\overline{x}t) \cdot \overline{A}(\overline{x}t)\, d\overline{x}$$

líkindastraumur í H_0

Sem verður

$$H_{int} = -\frac{e}{c}\sum_{\bar{k}\tau}\Big\{ A_{\bar{k}\tau}\, j(-\bar{k})\cdot \hat{e}_{k\tau}\,\frac{e^{-i\omega t}}{\sqrt{V}}$$

$$+ A^*_{\bar{k}\tau}\, j(\bar{k})\cdot \hat{e}^*_{k\tau}\,\frac{e^{i\omega t}}{\sqrt{V}}\Big\}$$

þar sem

$$j(\bar{k}) = \int d\bar{x}\, e^{-i\bar{k}\cdot\bar{x}}\, j(\bar{x})$$

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atómið $|0\rangle \longrightarrow |n\rangle$ | engin fasa-samheldni á milli mismunandi $\bar{k}$- pötta

Fermi gullna reglan:

$$\Gamma^{abs}_{0\to n;\bar{k}\tau} = \frac{2\pi}{\hbar}\delta(E_n - E_0 - \hbar\omega)\frac{e^2}{Vc^2}|A_{\bar{k}\tau}|^2$$

$$\cdot\left|\langle n|\, j(-k)\cdot\hat{e}_{\bar{k}\tau}|0\rangle\right|^2$$

$$A(\bar{x}) = \sum_{\bar{k}\tau}\Big\{ A_{\bar{k}\tau}\,\hat{e}_{\bar{k}\tau}\,\frac{e^{i\bar{k}\cdot\bar{x}}}{\sqrt{V}} + A^+_{\bar{k}\tau}\,\hat{e}^*_{\bar{k}\tau}\,\frac{e^{-i\bar{k}\cdot\bar{x}}}{\sqrt{V}}\Big\}$$

$$A(\bar{x}t) = e^{iH_{em}t/\hbar}\, A(\bar{x})\, e^{-iH_{em}t/\hbar}$$

$$\rightarrow A(\bar{x}t) = \sum_{\bar{k}\tau}\Big\{ A_{\bar{k}\tau}\,\hat{e}_{\bar{k}\tau}\,\frac{e^{i\bar{k}\cdot\bar{x} - i\omega t}}{\sqrt{V}} + h.c.\Big\}$$

Ísog

$$\frac{2\pi}{\hbar}\delta(E_n - E_0 - \hbar ck)$$

$$* \left|\langle n; \cdots N_{k\tau}-1 \cdots | H_{int} | 0; \cdots N_{\bar{k}\tau}\cdots\rangle\right|^2$$

$$= \frac{2\pi}{\hbar}\delta(E_n - E_0 - \hbar ck)\,\frac{e^2}{c^2V}\left|\langle n|\, j(-k)\,\hat{e}_{\bar{k}\tau}|0\rangle\right|^2$$

$$\cdot\underbrace{\left|\langle \cdots N_{\bar{k}\tau}-1 \cdots | A_{k\tau} | \cdots N_{k\tau}\cdots\rangle\right|^2}_{\frac{2\pi\hbar c^2}{\omega} N_{\bar{k}\tau}}$$

$$\hbar\omega = \hbar c k$$

$$\to \frac{1}{V}\sum_{\vec{k}} \to \int \frac{k^2\, dk\, d\Omega}{(2\pi)^3} = \int \frac{\omega^2\, d\omega\, d\Omega}{(2\pi c)^3}$$

summa $\vec{k}$ og τ

$$\to \Gamma^{abs}_{0\to n} = \frac{2\pi e^2}{\hbar^2 c^2}\,\frac{\omega_{n0}^2}{(2\pi c)^3}\int d\Omega \sum_{\tau} \left|\langle n|\, j(-\vec{k})\cdot \hat{e}_{\vec{k}\tau}\,|0\rangle\right|^2 \cdot \left|A_{\vec{k}\tau}\right|^2$$

með

$$\omega_{n0} = \frac{E_n - E_0}{\hbar}$$

Ef nú inngeislinn þekur $d\Omega$ og er skautaður í τ þá er heildar afl geislans

$$\frac{1}{V}\sum_{\vec{k}} \frac{\omega^2}{2\pi c}\left|A_{\vec{k}\tau}\right|^2 = d\Omega \int d\omega\, \frac{\omega^4}{(2\pi c)^4}\left|A_{\vec{k}\tau}\right|^2$$

→ styrkleiki á einingar tíðni

$$I(\omega) = \frac{d\Omega\, \omega^4 \left|A_{\vec{k}\tau}\right|^2}{(2\pi c)^4} \qquad \left(\frac{\text{erg}}{\text{cm}^2\text{-rad}}\right)$$

því fæst fyrir ísogs líkindin

$$\Gamma^{abs}_{0\to n} = \frac{4\pi^2 e^2}{\hbar^2 c\, \omega_{n0}^2}\, I(\omega_{n0}) \left|\langle n|\, j(-\vec{k})\, \hat{e}_{\vec{k}\tau}\,|0\rangle\right|^2$$

eins hefði mátt finna líkindin fyrir örvaðri útgeislun

$$\Gamma^{ind.emm.}_{n\to 0} = \frac{4\pi^2 e^2}{\hbar^2 c\, \omega_{n0}^2}\, I(\omega_{n0}) \left|\langle 0|\, j(\vec{k})\, \hat{e}^*_{\vec{k}\tau}\,|n\rangle\right|^2$$

$$\to \boxed{\Gamma^{abs}_{0\to n} = \Gamma^{ind.emm.}_{n\to 0}}$$

þar sem $\langle 0|\, j(\vec{k})\, \hat{e}^*_{\vec{k}\tau}\,|n\rangle = \langle n|\, j(-\vec{k})\, \hat{e}_{\vec{k}\tau}\,|0\rangle^*$

Skammtað rafsegulsvið

Viljum athuga færslur á milli ástandanna (t.d.)

$$|0;\, N_{\bar{k}_1\sigma_1},\, N_{\bar{k}_2\sigma_2} \cdots N_{\bar{k}\sigma} \cdots \rangle \quad i$$

$$|n;\, N_{\bar{k}_1\sigma_1},\, N_{\bar{k}_2\sigma_2} \cdots N_{\bar{k}\sigma}-1 \cdots \rangle \quad f$$

Í ástöndunum er engin víxlverkun á milli efnis og rafsegulsviðs.

Orka upphafs:

$$E_0 + \sum_{\bar{k}'\sigma'} \hbar c k' N_{\bar{k}'\sigma'}$$

Orka loka...:

$$E_n + \left(\sum_{\bar{k}'\sigma'} \hbar c k' N_{\bar{k}'\sigma'}\right) - \hbar c k$$

sem gefur það sama og áður fyrir

$$\Gamma^{abs}_{0\to n} \quad \text{ef} \quad |A_{\bar{k}\lambda}|^2 = \frac{2\pi\hbar c^2}{\omega} N_{\bar{k}\lambda}$$

útgeislun:

frá $\langle n; \cdots N_{\bar{k}\sigma} \cdots \rangle$

til $\langle 0; \cdots N_{\bar{k}\sigma}+1 \cdots \rangle$

$$|\langle 0; \cdots N_{\bar{k}\sigma}+1 \cdots | H_{int} | n; \cdots N_{\bar{k}\sigma} \cdots \rangle|^2$$

$$= \frac{e^2}{c^2 V} |\langle 0 | \bar{j}(\bar{k}) \cdot \hat{e}^{*}_{\bar{k}\sigma} | n \rangle|^2 \; \frac{2\pi\hbar c^2}{\omega V} (N_{\bar{k}\sigma}+1)$$

viðbót við það sem áður þekktist

$$\Gamma^{\text{emm.}}_{n\to 0;k\sigma} = \frac{4\pi \mathcal{E}^2}{\omega V}\,\delta(E_n - E_0 - \hbar\omega)\,|\langle 0|\,\bar{j}(\bar{k})\cdot \hat{e}^{*}_{k\sigma}\,|n\rangle|^2 \cdot (N_{k\sigma} + 1)$$

örvuð útgeislun

sjálf geislun

Einstein fann með hugleiðingum um jafnvægi

P_0 P_N setni 0 og N

$$(d_t N)_{\text{abs.}} = -BNP_0$$

$$(d_t N)_{\text{örð. emm.}} = BNP_n$$

$P_0 > P_n \to$ allar ljóseindir myndu hverfa

$$\to \quad (d_t N)_{\text{sp. emm.}} = A P_n$$

$$A = B \quad \to \quad (d_t N)_{\text{emm.}} = \underline{B P_n (N+1)}$$

Hér hefur verið fjallað um

<u>misfasa</u> rafsegulsvið með <u>vissa</u> ljóseindatölu

finna má

$$[E^{(+)}(\bar{x}), N] = E^{(+)}(\bar{x})$$

$$[E^{(-)}(\bar{x}), N] = -E^{(-)}(\bar{x})$$

$$\to \quad \Delta N \cdot \Delta E \geq |E|$$

ef N er nákvæmlega þekkt þá er alger óvissa í styrk rafsviðs

$$\boxed{\text{ef } N=0 \to \text{ mikið flökkt á } E}$$

$$\frac{\Delta N}{N}\,\Delta E \geq \frac{|E|}{N}$$

$N \to \infty$ $\quad \lambda$: stórt $\to$ klassískt markgildi ef $a_{\vec{k}T} = e^{i\phi_{\vec{k}T}}\sqrt{N_{\vec{k}T}}$

$$\rightarrow \quad [\phi_{\vec{k}T}, N_{\vec{k}T}] = -i$$

$$\rightarrow \quad \Delta N_{\vec{k}T} \cdot \frac{\Delta\phi_{\vec{k}T}}{1-3(\Delta\phi_{\vec{k}T})^2\frac{1}{\pi}} \geq \frac{1}{2}$$

ekki er hægt að þekkja samtímis
N og ϕ með mikilli nákvæmni

$\left\{ \begin{array}{l} \text{samfasa ástönd} \leftrightarrow \text{sett saman} \\ \text{úr ástöndum með öll möguleg } N \end{array} \right\}$

Dreifing

(2) Tímaháð : heppileg fyrir formlegan
skilning á dreifingu

(1) tímaóháð : heppileg til útreikninga

Gordon Baym, Cohen-Tannoudji (1)
Capri (1) + (2)

Athugum dreifingu skammdrægra mættis
Coulomb mættið er athugað síðar

Fyrsta Stömmtun fyrir
einda kerfi

Dreifi þversnið

hvað er mælt : fjöldi einda ~~með~~ ~~þessa orku~~ sem dreifast í hornið $(\Omega, \Omega + d\Omega)$ $\sim$ $\sigma(\theta, \varphi)$: diffursnið

eða

dN: fjöldi dreifðra einda á tíma einingu sem ferðast um flötinn $d\bar{s}$ er $\bar{J}_{scat} \cdot d\bar{s}$

$$dN = |\bar{J}_{inc}| \, \sigma(\theta, \varphi) \, d\Omega = \bar{J}_{scat} \cdot d\bar{s}$$

þar sem

$$d\Omega = \frac{\bar{n} \cdot d\bar{s}}{r^2}$$

$\bar{n}$: einingarvigur í stefnu $\bar{J}_{scat}$

Heildar árekstrar þversniðið er

$$\sigma_t = \int \sigma(\theta, \varphi) \, d\Omega$$

$$H = \frac{p^2}{2m} + V(\bar{r})$$

Samsvarandi Schrödinger jafna :

$$(\nabla^2 + k^2)\psi(\bar{r}) = U(\bar{r})\psi(\bar{r})$$

með

$$k^2 = \frac{2mE}{\hbar^2} \, , \quad U(\bar{r}) = \frac{2mV(\bar{r})}{\hbar^2}$$

skammdrægt mætti

$$r^2 V(\bar{r}) \xrightarrow[r \to \infty]{} 0$$

því er leitað að lausnum
á forminu

$$\psi(\bar{x}) = \underbrace{e^{i\bar{k}\cdot\bar{r}}}_{\text{innbylgja}} + \underbrace{\psi_{scat}(\bar{r})}_{\text{dreifð bylgja}}$$

$$\longrightarrow \quad \bar{J}_{inc} = \frac{\hbar \bar{k}}{m}$$

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flæðið (agna) í gegnum kúlu yfir borð
með stórann geisla R er

$$4\pi R^2 \langle \bar{J}_{scat} \rangle_{ave} \cdot \hat{r}$$

með

$$\bar{J}_{scat}\cdot\hat{r} = \frac{\hbar}{2im}\left\{\psi^*_{scat}\frac{\partial}{\partial r}\psi_{scat} - \left(\frac{\partial}{\partial r}\psi^*_{scat}\right)\psi_{scat}\right\}$$

Þetta flæði út ⟶ fasta fyrir $R \to \infty$

$$\longrightarrow \langle \psi^*_{scat}\,\partial_r\,\psi_{scat}\rangle_{ave} \xrightarrow[R\to\infty]{} \frac{\text{fasti}}{R^2}$$

því er vani að skilgreina $T(\bar{k},\bar{k}')$ eða
$f(k,\theta,\varphi)$ þ.a.

$$\lim_{R\to\infty} \psi_{scat}(R) = -\frac{1}{4\pi}T(\bar{k},\bar{k}')\frac{e^{ikR}}{R}$$

$$= f(k,\theta,\varphi)\frac{e^{ikR}}{R}$$

fyrir miðjumætti er $f(k,\theta,\varphi) = f(k,\theta)$

$\uparrow$

dreifivísir

flæði dreifðra einda um $d\Omega$

$$\underbrace{R^2 d\Omega \ \frac{\hbar k}{m} |f(k,\theta,\varphi)|^2 \frac{1}{R^2}}_{J_{scat}}$$

$$= \underbrace{\frac{\hbar k}{m}}_{J_{inc}} |f(k,\theta,\varphi)|^2 d\Omega$$

$$\longrightarrow \ \sigma(\theta,\varphi) = \underbrace{|f(k,\theta,\varphi)|^2}$$

það sem er mælt

φ – fylgi hverfur fyrir miðjumætti

en getur komið vegna

spuna – spuna víxverkunar

athugið !!

oft er notað í stað $\sigma(\theta,\varphi)$ hér skilgreiningin

$$dN = |\bar{J}_{inc}| \frac{d\sigma}{d\Omega} d\Omega$$

$$\sigma_t = \int \frac{d\sigma}{d\Omega} d\Omega$$

og því

$$\frac{d\sigma}{d\Omega} = |f(k,\theta,\varphi)|^2$$

þessum táknunum er því miður

líka ruglað saman (sjá Capri)

við notum

án breyta

$$\frac{d\sigma}{d\Omega} = \sigma(\theta,\varphi)$$

Greensföll og Lippmann-Schwinger jöfnurnar

Nú þarf að leysa

$$(\nabla^2+k^2)\psi(\bar r) = U(\bar r)\psi(\bar r) \qquad (1)$$

athuga síðan aðfellu form lausnarinnar til þess að finna $|f(k,\theta,\varphi)|^2 = \sigma(\theta,\varphi)$

heppilegast er að nota Greensföll til þess að skrifa lausnina

Við munum því sýna að lausnina má skrifa sem (finna G þ.a. (1) megi skrifa sem heild...)

$$\psi(\bar r) = \psi^{(0)}(\bar r) + \int G(\bar r-\bar r')U(\bar r')\psi(\bar r')d\bar r'$$

ef

óháð U

$$(\nabla^2+k^2)G(\bar r-\bar r') = \delta(\bar r-\bar r')$$

$$(\nabla^2+k^2)\psi^{(0)}(\bar r) = 0 \quad \text{frjáls innbylgja}$$

kostur við heildis).

finnum fyrst G :

fourier ummyndum, miðju mætti

$$G(\bar r-\bar r') = \frac{1}{(2\pi)^3}\int d\bar q\; e^{i\bar q\cdot(\bar r-\bar r')}G(\bar q)$$

$$\delta(\bar r-\bar r') = \frac{1}{(2\pi)^3}\int d\bar q\; e^{i\bar q\cdot(\bar r-\bar r')}$$

Sett inn í jöfnuna fyrir G

$$\longrightarrow \quad G(\bar q) = \frac{1}{k^2-q^2}$$

og því

$$G(\bar r) = -\frac{1}{(2\pi)^3}\int d\bar q\; \frac{e^{i\bar q\cdot\bar r}}{q^2-k^2}$$

$G(\bar q)$ hefur sérstöðupunkta því eru til mismunandi leiðir til þess að reikna heildið sem gefa <u>mismunandi svör</u>

athugum tvo möguleika

$$G^{\pm}(\bar{r}) = \lim_{\epsilon\to 0^+}\left\{-\frac{1}{(2\pi)^3}\int d\bar{q}\, \frac{e^{i\bar{q}\cdot\bar{r}}}{q^2-(k^2\pm i\epsilon)}\right\}$$

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veljum $\bar{q} \parallel \hat{z}$

$$\to \int d\bar{q}\, \frac{e^{i\bar{q}\cdot\bar{r}}}{q^2-(k^2\pm i\epsilon)} = 2\pi\int q^2 dq\, d\cos\theta\, \frac{e^{iqr\cos\theta}}{q^2-k^2\mp i\epsilon}$$

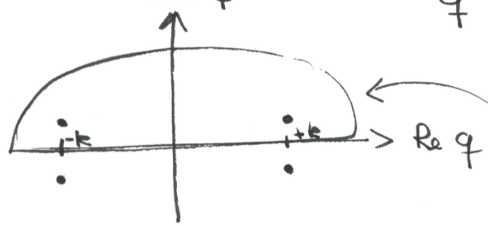
$$= 2\pi\int_0^\infty q^2 dq\, \frac{1}{q^2-k^2\mp i\epsilon}\int_{-1}^{1} du\, e^{iqru}$$

$$= \frac{2\pi}{ir}\int_0^\infty \frac{q\,dq}{q^2-k^2\mp i\epsilon}\left\{e^{iqr}-e^{-iqr}\right\}$$

$$\to G^{\pm}(\bar{r}) = -\frac{1}{(2\pi)^2}\,\frac{1}{ir}\int_{-\infty}^{\infty} q\,dq\, \frac{e^{iqr}}{q^2-k^2\mp i\epsilon}$$

sérstöðupunktar

$$q = \pm(k+i\epsilon)$$
$$q = \pm(k-i\epsilon)$$



lokað í efra plani þar sem e^{iqr} deyr út

$r>0$

því fæst

$$G^{\pm}(\bar{r}) = -\frac{1}{4\pi}\,\frac{e^{\pm ikr}}{r}$$

sem er eiginlausn schrödinger jöfnunnar

$\to$ tímaháðu lausnirnar eru

$$-\frac{1}{4\pi r}\,e^{\pm ikr - iE_k t/\hbar}$$

$\to$ G^+ er Greensfall fyrir útstreymi agna

G^- — || — innstreymi agna

við leitum lausna þar sem eindir dreifast út frá marki

$\longrightarrow$ notum G^{+} hér

sem er í raun seinkaða Greens fallið

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lausnirnar eru

$$\psi^{(\pm)}(\vec r) = \psi^{(0)}(\vec r) + \int G^{\pm}(\vec r-\vec r')\,U(\vec r')\,\psi^{(\pm)}(\vec r')\,d\vec r'$$

athugum látum (∇^2+k^2) verka báðum megin

$$(\nabla^2+k^2)\,\psi^{(\pm)}(\vec r) = 0 + \int d\vec r'\,\delta(\vec r-\vec r')\,U(\vec r')\,\psi^{(\pm)}(\vec r')$$

$$= U(\vec r)\,\psi^{(\pm)}(\vec r)$$

því eru $\psi^{(\pm)}(\vec r)$ lausnir á

$$(\nabla^2+k^2)\,\psi = U\psi$$

Lippmann framsetn.

Í stað

$$(\nabla^2+k^2)\,G(\vec r-\vec r') = \delta(\vec r-\vec r')$$

má finna virkjann $G(E)$:

$$\boxed{(-H_0+E)\,G(E) = 1}$$

höfum einnig

$$(E-H_0)\,|\psi(E)\rangle = V\,|\psi(E)\rangle$$

$$(E-H_0)\,|\psi^0(E)\rangle = 0$$

$\longrightarrow$ fyrir $E = z \in \mathbb{C}$ höfum við þá

$$G(z) = (z-H_0)^{-1}$$

sem er vel skilgreint því $|\psi_0\rangle$ er fullkomið mengi og

$$G(z)\,|\psi^0(E)\rangle = \frac{1}{z-E}\,|\psi^0(E)\rangle$$

Greens föllin (virkjarnir)

$$G^{\pm}(E) = \frac{1}{E - H_0 \pm i\epsilon}$$

hafa því staðarrúms framsetninguna

$$G^{\pm}(r) = -\frac{1}{4\pi r} e^{ikr}$$

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hliðraða jafnan

$$(E - H_0)|\psi(E)\rangle = V|\psi(E)\rangle$$

hefur lausnina

$$|\psi(E)\rangle = (E - H_0 \pm i\epsilon)^{-1} V|\psi(E)\rangle$$

auk þess sem óhliðraða jafnan hefur lausnina

$$|\psi_0\rangle$$

því er heildar lausnin (Lippmann Schwinger)

$$|\psi^{\pm}(E)\rangle = |\psi_0(E)\rangle + \frac{1}{E - H_0 \pm i\epsilon} V|\psi^{\pm}(E)\rangle$$

Hægt er að sýna að þessi jafna jafngildi

$$\psi^{(\pm)}(\bar{r}) = \psi^{(0)}(\bar{r}) + \int d\bar{r}' G(\bar{r}-\bar{r}') U(\bar{r}') \psi^{\pm}(\bar{r}')$$

í staðar rúminu:

$$\psi^{(\pm)}(\bar{r}) = \langle \bar{r}|\psi^{\pm}(E)\rangle \quad , E = \frac{\hbar^2 p^2}{2m}$$

$$\psi^{(\pm)}(\bar{r}) = \psi^{(0)}(\bar{r}) + \langle \bar{r}|\frac{1}{E - H_0 \pm i\epsilon} V|\psi^{\pm}(E)\rangle$$

$$= \psi^{(0)}(\bar{r}) + \int d\bar{k}\, d\bar{q}\, d\bar{r}'\, d\bar{r}'' \langle \bar{r}|\bar{k}\rangle$$

$$\cdot \langle \bar{k}|\frac{1}{E - H_0 \pm i\epsilon}|\bar{q}\rangle\langle \bar{q}|\bar{r}'\rangle$$

$$\langle \bar{r}'|V|\bar{r}''\rangle\langle \bar{r}''|\psi^{\pm}(E)\rangle$$

p.a.

$$\psi^{(\pm)}(\bar r) = \psi^{(0)}(\bar r) + \frac{1}{(2\pi)^3}\int \frac{d\bar r'\, d\bar k}{E - \frac{\hbar^2 k^2}{2m} \pm i\epsilon}$$

$$\cdot\, e^{i\bar k\cdot(\bar r - \bar r')}\, V(\bar r')\, \psi^{(\pm)}(\bar r')$$

þar

local

$$\langle \bar r' | V | \bar r'' \rangle = \delta(\bar r' - \bar r'')\, V(\bar r')$$

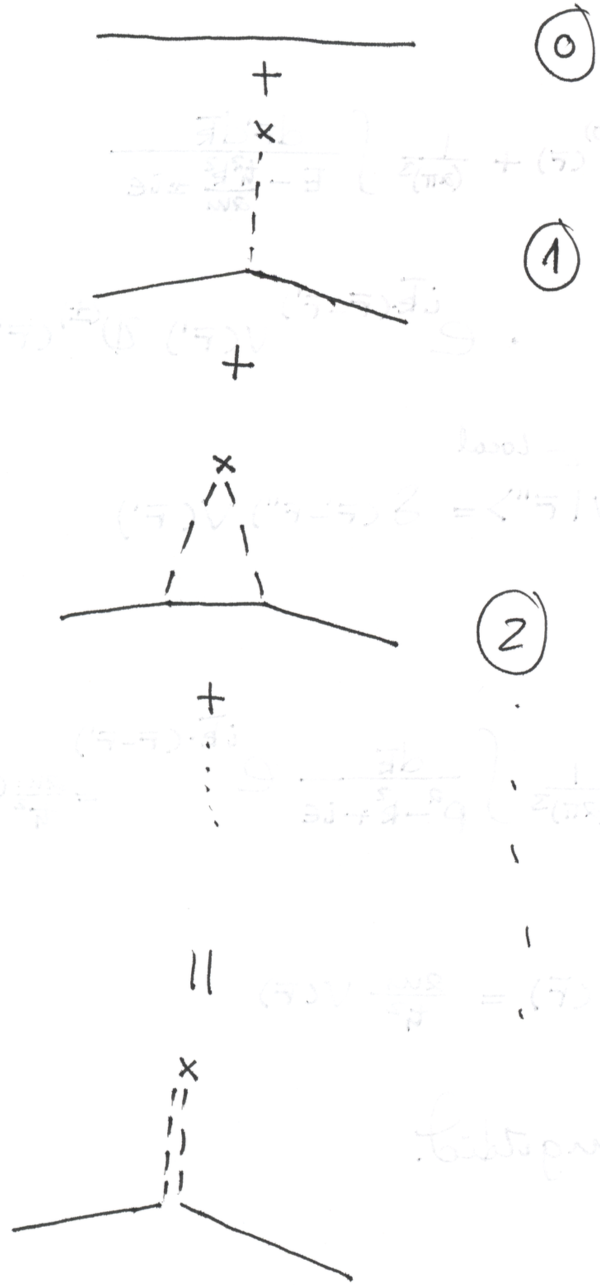
nota

$$\frac{2m}{\hbar^2}\,\frac{1}{(2\pi)^3}\int \frac{d\bar k}{p^2 - k^2 + i\epsilon}\, e^{i\bar k\cdot(\bar r - \bar r')} = \frac{2m}{\hbar^2}\, G^{\pm}(\bar r - \bar r')$$

og

$$U(\bar r) = \frac{2m}{\hbar^2}\, V(\bar r)$$

sýnir jafngildið.



Born nálgunin

$$|\psi^{\pm}(E)\rangle = |\psi^{(0)}(E)\rangle + \frac{1}{E-H_0 \pm i\epsilon} V |\psi^{\pm}(E)\rangle$$

má ítra sem:

Einnig hægt að leysa heildisjöfnuna tölulega án þessarar legundar ófunnar

$$|\psi^{\pm}(E)\rangle = \sum_{n=0}^{\infty} \left(\frac{1}{E-H_0 \pm i\epsilon} V\right)^n |\psi^{(0)}(E)\rangle$$

ef „$V \ll E$" { Neumann röð, Born röð }

1. stigs nálgun: (mat á gæðum hennar sést í Morse og Feshbach bls 1689)

$$|\psi^{+}(E)\rangle = |\psi^{(0)}(E)\rangle + \frac{1}{E-H_0 + i\epsilon} V |\psi^{(0)}(E)\rangle$$

eða

$$\psi^{+}(\bar{r}) = e^{i\bar{k}\cdot\bar{r}} + \frac{2m}{\hbar^2}\int d\bar{r}'\, G^{+}(\bar{r}-\bar{r}')V(\bar{r}')e^{i\bar{k}\cdot\bar{r}'}$$

$$= e^{i\bar{k}\cdot\bar{r}} - \frac{2m}{\hbar^2}\frac{1}{4\pi}\int d\bar{r}'\, \frac{e^{ik|\bar{r}-\bar{r}'|}}{|\bar{r}-\bar{r}'|} V(\bar{r}') e^{i\bar{k}\cdot\bar{r}'}$$

en við leitum að markgildinu

$$\psi_{scat}(R) \xrightarrow[R\to\infty]{} f(k,\theta)\frac{e^{ikR}}{R}$$

$\boxed{\cos\alpha = \hat{r}\cdot\hat{r}'}$

leitum að lausn fyrir $|\bar{r}| \to \infty$

$$|\bar{r}-\bar{r}'|^2 = r^2 - 2rr'\cos\alpha + r'^2$$

$$= r^2\left(1 - 2\frac{r'}{r}\cos\alpha + \left(\frac{r'}{r}\right)^2\right)$$

$$\xrightarrow[r\to\infty]{} r^2\left(1 - \frac{2r'}{r}\cos\alpha\right) + O\left(\left(\frac{r'}{r}\right)^2\right)$$

þess vegna

$$|\bar{r}-\bar{r}'| \xrightarrow[r\to\infty]{} r\sqrt{\left(1 - \frac{2r'}{r}\cos\alpha\right)}$$

$$\xrightarrow[r\to\infty]{} r - r'\cos\alpha$$

og
$$\frac{1}{|\bar r - \bar r'|} \xrightarrow[r\to\infty]{} \frac{1}{r\sqrt{1-\frac{2r'}{r}\cos\alpha}}$$

$$\xrightarrow[r\to\infty]{} \frac{1}{r} + \frac{r'\cos\alpha}{r^2}$$

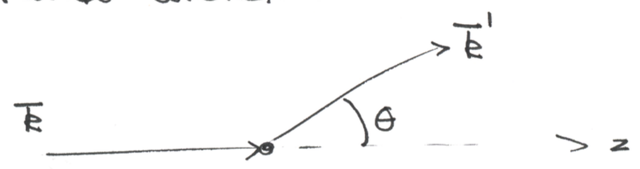
leitum lausna á forminu $\frac{e^{ikr}}{r}$ því nægir lægsta nálgun í nefnara en meira þarf í veldisvísi

$$\frac{e^{ik|\bar r-\bar r'|}}{|\bar r - \bar r'|} \xrightarrow[r\to\infty]{} \frac{e^{ik(r-r'\cos\alpha)}}{r}$$

þar sem $V(r)$ er staðsetið er leyft að skipta um röð á heildi og markgildi.

$$\psi^+_{Born}(\bar r) \xrightarrow[r\to\infty]{} e^{i\bar k\cdot\bar r} - \frac{2m}{\hbar^2}\frac{1}{4\pi}\frac{e^{ikr}}{r}\int d\bar r'\, e^{i\bar k\cdot\bar r' - ikr'\cos\alpha}\, V(r')$$

fjaðrandi árekstur



$\to$ $|\bar k| = |\bar k'|$ ef markið er fast.

$$i\bar k\cdot\bar r' - ikr'\cos\alpha = i\bar k\cdot\bar r' - ik'r'\cos\alpha$$

Ef nú innbyrðis lega $\bar k'$ og $\bar r$ er þannig valin að $\bar k' \parallel \bar r$ þá fæst

$$\bar k'\cdot\bar r' = k\hat r\cdot\bar r' = kr'\cos\alpha$$
$$= k'r'\cos\alpha$$

og
$$i\bar k\cdot\bar r' - ikr'\cos\alpha = i(\bar k-\bar k')\cdot\bar r'$$

skilgreinum skriðþunga breytingu í árekstri

$$\bar q = \bar k - \bar k'$$

því fæst

$$\psi^{+}_{Born}(\bar{r}) \xrightarrow[r\to\infty]{} e^{i\bar{k}\cdot\bar{r}} - \frac{1}{4\pi}\frac{e^{ikr}}{r}\int d\bar{r}' e^{i\bar{q}\cdot\bar{r}'} U(r')$$

þar sem $U = \frac{2m}{\hbar}V$, með fourier ummyndun

$$U(\bar{q}) = \int d\bar{r}' e^{i\bar{q}\cdot\bar{r}'} U(r')$$

fæst

$$\psi^{+}_{Born}(\bar{r}) \xrightarrow[r\to\infty]{} e^{i\bar{k}\cdot\bar{r}} - \frac{1}{4\pi} U(\bar{q}) \frac{e^{ikr}}{r}$$

og því samkvæmt skilgreiningu

$$f(k,\theta) = -\frac{1}{4\pi} U(\bar{k}-\bar{k}')$$

$$\longrightarrow \frac{d\sigma}{d\Omega} = \frac{1}{16\pi^2}\left|U(\bar{k}-\bar{k}')\right|^2$$

Athugasemd

Heildar dreifiþversnið

$$\sigma_t = \int \frac{d\sigma}{d\Omega} d\Omega \neq 1$$

því

$$dN = |\bar{J}_{inc}| \frac{d\sigma}{d\Omega} d\Omega = \bar{J}_{scat} \cdot d\bar{s}$$

σ_t: er heildar líkur á því að ögn „dreifist" deilt með heildar líkinda flæði agna á einingarflöt fyrir framan mark

$$||$$

líkindi fyrir því að ögn sé tekin úr inn geislanum

Yukawa-mætti

$$V(r) = -V_0 \frac{e^{-\alpha r}}{r}$$

Born nálgun

$$\longrightarrow u(\bar{q}) = -\frac{2mV_0}{\hbar^2}\int d\bar{r}\, e^{i\bar{q}\cdot\bar{r}-\alpha r}\frac{1}{r}$$

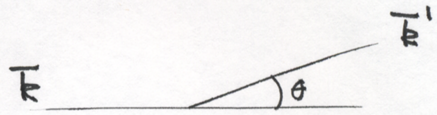
$$= -\frac{2mV_0}{\hbar^2}\int r\,dr\,d\varphi\int_{-1}^{1} d\cos\theta'\, e^{iqr\cos\theta' - \alpha r}$$

$$= -\frac{2mV_0}{\hbar^2}\frac{2\pi}{iq}\int_0^\infty dr\left\{e^{-r(\alpha-iq)} - e^{-r(\alpha+iq)}\right\}$$

$$= -\frac{2mV_0}{\hbar^2}\frac{4\pi}{q^2+\alpha^2}$$

$$\longrightarrow f(k,\theta) = \frac{2mV_0}{\hbar^2}\frac{1}{(\bar{k}-\bar{k}')^2+\alpha^2}$$

$$\bar{q} = \bar{k} - \bar{k}' \qquad |\bar{k}| = |\bar{k}'| = k$$



$$|\bar{q}|^2 = k^2 + k^2 - 2k^2\cos\theta$$

$$= 2k^2(1-\cos\theta) = 4k^2\sin^2\frac{\theta}{2}$$

$$\frac{d\sigma}{d\Omega} = |f(k,\theta)|^2$$

$$= \frac{4m^2V_0^2}{\hbar^4}\frac{1}{\left\{4k^2\sin^2\frac{\theta}{2}+\alpha^2\right\}^2}$$

óháð φ $\longrightarrow$

$$\frac{d\sigma}{d\cos\theta} = 2\pi\frac{4m^2V_0^2}{\hbar^4}\frac{1}{\left\{4k^2\sin^2\frac{\theta}{2}+\alpha^2\right\}^2}$$

ef $V_0 = Ze^2$

$$\lim_{\alpha \to 0} \frac{d\sigma}{d\cos\theta} = 2\pi \frac{4m^2 Z^2 e^4}{16\hbar^4 k^4 \sin^4\frac{\theta}{2}}$$

$$= \frac{Z^2 e^4}{16 E^2 \sin^4\frac{\theta}{2}}$$

<u>Rutherford þversnidid</u>

Er rétt og kemur heim og saman við niðurstöður sígildrar eðlisfræði en aðferðin er ekki leyfileg fyrir $\alpha = 0$

<u>athugum</u>

fyrir skammseilin mætti gildir

$$R(r) \underset{r\to\infty}{\sim\!\!>} \frac{e^{\pm ikr}}{r}$$

þ.e. frjálar kúlu bylgur

fyrir Coulomb mætti gildir hins vegar (sést frá Schrödinger jöfnu)

$$R(r) \underset{r\to\infty}{\sim\!\!>} \frac{e^{\pm i(kr - \gamma \ln r)}}{r}, \quad E>0$$

sem sé ekki frjálsar kúlu bylgjur

leysa part $\left(-\frac{\hbar^2}{2m}\nabla^2 - \frac{e^2}{r}\right)\psi(r) = E\psi(r)$

fyrir $E>0$ $\quad \gamma = -\frac{me^2}{\hbar^2 k}$

nota fleygboga hnit (α, β, z)

$$x = \frac{c}{2}(\alpha^2 - \beta^2) \qquad -\infty < \alpha < \infty$$

$$y = c\alpha\beta \qquad 0 \leq \beta < \infty$$

$$z = z \qquad -\infty < z < \infty$$

$$\to \psi(\bar{r}) = \Gamma(1+i\gamma)\, e^{-\pi\gamma/2}\, e^{ikz}\, {}_1F_1(-i\gamma; 1; ik(r-z))$$

$(r-z) \to \infty$

$$\psi(\bar r) := \psi_i(\bar r) + \psi_{sc}(\bar r)$$

$$\psi_i(\bar r) \to e^{ikz + i\gamma \ln k(r-z)}\left(1 - \frac{\gamma^2}{ik(r-z)}\right)$$

$$\psi_{sc}(\bar r) \to \frac{e^{ikr - i\gamma \ln k(r-z)}}{ik(r-z)} \frac{\Gamma(1+i\gamma)}{\Gamma(-i\gamma)}$$

$$\to \psi_{sc}(r) \to \frac{e^{i(kr - \gamma \ln 2kr)}}{r} f_c(\theta)$$

med

$$f_c(\theta) = \frac{e^{-i\gamma \ln \sin^2\theta/2}}{2ik \sin^2\theta/2} \frac{\Gamma(1+i\gamma)}{\Gamma(-i\gamma)}$$

$$= -\frac{\gamma}{2k} e^{2i\delta_0} \left(\sin^2\frac{\theta}{2}\right)^{-i\gamma-1}$$

med

$$e^{2i\delta_0} = \frac{\Gamma(1+i\gamma)}{\Gamma(1-i\gamma)}$$

$$\to \frac{d\sigma}{d\Omega} = |f_c(\theta)|^2 = \frac{e^4}{16E^2} \frac{1}{\sin^4\frac{\theta}{2}}$$

$$u(r) = \frac{2m}{\hbar^2} V(r)$$

$$V(r) = V_0\,\theta(a-r)$$ „mjäk kula"

$$\rightarrow u(r) = \frac{2m}{\hbar^2} V_0\,\theta(a-r) = u_0\,\theta(a-r)$$

$$\tilde{u}(\bar{q}) = \int d\bar{r}\, e^{i\bar{q}\cdot\bar{r}}\, u(\bar{r})$$

uzmimo $\bar{q} = \hat{z} q$

$$\rightarrow \tilde{u}(\bar{q}) = \int d\bar{r}\, e^{iqr\cos\theta}\, u(r)$$

$$= \int_0^a r^2 dr \int_0^{2\pi} d\varphi \int_{-1}^{1} d(\cos\theta)\, e^{iqr\cos\theta}\, u_0$$

$$= 2\pi u_0 \int_0^a r^2 dr\, \frac{1}{iqr}\left\{ e^{iqr} - e^{-iqr} \right\}$$

$$= 2\pi u_0\, \frac{2}{q} \int_0^a r^2 dr\, \sin(qr)$$

---

nota

$$\int x^2 dx \sin ax = -\sum_{k=0}^{2} k! \binom{2}{k} \frac{x^{2-k}}{a^{k+1}} \cos\left(ax + \frac{k\pi}{2}\right)$$

$$= -\cancel{\frac{x^2}{a}\cos(ax)} - 2\frac{x}{a^2}\cos\left(ax + \frac{\pi}{2}\right)$$

$$- 2\frac{1}{a^3}\cos(ax + \pi)$$

$$= -\cancel{\frac{x^2}{a}\cos(ax)} - \frac{2x}{a^2}\sin(ax)$$

$$+ \frac{2}{a^3}\cos(ax)$$

$$\rightarrow \tilde{u}(q) = 2\pi u_0 \frac{2}{q}\left\{ -\frac{r^2}{q}\cos(qr) - \frac{2r}{q^2}\sin(qr) + \frac{2}{q^3}\cos(qr) \right\}\Bigg|_0^a$$

$$\rightarrow \tilde{u}(q) = 2\pi u_0 \frac{2}{q}\left\{-\frac{a^2}{q}\cos(aq) - \frac{2a}{q^2}\sin(aq) + \frac{2}{q^3}\cos(aq) - \frac{2}{q^3}\right\}$$

$$= 2\pi u_0 \frac{2}{[illegible]}\left\{\left(\frac{2}{q^2} - a^2\right)\cos(aq) - \frac{2a}{q}\sin(aq) - \frac{2}{q^2}\right\}$$

$$\tilde{u}(q) = \frac{4\pi u_0}{[illegible]}\left\{\left(\frac{2}{q^2} - a^2\right)\cos(aq) - \frac{2a}{q}\left(\sin(aq) + \frac{1}{qa}\right)\right\}$$

bera saman við Yukawa

$$\tilde{u}(q) = -4\pi u_0 \frac{1}{q^2+\alpha^2}$$

ef $V(r) = -V_0 \frac{e^{-\alpha r}}{r}$

Born nálgunin er sérlega góð þegar $E >> V_0$ hér. Ekki er hægt að nota hana til þess að kanna markgildið þegar $V_0 \rightarrow \infty$

þá fæst

$$\sigma_t \rightarrow 2\pi a^2$$

(fyrir harða kúlu)

þessar niðurstöður verður að finna með hlutbylgju greiningu á Schrödinger jöfnunni

$\boxed{\text{skíra } 2}$

Hlutbylgju greining

frjálsa Schrödinger jafnan í kúluhnitum

$$(\nabla^2 + k^2)\psi(\bar{r}) = 0$$

hefur lausnir

$$\psi(\bar{r}) = R_\ell(r)\, Y_{\ell m}$$

þar sem $R_\ell(r)$ er einhver samantekt kúlu Bessel falla $j_\ell(kr)$ og $n_\ell(kr)$ með:

$$j_\ell(x) = \sqrt{\frac{\pi}{2x}}\, J_{\ell+1/2}(x)$$

$$n_\ell(x) = \sqrt{\frac{\pi}{2x}}\, N_{\ell+1/2}(x)$$

$$j_n(x) = (-x)^n \frac{d^n}{(x\,dx)^n}\left(\frac{\sin x}{x}\right)$$

$$j_0(x) = \frac{\sin x}{x}$$

$$j_1(x) = \left(\frac{\sin x}{x^2} - \frac{\cos x}{x}\right)$$

$$n_0(x) = -\frac{1}{x}\cos x$$

$$n_1(x) = -\frac{1}{x}\left(\frac{\cos x}{x} + \sin x\right)$$

$$\vdots$$

Eins má mynda Hankel föll

$$h_\ell^{(1)}(x) = j_\ell(x) + i\, n_\ell(x)$$

$$h_\ell^{(2)}(x) = j_\ell(x) - i\, n_\ell(x)$$

Aðfellur eru:

$$j_\ell(x) \xrightarrow[x\to\infty]{} \frac{1}{x}\sin\left(x - \frac{\ell\pi}{2}\right)$$

$$n_\ell(x) \xrightarrow[x\to\infty]{} -\frac{1}{x}\cos\left(x - \frac{\ell\pi}{2}\right)$$

$$h_\ell^{(1)}(x) \xrightarrow[x\to\infty]{} \frac{1}{x}\, e^{i\left(x - \frac{\ell+1}{2}\pi\right)}$$

$$h_\ell^{(2)}(x) \xrightarrow[x\to\infty]{} \frac{1}{x}\, e^{-i\left(x - \frac{\ell+1}{2}\pi\right)}$$

og

$$j_\ell(x) \xrightarrow[x\to 0]{} \frac{x^\ell}{(2\ell+1)!!}$$

$$n_\ell(x) \xrightarrow[x\to 0]{} -\frac{(2\ell-1)!!}{x^{\ell+1}}$$

$$h_\ell^{(1)}(x) \xrightarrow[x\to 0]{} -i\,\frac{(2\ell-1)!!}{x^{\ell+1}}$$

$$h_\ell^{(2)}(x) \xrightarrow[x\to 0]{} i\,\frac{(2\ell-1)!!}{x^{\ell+1}}$$

Ef $\bar{k} = k\hat{z}$ þá fæst

$$\boxed{e^{i\bar{k}\cdot\bar{r}} = \sum_{\ell=0}^{\infty} i^\ell (2\ell+1) P_\ell(\cos\theta)\, j_\ell(kr)}$$

$$e^{i\bar{k}\cdot\bar{r}} = e^{ikr\cos\theta} \qquad P_\ell(\cos\theta) = \sqrt{\frac{4\pi}{2\ell+1}}\, Y_{\ell 0}(\theta,\varphi)$$

Dreifing agna vegna mættis $U(r)$

Schrödinger jafnan er

$$\left(\nabla^2 + k^2 - U(r)\right)\psi(\bar{r}) = 0$$

mættið er kúlusamhverft $\longrightarrow$ skammtatölurnar (m,ℓ) haldast óbreyttar við árekstur. Því nægir að kanna dreifingu hvers þáttar sér.

upphafs bylgjan $e^{i\bar{k}\cdot\bar{r}}$ hefur engann $m \neq 0$ þátt (engin hverfiþunga um $\bar{k} \parallel \hat{z}$)

því er hægt að leita lausna á forminu

$$\psi(\bar{r}) = \sum_{\ell=0}^{\infty} \underbrace{i^\ell (2\ell+1)}_{\text{Valið til einföldunar síðar}} R_\ell(r)\, P_\ell(\cos\theta)$$