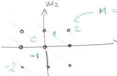
Dæmi um samlagningu hverfiþunga

$$j_1 = 1, \quad j_2 = 1$$

$M = m_1 + m_2$

$|j_1 - j_2| \leq J \leq j_1 + j_2$

$\rightarrow J = 2, 1, 0$



$$|J, M\rangle = \sum_{m_1=-j_1}^{j_1} \sum_{m_2=-j_2}^{j_2} |j_1 j_2; m_1 m_2\rangle\langle j_1, j_2; m_1 m_2 | JM\rangle$$

þurfum að finna, með

$$J_- = (J_{1-} + J_{2-})$$

Notum

$$j_{\pm} |j, m\rangle = \hbar\sqrt{j(j+1) - m(m \pm 1)}\, |j, m-1\rangle$$

Byrjum með almenna

$$J_- |J, M\rangle = (J_{1-} + J_{2-}) |j_1 + j_2, \overbrace{j_1 + j_2}^{M}\rangle = \hbar\sqrt{2(j_1 + j_2)}\, |\overbrace{j_1 + j_2}^{J}, \overbrace{j_1 + j_2 - 1}^{M-1}\rangle$$

$$= \sqrt{J(J+1) - J(J-1)}\, |J, M-1\rangle$$

Ef $M = j_1 + j_2$

(a)

$j_1 = 1 \;,\; j_2 = 1$

Ülekirjutus $J = 2$

ainuvõimalus $|j_1 + j_2, j_1 + j_2\rangle$

$|2,2\rangle = |1,1; 1,1\rangle$

$J_- |2,2\rangle = 2\hbar |2,1\rangle \longrightarrow |2,1\rangle = \frac{1}{2\hbar} J_- |2,2\rangle$

$$|2,1\rangle = \frac{1}{2\hbar}(J_{1-} + J_{2-}) |1,1; 1,1\rangle$$

$$= \frac{1}{2\hbar}\left\{\hbar\sqrt{2}\,|1,1; 0,1\rangle + \hbar\sqrt{2}\,|1,1; 1,0\rangle\right\}$$

$$= \frac{1}{\sqrt{2}}\left\{|1,1; 1,0\rangle + |1,1; 0,1\rangle\right\}$$

Kõrvuti ülekirjutud meie J_-

$$J_-|2,1\rangle = \hbar\sqrt{3}\,|2,0\rangle \quad \rightarrow \quad |2,0\rangle = \frac{1}{\hbar\sqrt{3}} J_-|2,1\rangle$$

$$\rightarrow |2,0\rangle = \frac{1}{\hbar\sqrt{3}}(J_{1-}+J_{2-})\frac{1}{\sqrt{2}}\Big\{|1,1;1,0\rangle + |1,1;0,1\rangle\Big\}$$

$$= \frac{1}{\hbar\sqrt{6}}\Big\{\hbar\sqrt{2}\,|1,1;0,0\rangle + \hbar\sqrt{2}\,|1,1;1,-1\rangle$$

$$\hbar\sqrt{2}\,|1,1;-1,1\rangle + \hbar\sqrt{2}\,|1,1;0,0\rangle\Big\}$$

$$= \frac{1}{\sqrt{6}}\Big\{|1,1;-1,1\rangle + 2\,|1,1;0,0\rangle + |1,1;1,-1\rangle\Big\}$$

↑ $\sqrt{2}$ tetim i Leid t.p.a ästentid së ucned

(c)

$$J_-|2,0\rangle = \hbar\sqrt{6}\,|2,-1\rangle$$

$$\rightarrow \quad |2,-1\rangle = \frac{1}{\hbar\sqrt{6}}\left(J_{1-}+J_{2-}\right)|2,0\rangle$$

$$= \frac{1}{\hbar\sqrt{6}}\frac{1}{\sqrt{6}}\left(J_{1-}+J_{2-}\right)\left\{|1,1;-1,1\rangle + 2|1,1;0,0\rangle + |1,1;1,-1\rangle\right\}$$

$$= \frac{1}{\hbar 6}\left\{0 + \hbar\sqrt{2}|1,1;-1,0\rangle + \hbar 2\sqrt{2}|1,1,-1,0\rangle + \hbar 2\sqrt{2}|1,1,0,-1\rangle + \sqrt{2}\hbar|1,1,0,-1\rangle + 0\right\}$$

$$= \frac{\sqrt{2}}{\hbar 6}\left\{3\hbar|1,1;-1,0\rangle + 3\hbar|1,1;0,-1\rangle\right\}$$

$$= \frac{1}{\sqrt{2}}\left\{|1,1;-1,0\rangle + |1,1;0,-1\rangle\right\}$$

↑ [illegible]

$$J_- |2,-1\rangle = \hbar 2 \, |2,-2\rangle$$

$$|2,-2\rangle = \frac{1}{2\hbar}(J_{1-} + J_{2-})\frac{1}{\sqrt{2}}\{|1,1;-1,0\rangle + |1,1;0,-1\rangle\}$$

$$= \frac{1}{2\hbar\sqrt{2}}\{0 + 0 + \hbar\sqrt{2}\,|1,1;-1,-1\rangle \cdot 2\}$$

$$= |1,1;-1,-1\rangle$$

$J = 1$ hluttvein

Byrjum með $|1,1\rangle$ getur ~~aðeins~~ ~~verið~~ sett saman sem

$$|1,1\rangle = \alpha|1,1;1,0\rangle + \beta|1,1;0,1\rangle$$

verður að vera hornrétt

$$\langle 1,1|2,1\rangle = 0$$

en ~~þessi~~ ástand fundum við í

$$|2,1\rangle = \frac{1}{\sqrt{2}}\{|1,1;1,0\rangle + |1,1;0,1\rangle\}$$

$$\rightarrow \alpha + \beta = 0$$

setjum $\alpha = \frac{1}{\sqrt{2}}$

$\beta = -\frac{1}{\sqrt{2}}$

(e)

$$J_- |1,1\rangle = \hbar\sqrt{2}\,|1,0\rangle$$

$$\rightarrow \underline{|1,0\rangle} = \frac{1}{\hbar\sqrt{2}}(J_{1-}+J_{2-})\frac{1}{\sqrt{2}}\Big\{|1,1;1,0\rangle - |1,1;0,1\rangle\Big\}$$

$|1,1;0,0\rangle$ dette ut vegna

$$= \frac{1}{2\hbar}\Big\{0 + \hbar\sqrt{2}\,|1,1;1,-1\rangle - \hbar\sqrt{2}\,|1,1;-1,1\rangle$$

$$= \frac{1}{\sqrt{2}}\Big\{|1,1;1,-1\rangle - |1,1;-1,1\rangle\Big\}$$

$$J_- |1,0\rangle = \hbar\sqrt{2}\,|1,-1\rangle$$

$$\rightarrow \underline{|1,-1\rangle} = \frac{1}{\hbar\sqrt{2}}(J_{1-}+J_{2-})\frac{1}{\sqrt{2}}\Big\{|1,1;1,-1\rangle - |1,1;-1,1\rangle\Big\}$$

$$= \frac{1}{2\hbar}\Big\{\hbar\sqrt{2}\,|1,1;0,-1\rangle + 0 - 0 - \hbar\sqrt{2}\,|1,1;-1,0\rangle\Big\}$$

$$= \frac{1}{\sqrt{2}}\Big\{|1,1;0,-1\rangle - |1,1;-1,0\rangle\Big\}$$

Ettir er $|0,0\rangle$, som einungis er kopt við nyndu úr (f)

$$|0,0\rangle = a\,|1,1;1,-1\rangle + b\,|1,1;0,0\rangle + c\,|1,1;-1,1\rangle$$

er kornrétter á

$$|2,0\rangle = \frac{1}{\sqrt{6}}\left\{ |1,1;-1,1\rangle + 2|1,1;0,0\rangle + |1,1;1,-1\rangle \right\}$$

og

$$|1,0\rangle = \frac{1}{\sqrt{2}}\left\{ |1,1;1,-1\rangle - |1,1;-1,1\rangle \right\}$$

$$\rightarrow \quad \left.\begin{array}{l} a + 2b + c = 0 \\ a - c = 0 \end{array}\right\} \rightarrow \begin{array}{l} a = c \\ b = -a \end{array}$$

$$\rightarrow \quad |0,0\rangle = \frac{1}{\sqrt{3}}\left\{ |1,1;1,-1\rangle - |1,1;0,0\rangle + |1,1;-1,1\rangle \right\}$$

Niederstädter

J=2

$$|2,2\rangle = |1,1;1,1\rangle$$

$$|2,1\rangle = \frac{1}{\sqrt{2}}\{|1,1;1,0\rangle + |1,1;0,1\rangle\}$$

$$|2,0\rangle = \frac{1}{\sqrt{6}}\{|1,1;-1,1\rangle + 2|1,1;0,0\rangle + |1,1;1,-1\rangle\}$$

$$|2,-2\rangle = |1,1;-1,-1\rangle$$

$$|2,-1\rangle = \frac{1}{\sqrt{2}}\{|1,1;-1,0\rangle + |1,1;0,-1\rangle\}$$

$$|1,1\rangle = \frac{1}{\sqrt{2}}\{|1,1;1,0\rangle - |1,1;0,1\rangle\}$$

$$|1,0\rangle = \frac{1}{\sqrt{2}}\{|1,1;1,-1\rangle - |1,1;-1,1\rangle\}$$

$$|1,-1\rangle = \frac{1}{\sqrt{2}}\{|1,1;0,-1\rangle - |1,1;-1,0\rangle\}$$

$$|0,0\rangle = \frac{1}{\sqrt{3}}\{|1,1;1,-1\rangle - |1,1;0,0\rangle + |1,1;-1,1\rangle\}$$

Tafel afvef

m_1,m_2 \ J	2
1,1	1

m_1,m_2 \ J	2	1
1,0	$\sqrt{\frac{1}{2}}$	$\sqrt{\frac{1}{2}}$
0,1	$\sqrt{\frac{1}{2}}$	$-\sqrt{\frac{1}{2}}$

m_1,m_2 \ J	2	1	0
1,-1	$\sqrt{\frac{1}{6}}$	$\sqrt{\frac{1}{2}}$	$\sqrt{\frac{1}{3}}$
0,0	$\sqrt{\frac{2}{3}}$	0	$-\sqrt{\frac{1}{3}}$
-1,1	$\sqrt{\frac{1}{6}}$	$-\sqrt{\frac{1}{2}}$	$\sqrt{\frac{1}{3}}$