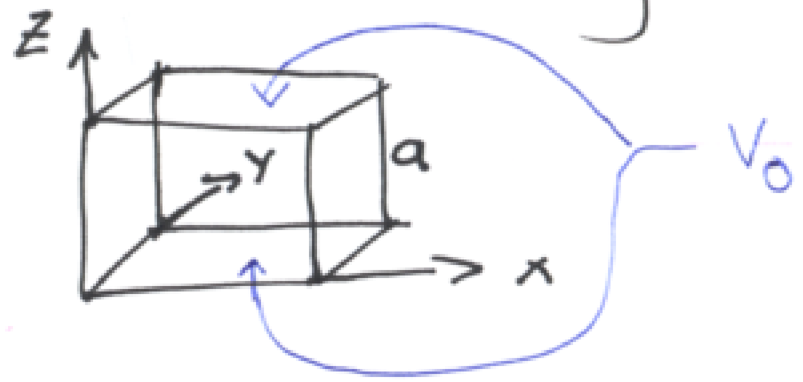
① ①

Holur teningur gerður úr kjörleiðandi ferningum



hinar hliðarnar eru með $V = 0$

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① Finna $V(x, y, z)$

V er lausn $\nabla^2 V = 0$, Laplace jöfnu, eða

$$\left\{\partial_x^2 + \partial_y^2 + \partial_z^2\right\} V(x, y, z) = 0$$

Veljum hnitakerfi eins og myndin sýnir, jaðarskilyrðin eru þannig að $V = 0$ á öllum fjórum lóðréttu flötunum og V_0 á þeim láréttu.

Lausnin er samskonar í x og y-stefnu
þar verður lausnin að vera lotubundið fall
til þess að uppfylla $k_x^2 + k_y^2 + k_z^2 = 0$
verður þá lausnin í z-átt að vera sett saman
úr dofnandi og rísandi veldisvísis föllum

$$V(x,y,z) = V_x(x)V_y(y)V_z(z)$$

V_z er valið þannig að þetta skilyrði sé sjálfkrafa uppfyllt

$$V_x(x) = A\sin(k_x x)$$

$$V_y(y) = B\sin(k_y y)$$

$$V_z(z) = C\exp\left\{+\sqrt{k_x^2+k_y^2}\,z\right\} + D\exp\left\{-\sqrt{k_x^2+k_y^2}\,z\right\}$$

Til að uppfylla jaðarskilyrðin á lóðréttu flötunum verður að gilda

$$k_x = \frac{n\pi}{a} \qquad n = 1, 2, \ldots\ldots$$

$$k_y = \frac{m\pi}{a} \qquad m = 1, 2, \ldots\ldots$$

Ef $V_0 = 0$ þá gæti $n = 0$ og $m = 0$ líka verið mögulegt. Veljum hér að $V_0 \neq 0$

$$\gamma_{nm} \equiv +\sqrt{\left(k_x^2 + k_y^2\right)} = \frac{\pi}{a}\sqrt{n^2 + m^2}$$

$$V_z(z) = C\exp\left[\gamma z\right] + D\exp\left\{-\gamma z\right\}$$

Við erum ekki búin enn að uppfylla öll jaðarst.
en setjum saman lausnina

$$V(x,y,z) = \sum_{n,m} \sin\left(\frac{n\pi x}{a}\right)\sin\left(\frac{m\pi y}{a}\right)\left\{C_{nm}\, e^{\gamma_{nm} z} + D_{nm}\, e^{-\gamma_{nm} z}\right\}$$

Hér eru ~~stuðlarnir~~ A_n og B_n teknir inn í C_{nm} og D_{nm}

Jaðar í $z=0$

$$V_0 = \sum_{n,m} \sin\left(\frac{n\pi x}{a}\right)\sin\left(\frac{m\pi y}{a}\right)\left\{C_{nm} + D_{nm}\right\}$$

Notum að föllin í x- og y-stefnu mynda hornrétt grunn á bilinu $[0,a]$

$\longrightarrow$

$$V_0 \int_0^a dx\,dy\, \sin\left(\frac{p\pi x}{a}\right) \sin\left(\frac{q\pi y}{a}\right)$$

$$= \sum_{nm} \left\{ C_{nm} + D_{nm} \right\} \int_0^a dx\, \sin\left(\frac{n\pi x}{a}\right) \sin\left(\frac{p\pi x}{a}\right) \int_0^a dy\, \sin\left(\frac{m\pi y}{a}\right) \sin\left(\frac{q\pi y}{a}\right)$$

almeunt gizdir

$$\int_0^a dx\, \sin\left(\frac{n\pi x}{a}\right) \sin\left(\frac{p\pi x}{a}\right) = \frac{a}{2} \delta_{n,p}$$

$$\int_0^a dx\, \sin\left(\frac{n\pi x}{a}\right) = \frac{a}{n\pi} \left\{ 1 - (-1)^n \right\}$$

og því fæst

$$V_0 \frac{a}{p\pi}\left\{1-(-1)^p\right\}\frac{a}{q\pi}\left\{1-(-1)^q\right\} = \sum_{n,m}\left\{C_{nm}+D_{nm}\right\}\frac{a^2}{4}\delta_{n,p}\delta_{m,q}$$

$$\rightarrow V_0 \frac{a^2}{pq\pi^2}\left\{1-(-1)^p\right\}\left\{1-(-1)^q\right\} = \frac{a^2}{4}\left\{C_{pq}+D_{pq}\right\}$$

$$\rightarrow C_{pq}+D_{pq} = \begin{cases} 0 & \text{f. } p \text{ og } q \text{ jafnartölur} \\ \dfrac{16V_0}{pq\pi^2} & \text{f. } p \text{ og } q \text{ oddatölur} \end{cases} \quad (*)$$

<u>Jaðar í $z=a$</u>

$$V_0 = \sum_{n,m}\sin\left(\frac{n\pi x}{a}\right)\sin\left(\frac{m\pi y}{a}\right)\left[C_{nm}e^{\gamma_{nm}a}+D_{nm}e^{-\gamma_{nm}a}\right]$$

Notum aftur grunnføllin í x- og y- stefnu eru fullkomin grunnur

$$\rightarrow C_{pq}\, e^{\gamma_{pq} a} + D_{pq}\, e^{-\gamma_{pq} a} = \begin{cases} 0 & \text{jøfn } p,q \\ \dfrac{16 V_0}{pq\pi^2} & \text{odda } p,q \end{cases} \qquad (**)$$

Loysum saman (*) og (**)

$$\begin{pmatrix} 1 & 1 \\ e^{\gamma_{pq} a} & e^{-\gamma_{pq} a} \end{pmatrix} \begin{pmatrix} C_{pq} \\ D_{pq} \end{pmatrix} = \begin{pmatrix} \dfrac{16 V_0}{pq\pi^2} \\ \dfrac{16 V_0}{pq\pi^2} \end{pmatrix}$$

$$C_{pq} = \frac{16 V_0}{pq\pi^2 \left(e^{\gamma_{pq} a} + 1\right)} \qquad D_{pq} = \frac{16 V_0\, e^{\gamma_{pq} a}}{pq\pi^2 \left(e^{\gamma_{pq} a} + 1\right)}$$

því er lausnin

$$V(x,y,z) = \sum_{\substack{n=0\\m=0}}^{\infty} \sin\left(\frac{(2n+1)\pi x}{a}\right)\sin\left(\frac{(2m+1)\pi y}{a}\right)\left\{C_{(2n+1)(2m+1)}\, e^{\gamma_{(2n+1)(2m+1)} z} + D_{(2n+1)(2m+1)}\, e^{-\gamma_{(2n+1)(2m+1)} z}\right\}$$

með

$$C_{pq} = \frac{16 V_0}{pq\pi^2\left(e^{\gamma_{pq} a}+1\right)} \qquad D_{pq} = \frac{16 V_0\, e^{\gamma_{pq} a}}{pq\pi^2\left(e^{\gamma_{pq} a}+1\right)}$$

$$\gamma_{pq}\, a = \pi\sqrt{p^2+q^2}$$

$$\gamma_{pq}\, z = \pi\frac{z}{a}\sqrt{p^2+q^2}$$

Tilbúið fyrir grafík

② Yfirborðshleðsluþéttleikinn á toppplötunni er í réttu hlutfalli við normalþátt rafsviðsins við plötuna

$$\rho_s(x,y,a^-) = \epsilon_0 E_n(x,y,a^-)$$

Hleðslan er neðan á plötunni því við þekkjum bara rafstöðumættið innan teningsins.

$$\rho_s(x,y,a^-) = \epsilon_0 \bar{n}\cdot\bar{E}(x,y,a^-) = -\epsilon_0 \hat{a}_z\cdot\bar{E}(x,y,a^-)$$

$$= \epsilon_0 \partial_z V(x,y,z)\Big|_{z=a^-}$$

$$\rho_s(x,y) = \epsilon_0 \sum_{\substack{n=0\\m=0}}^{\infty} \sin\left(\frac{(2n+1)\pi x}{a}\right)\sin\left(\frac{(2m+1)\pi y}{a}\right)\gamma_{(2n+1)(2m+1)}\Big\{C_{(2n+1)(2m+1)}e^{\gamma_{(2n+1)(2m+1)}a} - D_{(2n+1)(2m+1)}e^{-\gamma_{(2n+1)(2m+1)}a}\Big\}$$

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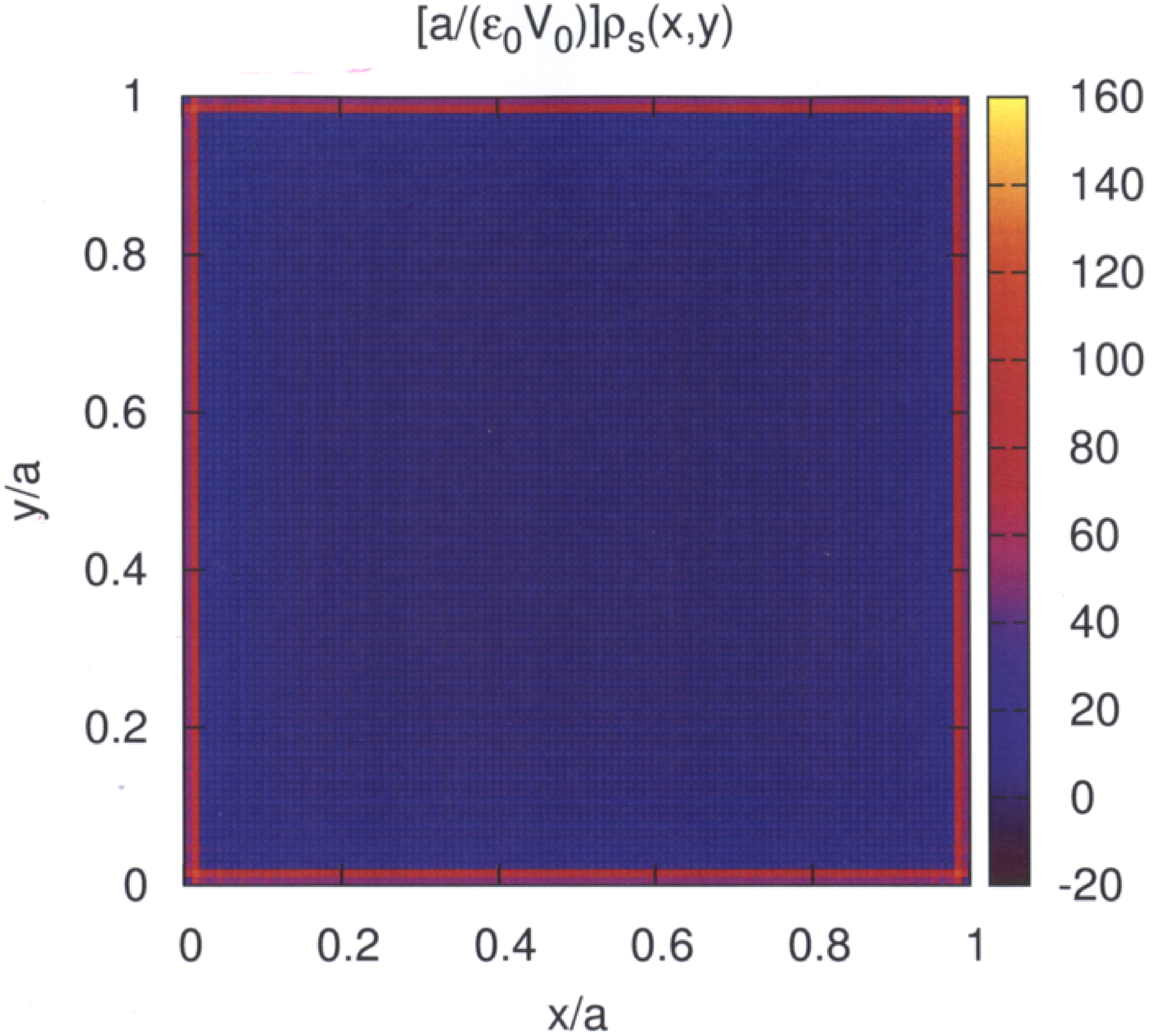
$n_{max} = 32$
$m_{max} = 32$



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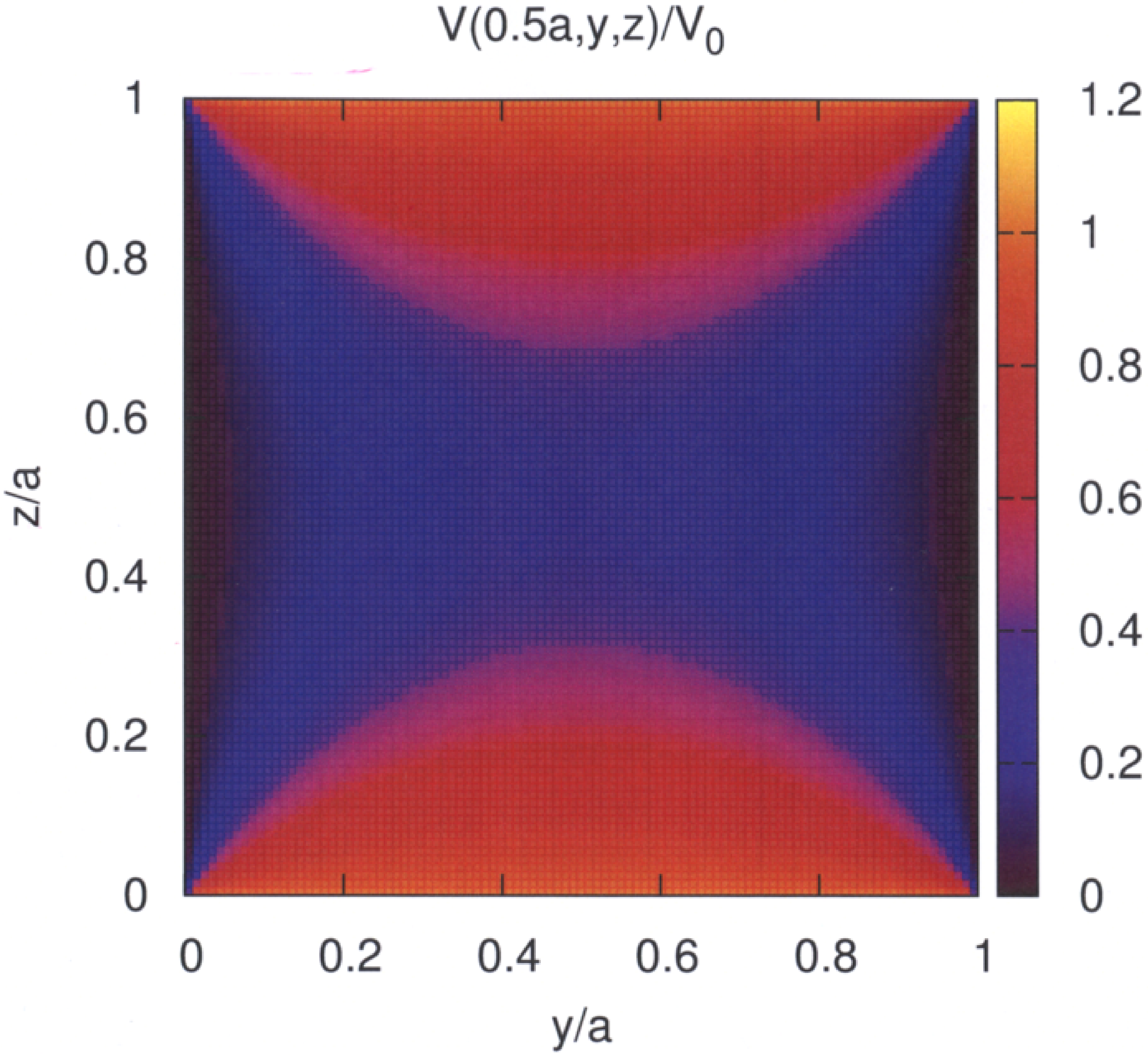

(12)

$n_{max} = 32$

$m_{max} = 32$