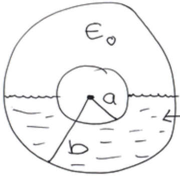
1 Tveir sammiðja sívalningar



$a, b << L$ lengd þeirra

Rafsvarinn brýtur ekki „radíal"-samhverfu $\bar{E}$

1 finna rýmd

Nota verður almenna framsetn Gauß lögmáls

hleðslan á innri skel

$$\oint_S \bar{D} \cdot d\bar{S} = Q$$

N: Norður
S: Suður

$$\rightarrow \epsilon_0 \pi R L E(R) + \epsilon_r \epsilon_0 \pi R L E(R) = Q$$

$$\rightarrow (1+\epsilon_r)\epsilon_0 \pi R L E(R) = Q$$

$$\rightarrow E(R) = \frac{Q}{\pi(1+\epsilon_r)\epsilon_0 R L} = \frac{(Q/L)}{\pi(1+\epsilon_r)\epsilon_0 R}$$

Við þurfum spennumun flatanna

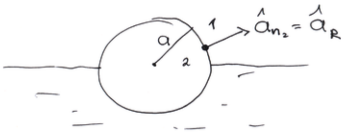
$$V_b - V_a = -\int_a^b \bar{E}\cdot d\bar{l} = -\int_a^b dR\, E(R) = -\frac{(Q/L)}{\pi(1+\epsilon_r)\epsilon_0}\int_a^b \frac{dR}{R}$$

$$= -\frac{(Q/L)}{\pi(1+\epsilon_r)\epsilon_0}\left\{\ln\left(\frac{b}{a}\right)\right\}$$

$$\frac{C}{L} = \frac{Q/L}{|V_b - V_a|} = \frac{\pi(1+\epsilon_r)\epsilon_0}{\ln\left(\frac{b}{a}\right)}$$

2 Hve stór hluti Q er á hvorum hálf sívalningi

$$\hat{a}_{n_2} \cdot (\bar{D}_1 - \bar{D}_2) = \rho_s$$



$$\hat{a}_R \cdot \bar{D}_N(a) = \rho_s$$

$$\rightarrow \quad \epsilon_0 E(a) = \rho_s$$

$$\rightarrow \frac{Q/L}{\pi(1+\epsilon_r)a} = \rho_s^N$$

og á ~~suður~~ sívalningi fast

$$\epsilon_0 \epsilon_r E(a) = \rho_s^S \rightarrow \frac{\epsilon_r Q/L}{\pi(1+\epsilon_r)a} = \rho_s^S$$

Heildarhleðslan á sívalningshelming er því

$$\left.\begin{aligned} Q_N &= \pi a L \cdot \rho_s^N = \frac{Q}{(1+\epsilon_r)} \\ Q_S &= \pi a L\, \rho_s^S = \frac{\epsilon_r Q}{(1+\epsilon_r)} \end{aligned}\right\} \quad Q_N + Q_S = Q$$

3 Rúmhleðsla?

Hún væri skilgreind með

$$\rho_P = -\bar{\nabla}\cdot\bar{P}$$

$$\bar{D} = \epsilon_0 \bar{E} + \bar{P}$$

$$\rightarrow \bar{P} = \bar{D} - \epsilon_0 \bar{E}$$

Í N-helmingi er $\bar{P} = 0$

Í S-helmingi, í rafsvara er

$$E(R) = \frac{(Q/L)}{\pi(1+\epsilon_r)\epsilon_0 R}$$

$$D_s(R) = \epsilon_r E(R)$$

$$\bar{P}_N(R) = \frac{(Q/L)}{\pi(1+\epsilon_r)}\left(\frac{\epsilon_r}{\epsilon_0}-1\right)\frac{1}{R}\,\hat{a}_R$$

$$\bar{\nabla}\cdot\bar{P}_N(R) = \frac{1}{R}\frac{\partial}{\partial R}\left(R\,P_R^N\right) = 0$$

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(4) Af þessu sést að hleðslan á ytri sívalningi er $-Q$, $\rho_p = 0$

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(5) Hér er eðlilegt að nota

$$W_e = \frac{1}{2}\int_V dv\,\bar{D}\cdot\bar{E} = \frac{1}{2}\int_N dv\,\epsilon_0 E^2 + \frac{1}{2}\int_S dv\,\epsilon_r\epsilon_0 E^2$$

$$= \frac{1}{2}\epsilon_0\frac{(1+\epsilon_r)}{2}\int_V dv\,E^2 = \epsilon_0\frac{(1+\epsilon_r)}{4}2\pi\int_a^b R\,dR\,\frac{1}{R^2}\frac{(Q/L)^2 L}{\{\pi(1+\epsilon_r)\epsilon_0\}^2}$$

$$W_e = \frac{\epsilon_0(1+\epsilon_r)}{2\{\pi(1+\epsilon_r)\epsilon_0\}^2}\pi\left(\frac{Q}{L}\right)^2 L\int_a^b dR\,\frac{1}{R} = \frac{\epsilon_0(1+\epsilon_r)}{\{\pi(1+\epsilon_r)\epsilon_0\}^2}\pi\left(\frac{Q}{L}\right)^2 \ln\left(\frac{b}{a}\right)$$

Ef Q er haldið föstu og hið rafsvarandi vökva haldið föstu þá er ljóst að W_e eykst ef b ykist aðeins eða a minnkaði aðeins

$$W_e = \frac{1}{2\pi(1+\epsilon_r)\epsilon_0}\left(\frac{Q^2}{L}\right)\ln\left(\frac{b}{a}\right)$$

sem líka mátti finna með því að nota (3-180 a-c)

Krafturinn á ytra sívalningnum er

$$\overline{F}_Q = -\overline{\nabla} W_e = -\hat{a}_R\left\{\frac{Q^2}{2\pi(1+\epsilon_r)\epsilon_0\, b\, L}\right\}$$

m.t.t. b

með stefnu að innri sívalningi