1

Problem 1: Estimate surface area A of human being with mass M and height h

Cylindrical human should give us lower bounds for A, ("spherical cow" :))

$M = \rho \cdot V = \rho \cdot h \cdot \pi r^2$, V: volume, ρ: density
r: radius

$A = \underbrace{h \cdot 2\pi r}_{\text{side}} + \underbrace{2 \cdot \pi r^2}_{\text{top + bottom}}$

$r^2 = \frac{M}{\rho h \pi} \longrightarrow r = \sqrt{\frac{M}{\rho h \pi}}$

$$\boxed{A} = h 2\pi \sqrt{\frac{M}{\rho h \pi}} + \frac{2\pi M}{\rho h \pi}$$

$$\boxed{= 2\sqrt{\frac{h M \pi}{\rho}} + \frac{2M}{\rho h}}$$

2

here is good to check the dimension

$$[A] = L^2 = \sqrt{\frac{L M L^3}{M}} + \frac{M L^3}{M L} = \underline{L^2}$$

For fun we test numbers

$M = 80$ kg, $h = 1.90$ m, $\rho \approx 1000$ kg/m^3

$$\rightarrow A = \left\{2\sqrt{\frac{1.90 \cdot 80 \cdot \pi}{1000}} + \frac{2 \cdot 80}{1900}\right\} m^2$$

$$\approx \{1{,}382 + 0{,}084\} m^2 \approx \underline{1{,}5 \ m^2}$$

3

Problem 2: (1-01-70)

72.0 beats/min

a) Beats in 2.0 yr

$\{2 \text{ yr} \approx 2 \cdot 365 \cdot 24 \cdot 60 \text{ min} = 1.0512 \cdot 10^6 \text{ min}\}$

$2.0 \text{ yr} \longrightarrow N = 7.20 \cdot 10^1 \text{ beat/min} \cdot 1.1 \cdot 10^6 \text{ min}$

$\approx \underline{7.6 \cdot 10^7}$

b) 2.00 yr

$N = 7.20 \cdot 10^1 \text{ beats/min} \cdot 1.0512 \cdot 10^7 \text{ min}$

$\approx \underline{7.57 \cdot 10^7}$

c) 2.000 yr

even though the time is known with more accuracy, the heart rate only has 3 significant digits $\longrightarrow$ $\underline{N \simeq 7.57 \cdot 10^7}$

4

Problem 3: (1-01-84)

Box :

$a = 1.80 \pm 0.1$ cm
$b = 2.05 \pm 0.02$ cm
$c = 3.1 \pm 0.1$ cm

discard terms with $\Delta \cdot \Delta \cdots$ and $\Delta \cdot \Delta \cdot \Delta$

$$V = abc = (a \pm \Delta a)(b \pm \Delta b)(c \pm \Delta c)$$
$$= abc\left(1 \pm \frac{\Delta a}{a}\right)\left(1 \pm \frac{\Delta b}{b}\right)\left(1 \pm \frac{\Delta c}{c}\right)$$
$$\simeq abc\left\{1 \pm \frac{\Delta a}{a} \pm \frac{\Delta b}{b} \pm \frac{\Delta c}{c}\right\}$$
$$= abc \pm \{bc\Delta a + ac\Delta b + ab\Delta c\}$$
$$= \{1.1 \cdot 10^1 \pm 2\} \text{ cm}^3 = \underline{\{11 \pm 2\} \text{ cm}^3}$$

5

Problem 4: (1-02-72)

Cartesian $(2, y)$ Find y and r

Polar $(r, \frac{\pi}{6})$ Many ways, I use:

$x = 2$ $x = r\cos\varphi$ $r = \sqrt{x^2 + y^2}$

$\varphi = \frac{\pi}{6}$ $y = r\sin\varphi$

$r = \sqrt{4 + y^2}$ and $y = \frac{r}{2}$, as $\sin\frac{\pi}{6} = \frac{1}{2}$

$\rightarrow r = \sqrt{4 + \frac{r^2}{4}} \rightarrow r^2 = 4 + \frac{r^2}{4}$

$\rightarrow \left\{1 - \frac{1}{4}\right\} r^2 = 4 \rightarrow r^2 = \frac{4}{1 - \frac{1}{4}} = \frac{16}{3}$

6

$\rightarrow \boxed{r = \frac{4}{\sqrt{3}}} \rightarrow \boxed{y = \frac{2}{\sqrt{3}}}$, as $y = \frac{r}{2}$

Test, we had

$x = r\cos\varphi = \frac{4}{\sqrt{3}} \cdot \underbrace{\frac{\sqrt{3}}{2}}_{\cos\left(\frac{\pi}{6}\right)} = 2$ as was given initially

Problem 1: (1-03-40)

a)

$t_0 = 0$ $\quad a = 1{,}40\ m/s^2 \quad$ $t_1 = ?$

$v_0 = 0$ $\quad\quad$ $v_1 = 2.00\ m/s$

a: Constant

$$v_1 = v_0 + at_1 = at_1 \rightarrow t_1 = \frac{v_1}{a} = \frac{2.00}{1.40}\ s \approx 1.43\ s$$

b) $v_0 = 2.00\ m/s$ $\quad$ $v_1 = 0$

$t_0 = 0$ $\quad a = ?$ $\quad t_1 = 0{,}800\ s$

$$v_1 = v_0 + at_1$$

$$0 = v_0 + at_1 \rightarrow a = -\frac{v_0}{t_1} = -\frac{2.00\ m/s}{0{,}800\ s} = -2.50\ \frac{m}{s^2}$$

Problem 2: (1-03-54)

a) $v_0 = 9.00\ m/s$ $\quad a = -2.00\ m/s^2$

$t_0 = 0$ $\quad\quad t_1 = 5.00\ s$

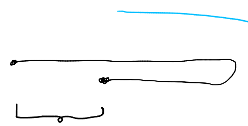
$S_0 = 0$ $\quad\quad S_1 = ?$

$$v_1 = v_0 + at_1 = \{9.00 - 2.00 \cdot 5.00\}\ m/s = -1.00\ m/s$$ (b)

so, she has turned around

$$S_1 = S_0 + v_0 t_1 + \frac{1}{2}at_1^2$$
$$= \{0 + 9.00 \cdot 5.00 - \frac{1}{2}2.00 \cdot (5.00)^2\}\ m = 20.0\ m$$

So, she turns around, that is probably not a usual behavior here...



20.0 m

Problem 3, (1-03-60)

$a = 0{,}35\ m/s^2$

$t_0 = 0$ $\quad\quad t_1 = ?$

$S_0 = 0$ $\quad\quad v_1 = 6.00\ m/s$

$S_1 = ?$

$$S_1 = S_0 + v_0 t_1 + \frac{1}{2}at_1^2 = \frac{1}{2}at_1^2$$

$$v_1 = v_0 + at_1 = at_1 \rightarrow t_1 = \frac{v_1}{a}$$

$$\rightarrow S_1 = \frac{1}{2}a\left(\frac{v_1}{a}\right)^2 = \frac{v_1^2}{2a}$$

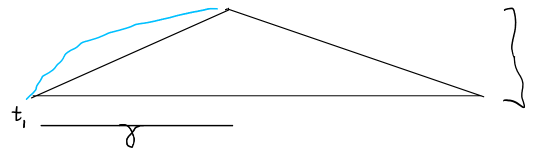
$$S_1 = \frac{(6.00)^2}{2 \cdot 0.35}\ m \approx 51\ m$$

b) $$t_1 = \frac{v_1}{a} = \frac{6.00}{0.35}\ s \simeq 17\ s$$

The results look resonable after observing this :)

Problem 4: (1-04-56)

$v_{x0} = v_0 = ?$



$h = 25\ km$

t_1

$R = 312\ km$ $\quad\quad g = -3.7\ m/s^2$

Vertical motion

$$h_1 = 0 = h_0 + \frac{1}{2}gt_1^2$$

Horizontal motion

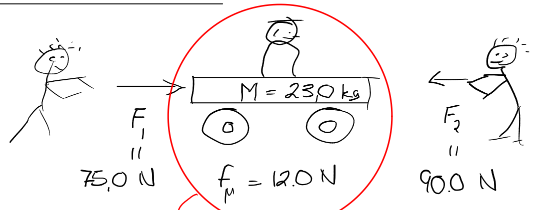
$$R = 0 + v_0 t_1 \rightarrow v_0 = \frac{R}{t_1} \quad \text{or} \quad t_1 = \frac{R}{v_0}$$

$$0 = h_0 + \frac{g}{2}\left(\frac{R}{v_0}\right)^2 \rightarrow v_0^2 h_0 + \frac{g}{2}R^2 = 0$$

$$v_0^2 = -\frac{gR^2}{2h_0}$$

$$\rightarrow v_0 = \sqrt{-\frac{gR^2}{2h_0}} \simeq 2.7 \cdot 10^3\ m/s$$

Problem 1: (1-05-32)



a) System of interest if $a_{wagon} = a$ is needed

b) $aM = F_1 - F_2 + f_\mu$

If $v_0 = 0$ and as $F_1 - F_2 < 0$

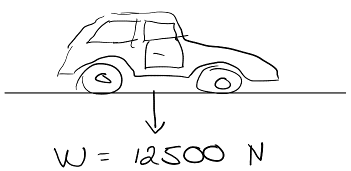
the wagon is accelerated to the left

$\rightarrow f_\mu = 12.0$ N (to the right)

$$\rightarrow a = \frac{F_1 - F_2 + F_\mu}{M} = \frac{75.0 - 90.0 + 12.0}{23.0}\ m/s^2 \approx -1.30\ m/s^2$$

c) but if $f_\mu = 15.0$N and $v_0 = 0$ $\rightarrow a = 0$

Problem 2: (1-05-46)



$\rightarrow$ accelerates from $v_0 = 0$ at $t_0 = 0$ to $v_1 = 83.0$ km/h in $t_1 = 5.00$ s

$F_\mu = 1350$ N

Find the force produced by the motor

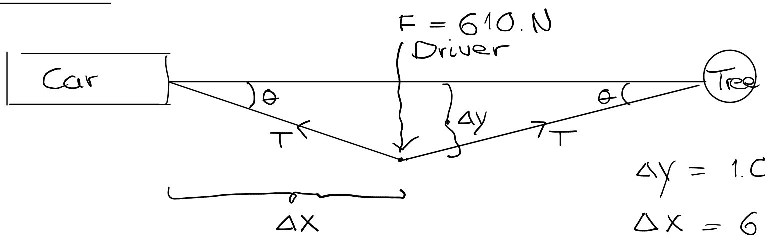
$$v_1 = v_0 + at_1 = at_1 \rightarrow a = \frac{v_1}{t_1}$$

$$a = \frac{83.0\ km/h \cdot 1000\ \frac{m}{km} \cdot \frac{h}{3600 s}}{5.00\ s} \approx 4.61\ m/s^2$$

$$W = gM \rightarrow M = \frac{W}{g} \rightarrow (F_{motor} - F_\mu) = aM = \frac{v_1}{t_1}\frac{W}{g}$$

$$\rightarrow F_{motor} = F_\mu + \frac{v_1 W}{t_1 g} = 1350\ N + \left(\frac{83.0}{3.6}\right)\frac{1}{5.00}\left(\frac{12500}{9.81}\right) N \simeq 7.23 \cdot 10^3\ N$$

Problem 3: (1-05-62)



$\Delta y = 1.00$ m

$\Delta x = 6.00$ m

Find T

$\tan\theta = \frac{\Delta y}{\Delta x}$, Equilibrium when $2T \sin\theta = F$

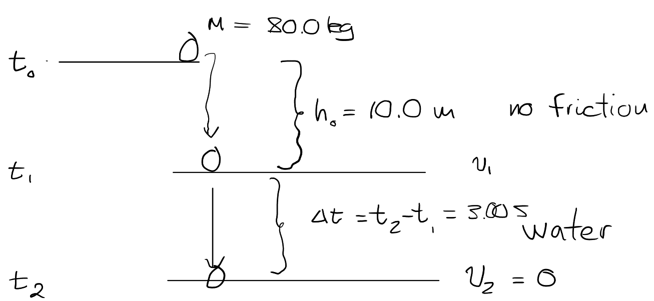
$$\rightarrow T = \frac{F}{2\sin\theta} = \frac{F}{2\sin\{\arctan(\frac{\Delta y}{\Delta x})\}}$$

$$= \frac{F}{2}\frac{\sqrt{1 - (\frac{\Delta y}{\Delta x})^2}}{(\frac{\Delta y}{\Delta x})}$$

$$\simeq 1.80 \cdot 10^3\ N$$

$$\sin\{\arctan(z)\} = \frac{z}{\sqrt{1 - z^2}}$$

Problem 4: (1-05-76)



$v_1 = v_0 - gt_1 = -gt_1$

$h = h_0 - \frac{1}{2}gt_1^2 + v_0 t_1$

$0 = h_0 - \frac{1}{2}gt_1^2 + 0$

$\rightarrow t_1 = \sqrt{\frac{2h_0}{g}}$

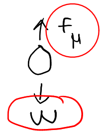
$v_1 = -g\sqrt{\frac{2h_0}{g}} = -\sqrt{2h_0 g}$

Find F acting on the swimmer in the water, that stops her

$v_2 = v_1 + a \cdot \Delta t$

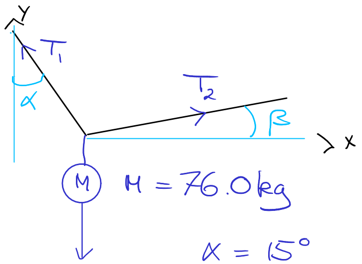
$0 = v_1 + a \cdot \Delta t \rightarrow a = -\frac{v_1}{\Delta t} = \frac{\sqrt{2h_0 g}}{\Delta t}$

In the water two forces work on her



$$\rightarrow F_\mu - Mg = Ma = M\frac{\sqrt{2h_0 g}}{\Delta t} \rightarrow F_\mu = M\left\{g + \frac{\sqrt{2h_0 g}}{\Delta t}\right\} \simeq 1.16 \cdot 10^3\ N$$

Problem 1: (1-6-30)



$M = 76.0\,kg$

$W = -gM$

$\alpha = 15°$

$\beta = 10°$

Equilibrium

$$y: \quad T_1 \cos\alpha + T_2 \sin\beta - gM = 0$$

$$x: \quad -T_1 \sin\alpha + T_2 \cos\beta = 0$$

$$x: \rightarrow \quad \frac{T_1}{T_2} = \frac{\cos\beta}{\sin\alpha}$$

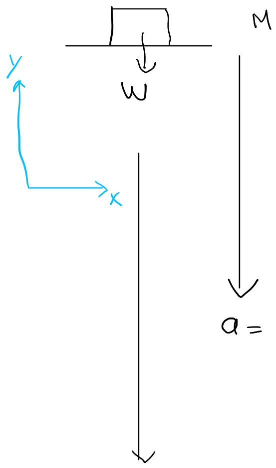
$$y: \rightarrow \quad T_2\left\{\frac{\cos\beta}{\sin\alpha}\cos\alpha + \sin\beta\right\} = gM$$

$$\rightarrow \quad T_2\left\{\cos\beta \cdot \cot\alpha + \sin\beta\right\} = gM$$

$$\rightarrow \quad T_2 = \frac{gM}{\cos\beta \cot\alpha + \sin\beta} \simeq 194\ N$$

$$T_1 = \frac{gM}{\cos\beta \cot\alpha + \sin\beta} \cdot \frac{\cos\beta}{\sin\alpha} \simeq 7.4 \cdot 10^2\ N$$

Problem 2: (1-06-40)



$M = 10.0\,kg$

$a = -1.2\ m/s^2$

Find the force of M on the floor of the elevator during the trip down with acceleration a

$$F = -M\left\{g - a\right\}$$

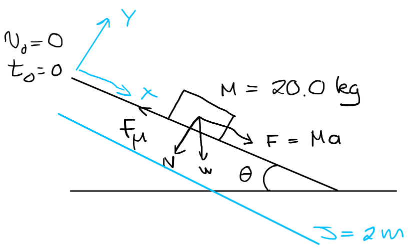
$$= -10.0 \cdot \left\{9.81 - 1.2\right\}\ N$$

$$= -86\ N$$

The force is downwards --> negative

The force is reduced by the acceleration of the elevator

Problem 3: (1-06-64)



$\theta = \frac{\pi}{6}$, $\mu = 0.0300$

a) Find a

$$F_\mu = \mu N = \mu g M \cos\theta$$

$$-F_\mu + gM\sin\theta = Ma$$

$$\rightarrow \quad -\cos\theta \cdot \mu g M + gM\sin\theta = Ma$$

$$\rightarrow \quad a = -\mu g \cos\theta + g\sin\theta = g\left\{\sin\theta - \mu\cos\theta\right\}$$

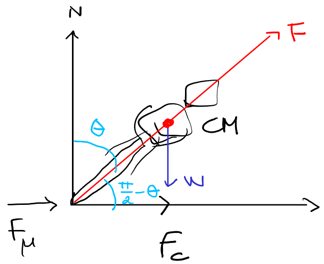
b) Find v at bottom

$$v = v_0 + at = at \longrightarrow v = a\sqrt{\frac{2S}{a}} = \sqrt{2Sa}$$

$$S = \frac{1}{2}at^2 \rightarrow t^2 = \frac{2S}{a}$$

$$\rightarrow \quad v = \sqrt{2Sg\left\{\sin\theta - \mu\cos\theta\right\}} \simeq 4.31\ m/s$$

Problem 4: (1-06-70)



$$F_c = M\frac{v^2}{r}$$

F has to pass through the CM to have equilibrium

N and F_μ supply F_i F_μ gives F_c

$$\arctan\left(\frac{F_y}{F_x}\right) = \frac{\pi}{2} - \theta$$

$$\rightarrow \quad \arctan\left(\frac{F_x}{F_y}\right) = \theta$$

$$F_x = M\frac{v^2}{r}$$

$$F_y = N = gM$$

$$\theta = \arctan\left(\frac{v^2}{rg}\right)$$

often this problem is solved using the torque of the forces around the touching point of the tire and the ground

1

Problem 1: (1-07-48)

$M_1 = 5.0$ kg
$M_2 = 8.0$ kg

$$\left(E_{kin}\right)_1 = 3\left(E_{kin}\right)_2$$

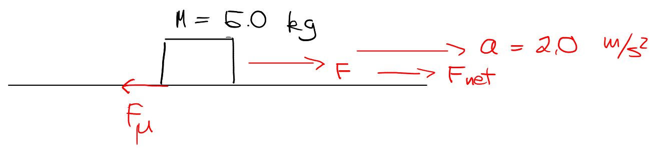
$$\rightarrow \tfrac{1}{2} M_1 v_1^2 = \tfrac{3}{2} M_2 v_2^2$$

$$\rightarrow \left(\frac{v_1^2}{v_2^2}\right) = 3\,\frac{M_2}{M_1}$$

$$\rightarrow \frac{v_1}{v_2} = \sqrt{3\,\frac{M_2}{M_1}} = \sqrt{\frac{3 \cdot 8.0}{5.0}} \approx 2.2$$

2

Problem 2: (1-07-54)



Translation $\Delta x = 0.10$ m

Use the general eq. $W_{AB} = \int_{C_{AB}} \vec{F} \cdot d\vec{r}$

$F_{net} = F - f_\mu$ (1D --> use a sign to indicate the direction of a vector)

b) $F_{net} = Ma \rightarrow F - f_\mu = F - \mu_k N = Ma$

$\rightarrow F - \mu_k Mg = Ma$

Work f_μ

$$W^{f_\mu} = \int_{\Delta x} \vec{f}_\mu \cdot d\vec{r} = -f_\mu \cdot \Delta x = -\mu_k Mg \cdot \Delta x$$

--> f dissipates energy from M

3

c) Work of F_{net}

$$W^{F_{net}} = Ma \cdot \Delta x$$

a) Work of F

$$W^F = F \cdot \Delta x = \{Ma + \mu_k Mg\}\,\Delta x$$

$$\rightarrow W^F = W^{F_{net}} - W^{f_\mu}$$

d) ΔE_{kin} ? $\Delta E_{pot} = 0$

$$\hookrightarrow \Delta E_{kin} = W^{F_{net}} = Ma \cdot \Delta x$$

$$= 5.0\,\text{kg} \cdot 2.0\,\text{m/s}^2 \cdot 0.1\,\text{m}$$

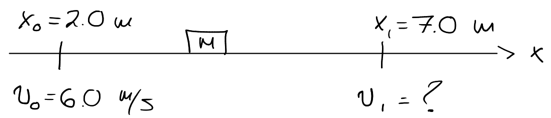
$$= 1.0\,\text{J}$$

4

Problem 3: (1-08-28)

1D motion in force field $F(x) = \left(\frac{3}{\sqrt{x}}\right)$ N

which in reality means that "3" has dimension...



$M = 2.0$ kg

No friction, no dissipation, 1D conservative force $\rightarrow \Delta E_{total} = 0$

$$F(x) = -\frac{dU(x)}{dx} = \left(\frac{3}{\sqrt{x}}\right) \text{N}$$

constant

$$\rightarrow U(x) = -6\sqrt{x} + U_0$$

$$U(x) - U_0 = \left(-6\sqrt{x}\right) \text{N}$$

$$\left.\begin{array}{l} U(7) = -6\sqrt{7} + U_0 \\ U(2) = -6\sqrt{2} + U_0 \end{array}\right\} \rightarrow \Delta\left[U(7) - U(2)\right] = -6\left[\sqrt{7} - \sqrt{2}\right]$$

$$\Delta E_{kin} = \frac{M}{2}\left\{ v_1^2 - v_0^2 \right\}$$

and

$$\Delta E_{kin} + \Delta U = 0$$

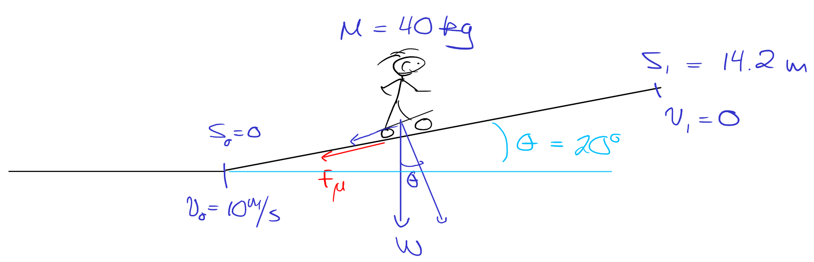
$$\rightarrow \frac{M}{2}\left\{ v_1^2 - v_0^2 \right\} - 6\left\{ \sqrt{7} - \sqrt{2} \right\} = 0$$

$$\rightarrow v_1^2 = v_0^2 + \frac{12}{M}\left\{ \sqrt{7} - \sqrt{2} \right\}$$

$$v_1 = \sqrt{v_0^2 + \frac{12}{M}\left\{ \sqrt{7} - \sqrt{2} \right\}} \simeq 6.59\ m/s$$

I consider the system to be the mass and the force field, thus there is no external force working on the mass. If the force field is considered to be an external one, then I have to calculate how the external force changes the kinetic energy of the mass by doing work on it

Problem 4: (1-08-40)



Find F_μ

component of gravity pulling the girl down the slope $-Mg\sin\theta$

the total force against her motion $-F_\mu - Mg\sin\theta$

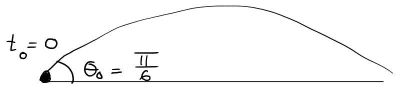
$$\rightarrow a = -\frac{F_\mu}{M} - g\sin\theta \;, \quad \text{use} \quad \begin{aligned} v^2 &= v_0^2 + 2aS \\ 0 &= v_0^2 + 2aS \end{aligned}$$

$$\rightarrow 0 = v_0^2 - 2S\frac{F_\mu}{M} - 2Sg\sin\theta$$

$$\rightarrow \boxed{-F_\mu = \frac{Mv_0^2}{2S} - Mg\sin\theta}$$

Problem 1: (1-09-34)

1



$v_0 = 25\ m/s$ No friction

$M = 0.250\ kg$

find $\bar{p}(t = 0.2\ s)$

$$v_{ox} = v_0 \cos\theta_0$$
$$v_{oy} = v_0 \sin\theta_0$$
$$v_x(t) = v_{ox}$$
$$v_y(t) = v_{oy} - gt$$
$$p_x = M \cdot v_{ox} = M v_0 \cos\theta_0$$
$$p_y = [v_{oy} - gt] M = M\{v_0 \sin\theta_0 - gt\}$$

$$|\bar{p}| = \sqrt{p_x^2 + p_y^2} = M \sqrt{v_0^2 \cos^2\theta_0 + \{v_0 \sin\theta_0 - gt\}^2}$$
$$= \sqrt{[v_0^2 - 2gt\, v_0 \sin\theta_0 + (gt)^2]}$$

The angle θ depends on time, $\theta_0 = \theta(0)$

2

$$\theta(t) = \arctan\left(\frac{p_y}{p_x}\right) = \arctan\left\{\frac{v_0 \sin\theta_0 - gt}{v_0 \cos\theta_0}\right\}$$

$$|\bar{p}| = 0.25 \sqrt{\left[(25)^2 - 2 \cdot 9.81 \cdot 0.2 \cdot 25 \cdot \sin\left(\frac{\pi}{6}\right) + (9.81 \cdot 0.2)^2\right]}\ kg\frac{m}{s}$$

$$\approx 6.0\ kg\,m/s$$

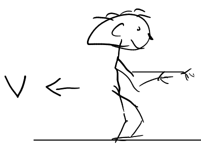
$$\theta(2s) = \arctan\left\{\frac{25 \cdot \sin\left(\frac{\pi}{6}\right) - 9.81 \cdot 0.2}{25 \cdot \cos\left(\frac{\pi}{6}\right)}\right\} \simeq 0.45$$

$$\simeq 26^\circ$$

So, the angle $\theta(t=0.2\ s)$ is reduced from the initial value, but is still positive.
At the top of the track it is 0, and then turns negative after that

Problem 2: (1-09-50)

3



$m = 1.0\ kg$

O ⟶ $v = 6{,}7\ m/s$

$M = 65\ kg$

I assume V_0 of M to be 0 initially

conservation of momentum

$$\bar{p}_M + \bar{p}_m = 0$$

$$\rightarrow MV + mv = 0 \rightarrow \boxed{V = -\frac{m}{M} v}$$

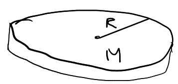
$$V = -\frac{1.0\ m/s}{65\ m/s}\ 6.7\ \frac{m}{s} \simeq -0.10\ \frac{m}{s}$$

You slip on the ice in opposite direction to the ball

Problem 3: (1-10-62)

4

Disk of a sander



$R = 0.10\ m$ $\nu = 15\ \frac{rev}{s}$

$M = 0.7\ kg$ $\omega = 2\pi\nu$

a) when sanding ω decreases by 20% $\rightarrow \omega_1 = \omega_0 \cdot 0.8$

Find $(E_{kin})_1 = \frac{1}{2} I \omega_1^2$, $I = \frac{1}{2} MR^2$ (Ex. 10.5)

$$(E_{kin})_1 = \frac{1}{4} MR^2 (\omega_0 \cdot 0.8)^2 = \frac{1}{4} 0.7 \cdot 0.10^2 (2\pi \cdot 15 \cdot 0.8)^2$$

$$\approx 9.95\ J$$

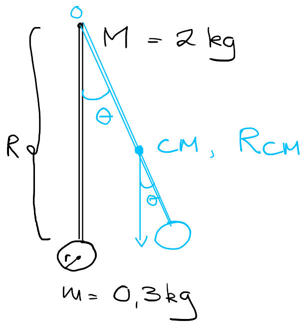
b) How large is the change in the kinetic energy from ω_0 to ω_1 ?

$$\Delta E_{kin} = (E_{kin})_1 - (E_{kin})_0 = \frac{1}{4} MR^2\omega_0^2 - \frac{1}{4} MR^2\omega_1^2$$
$$= \frac{1}{4} MR^2\omega_0^2 (1 - 0.8^2)$$
$$= (E_{kin})_0 \cdot 0.36$$

--> decreased by 64%

5

Problem 4: (1-10-68)



$R = 1m$, originally $\theta = \frac{\pi}{6}$, $v_0 = 0$

Find $\omega_1 = \omega(\theta = 0)$

$$I = I_r + I_{sph}$$

$$I_r = \frac{1}{3} MR^2, \quad I_{sph} = \frac{2}{5} mr^2 + m(R+r)^2$$

$$R_{CM} = \frac{\frac{1}{2}MR + (R+r)m}{M+m}$$

I use the energy conservation, as both the torque and the angular acceleration are not constant.

measured from bottom, where it is 0

$$E_{pot}(0) = R_{CM} \cdot \{1 - \cos\theta\} Mg \longrightarrow \Delta E_{pot} = R_{CM}\{1 - \cos\theta\} Mg$$

6

$$E_{kin}\left(\theta = \frac{\pi}{6}\right) = 0$$

$$\to \Delta E_{kin} = \frac{1}{2}\left[I_r + I_{sph}\right]\omega_1^2$$

conservation of the energy

$$\to \Delta E_{pot} = \Delta E_{kin} \to R_{CM}\left[1 - \cos\theta\right] Mg = \frac{1}{2}\left\{I_r + I_{sph}\right\}\omega_1^2$$

$$\to \omega_1 = \sqrt{\frac{2(1-\cos\theta) R_{CM} Mg}{(I_r + I_{sph})}}$$

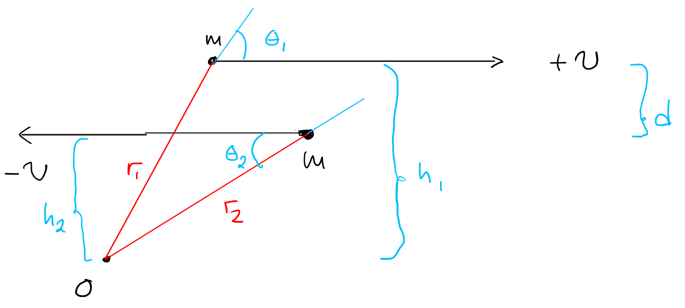
to check, dimension

$$[\omega_1] = \frac{1}{T} = \sqrt{\frac{L M L}{T^2 M L^2}} = \frac{1}{T} \quad \text{ok}$$

7

$$\omega_1 = \sqrt{\frac{2(1-\cos\theta) \cdot \left(\frac{M}{2}R + m(R+r)\right) Mg}{(M+m)\left\{\frac{MR^2}{3} + \frac{2mr^2}{5} + m(R+r)^2\right\}}} \approx 0.99 \ 1/s$$

Problem 5: (1-11-40)



Remember

$$\bar{L}_1 = \bar{r}_1 \times \bar{p}_1$$

$$L_1 = r_1 p_1 \sin\theta_1 = p_1 \cdot h_1$$

8

$$L = L_1 + L_2$$

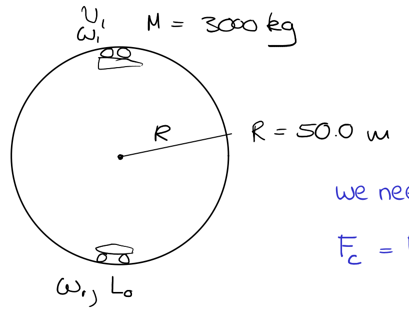
L_1 and L_2 have opposite directions

$$|L| = h_1 mv - h_2 mv = mv\{h_1 - h_2\}$$

$$\text{but,} \ |h_1 - h_2| = d$$

Thus we will always have the same angularmomentum for the system, independent of the choice we make for the reference point o

Problem 6: (1-11-50)



Find minimum L_0 for the roller coaster to stay on the track

we need at least gravity to supply

$$F_c = MR\omega_1^2 = M\frac{v_1^2}{R}$$

So, minimum angular frequency

$$W = Mg = MR\omega_1^2 \longrightarrow \omega_1^2 = \frac{g}{R} \longrightarrow L_1 = R^2 M\sqrt{\frac{g}{R}}$$

$$L_1 = R \cdot M\underbrace{(\omega_1 R)}_{v_1} = MR^2\omega_1$$

$$L_0 = MR^2\omega_0$$

Energy conservation

$$E_1 = E_0$$

$$\frac{1}{2}M(\omega_1 R)^2 + gM2R = \frac{1}{2}M(\omega_0 R)^2$$

$$\rightarrow (\omega_1 R)^2 + g4R = (\omega_0 R)^2$$

$$\rightarrow (\omega_0 R) = \sqrt{(\omega_1 R)^2 + g4R}$$

$$\rightarrow L_0 = MR^2\omega_0 = MR\sqrt{(\omega_1 R)^2 + 4gR}$$

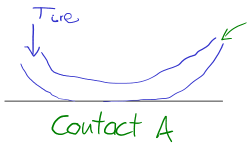
So,

$$L_0 \geq MR\sqrt{(\omega_1 R)^2 + 4gR} = MR\sqrt{gR + 4gR} = \underline{MR\sqrt{5gR}}$$

$$L_0 \geq 7.43 \cdot 10^6 \ \frac{kg\,M^2}{s}$$

1

Problem 1: (1-14-58)



$p = 3.50 \cdot 10^5\ \text{Pa}$, $\quad 1\ \text{Pa} = 1\ \frac{N}{m^2}$

$1\ \text{psi} = 6.895 \cdot 10^3\ \text{Pa}$

$p = 50.8\ \text{psi}$ (Familiar)

If only the tire pressure holds up the weight on the tire $\quad W = gM, \quad M = 80.0\ \text{kg}$

$$pA = Mg \rightarrow \boxed{A = \frac{Mg}{p}}$$

$$A = \frac{80 \cdot 9.81}{3.50 \cdot 10^5}\ m^2 \approx 2.24 \cdot 10^{-3}\ m^2 = \underline{22.4\ cm^2}$$

2

Problem 2: (1-14-68)

Buoyant force of a 2.00 L He balloon

$$\boxed{F_B = \rho_{air} V \cdot g}$$

He: $\rho_{He} = 1.80 \cdot 10^{-1}\ kg/m^3$

Air: $\rho_{air} = 1.29 \cdot 10^0\ kg/m^3$

a)

$$\rightarrow F_B = \{1.29\}\ \frac{kg}{m^3}\ 2(10^{-3}\ m^3)\ 9.81\ m/s^2$$

$$= \underline{0.025\ N}$$

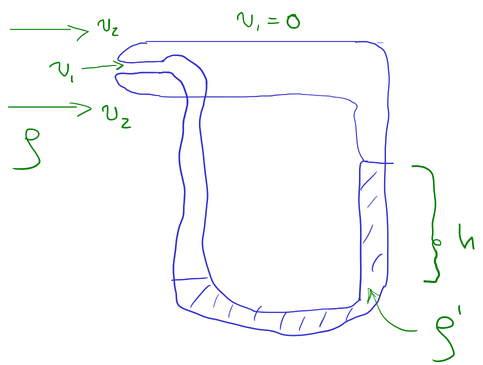
b) $M = 1.5\ g$ of balloon

$$F_B - W_b = F_B - \rho_{He} V g - gM = F^{Lift}$$

$$\rightarrow F^{lift} = \left(0.025 - (0.180 \cdot 0.002 + 0.0015) \cdot 9.81\right) N \simeq \underline{6.8 \cdot 10^{-3}\ N}$$

3

Problem 3: (1-14-88)



Show that

$$v_2 = \left(\frac{2\rho' g h}{\rho}\right)$$

Bernoulli

$$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$$

$$v_1 = 0 \rightarrow P_1 = P_2 + \frac{1}{2}\rho v_2^2 \rightarrow (P_1 - P_2) = \frac{1}{2}\rho v_2^2$$

but $\quad (P_1 - P_2) \cdot A = h \cdot \rho' g A \rightarrow \frac{1}{2}\rho v_2^2 = h\rho' g$

$$\rightarrow v_2^2 = \frac{2\rho' g h}{\rho} \rightarrow \boxed{v_2 = v = \sqrt{\left(\frac{2\rho' g h}{\rho}\right)}}$$

4

b) In Hg manometer if $h = 0.200\ m$

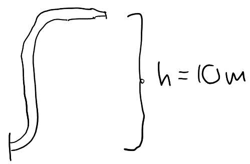
Find v for air

$$v = \sqrt{2gh\left(\frac{\rho'_{Hg}}{\rho_{air}}\right)} = \sqrt{2 \cdot 9.81 \cdot 0.2\left(\frac{1.36 \cdot 10^4}{1.29}\right)}\ m/s$$

$$= \underline{203\ m/s}$$

Problem 4 (1-14-102)

Estimate N_R for a fire hose and a nozzle



$P_0 = 1.62 \cdot 10^6\ \text{Pa}$

Flow $40.0\ L/s$

$d_h = 6.40\ cm$

$d_n = 3.00\ cm$

$$N_R = \frac{2\rho v r}{\eta}$$

$$\eta_{H_2O} = 1.002 \cdot 10^{-3}\ \text{Pa} \cdot s$$

5

with Ex. 14.7 in the book in mind and the eq. for N_R I see we only need the equation of continuity

$$Q_1 = A_1 v_1 = A_2 v_2 = Q_2$$

$$v_1 = \frac{Q_1}{A_1} = \frac{Q_1}{\pi\left(\frac{d_h}{2}\right)^2} = \frac{0{,}040\, m^3/s}{1} \cdot \frac{1}{\pi \cdot (0.032)^2\, m^2}$$

$$v_2 = v_1\left(\frac{A_1}{A_2}\right) \qquad \approx 12.4\ m/s$$

Hose: $$(N_R)_h = \frac{2\rho v_1 \left(\frac{d_h}{2}\right)}{\eta} = \frac{2\rho\left(\frac{Q_1}{\pi\left(\frac{d_h}{2}\right)^2}\right)\left(\frac{d_h}{2}\right)}{\eta}$$

$$= \frac{2 \cdot 10^3\ kg/m^3 \cdot 12.4\ m/s \cdot 0{,}032\ m}{1.002 \cdot 10^{-3}\ Pa \cdot s} \approx \underline{7.92 \cdot 10^5}$$

6

Nozzle: $$(N_R)_n = (N_R)_h \cdot \left(\frac{d_h}{d_n}\right) \approx \underline{1.69 \cdot 10^6}$$

So, both the flow in the hose and the nozzle could be considered turbulent

1

Problem 1: (11-01-50)

How much does the Eiffel tower lengthen when $\Delta T = 15.0\ ^\circ C$

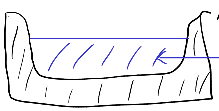
Table 1.2 steel $\longrightarrow$ $\alpha = 12 \cdot 10^{-6}\ 1/^\circ C$

$L = 321\ m$

$$\Delta L = \alpha \cdot L \cdot \Delta T$$

$$= 12 \cdot 10^{-6} \cdot 321 \cdot 15\ m = 0.058\ m = \underline{5.8\ cm}$$

Problem 2: (11-01-84)



Al, $M = 0.250\ kg$

Soup, $m = 0.800\ kg$ (H_2O)

at $T_0 = 25.0\ ^\circ C$

put in freezer and $Q = 388$ kJ taken away from the system, find the final temperature T_f

2

First, assume no freezing (we might be wrong on this point, but try)

$$Q = \{ m C_{H_2O} + M \cdot C_{Al} \} \Delta T$$

$$\to \Delta T = \frac{Q}{m C_{H_2O} + M \cdot C_{Al}} \approx \frac{388 \cdot 10^3\ J}{0.8\ kg \cdot 4186 \frac{J}{kg ^\circ C} + 0.25\ kg \cdot 900 \frac{J}{kg ^\circ C}}$$

$$\approx 109\ ^\circ C$$

.. so the no freezing is a silly assumption, try with freezing, but same specific heat for water and ice (correct that later)

$$Q = \{ m \cdot C_{H_2O} + M \cdot C_{Al} \} \Delta T + m L_f^{H_2O}$$

$$\to \Delta T = \frac{\{ Q - m L_f^{H_2O} \}}{m C_{H_2O} + M \cdot C_{Al}} = \frac{388 \cdot 10^3\ J - 0.8\ kg \cdot 334 \cdot 10^3\ J/kg}{\text{as before}}$$

$$\approx 33.8\ ^\circ C$$

$$\to \underline{T_f} = (25 - 33.8) \simeq \underline{-8.8\ ^\circ C}$$

3

Specific heat of ice is only 2090 J/(kg·K) See Table 1.3 in vol II

$$\longrightarrow Q = \{ m C_{H_2O} + M C_{Al} \} \cdot \Delta T_1 + m L_f^{H_2O} + \{ m C_{ice} + M C_{Al} \} \cdot \Delta T_2$$

where

$$\Delta T_1 = 25.0\ ^\circ C$$

but we need to find ΔT_2

$$\Delta T_2 = \frac{Q - \{ m C_{H_2O} + M C_{Al} \} \Delta T_1 - m L_f^{H_2O}}{m C_{ice} + M C_{Al}}$$

$$= 20.5\ ^\circ C$$

thus the final temperature of the soup and the pot in the freezer will be

$$\underline{T_f = -20.5\ ^\circ C}$$

4

Problem 3: (11-02-44)

$v_{rms} = 514\ m/s$

N_2 $\quad p?$

$V = 6.56\ m^3$

$n = 4.86 \cdot 10^4\ mol$

$$v_{rms} = \sqrt{\frac{3 k_B T}{m}} = \sqrt{\frac{3RT}{M}}$$

$$p = \frac{nRT}{V} \to T = \frac{pV}{nR} \quad \text{ideal gas}$$

$$\to (v_{rms})^2 = \frac{3R}{M} \left(\frac{pV}{nR} \right) = \frac{3pV}{nM}$$

$$\to p = \frac{(v_{rms})^2\ nM}{3V} \qquad M = 28.0\ g/mole = 28.0 \cdot 10^{-3} \frac{kg}{mole}$$

$$\to p = \frac{(514)^2 \cdot 4.86 \cdot 10^4\ mol \cdot 28.0 \cdot 10^{-3} \frac{kg}{mol}}{3 \cdot 6.56\ m^3} = 1.83 \cdot 10^7 \frac{kg}{m s^2}$$

but remember

$$1\,Pa = 1\,\frac{N}{m^2} = 1\,\frac{kg\; m/s^2}{m^2} = 1\,\frac{kg}{m\,s^2}$$

$$\rightarrow \quad \underline{p = 1.83 \cdot 10^7\ Pa}$$

Problem 4: (11-02-58)

How much Q (heat) is needed to raise $T_0 = 25.0\,°C$ --> $33.0\,°C = T_f$

a) for air $n = 1.5$ mol, diatomic, Table 2.3 : $C_V = 2.5R$

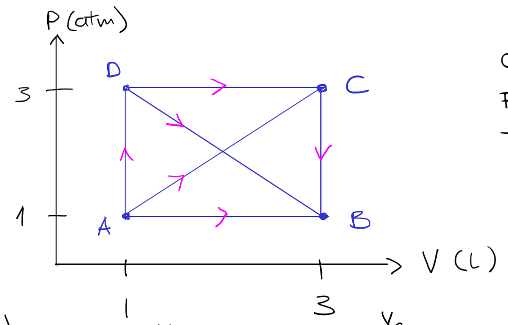
$$Q^{air} = nC_V\,\Delta T \approx n(2.5R)\Delta T \simeq 1.5\,\text{mole}\left(2.5 \cdot 8.31\,\frac{J}{mol \cdot K}\right) \cdot 8 \simeq \underline{250\,J}$$

b) For Xenon, ideal gas $C_V = \frac{3}{2}R$

$$\rightarrow Q^{Xenon} = n(1.5R)\,\Delta T \approx \underline{150\,J}$$

1

Problem 1: (11-03-28)



Quasi-static processes

Find w done by the gas

The type of gas is not specified

a) $W_{AB} = \int_{V_A}^{V_B} p\,dV = p_A \int_{V_A}^{V_B} dV$ as p is Constant

$= p_A (V_B - V_A) = 1.013 \cdot 10^5 \, Pa \cdot 2 \cdot 10^{-3} \, m^3$

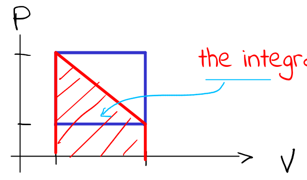
$= 1.013 \cdot 10^5 \frac{N}{m^2} \cdot 2 \cdot 10^{-3} \, m^3 = 203 Nm = \underline{203 J}$

2

b) $W_{ADB} = \underbrace{W_{AD}}_{=0} + W_{DB} = W_{DB}$

as $dV = 0$

$W_{DB} = \int_{V_D}^{V_B} p\,dV$ "="



the integral indicated

$= 2 \cdot W_{AB} = \underline{406 J}$

c) $W_{ACB} = W_{AC} + \underbrace{W_{CB}}_{=0} = W_{AC} = W_{DB} = 406 J$

$dV = 0$ Similar integral as in b)

d) $W_{ADCB} = \underbrace{W_{AD}}_{=0} + W_{DC} + \underbrace{W_{CB}}_{=0} = W_{DC} = 3 W_{AB}$

$dV = 0$ $dV = 0$

$= \underline{609 J}$

3

Problem 2: (11-03-76)

a) Adiabatic

$V_i = 2.0 \cdot 10^{-3} \, m^3$, $T_i = 300 K$, $p_i = 5.0 \cdot 10^5 \, Pa$

$V_f = 2.5 \cdot 10^{-3} \, m^3$

Find T_f and p_f $\gamma = 5/3$

$pV^\gamma = \text{Const.}$ (3.12)

$p^{1-\gamma} T^\gamma = \text{Const.}$ (3.13)

$TV^{\gamma-1} = \text{Const.}$ (3.14)

$\rightarrow p_f V_f^\gamma = p_i V_i^\gamma \rightarrow p_f = p_i \left(\frac{V_i}{V_f}\right)^\gamma$

$= 5.0 \cdot 10^5 \, Pa \cdot \left(\frac{2.0}{2.5}\right)^{5/3}$

$\approx \underline{3.45 \cdot 10^5 \, Pa}$

4

$T_f V_f^{\gamma-1} = T_i V_i^{\gamma-1} \rightarrow T_f = T_i \left(\frac{V_i}{V_f}\right)^{\gamma-1}$

$= 300 K \left(\frac{2.0}{2.5}\right)^{\frac{5}{3}-1} \approx \underline{260 K}$

b) isothermal

Find T_f and p_f

$T_f = T_i = 300 K$

$pV = nRT$, $T = \text{Const.} \rightarrow p_f V_f = p_i V_i$

$\rightarrow p_f = p_i \left(\frac{V_f}{V_i}\right) = 5.0 \cdot 10^5 \, Pa \left(\frac{2.0}{2.5}\right)$

$= \underline{4.0 \cdot 10^5 \, Pa}$

5

Problem 3: (11-04-50)

Ideal gas

$n = 1$ mol

$V_f = 2V_i$

Isothermal reversible expansion

a) Find ΔS_{gas}, $pV = nRT$, $E_{int} = \frac{3}{2} nRT$

$\Delta E_{int} = 0$

$\Delta E_{int} = Q - W = 0 \rightarrow W = Q$

$$W = \int_{V_i}^{V_f} p\,dV = nRT \int_{V_i}^{V_f} \frac{dV}{V} = nRT \ln\left(\frac{V_f}{V_i}\right) = nRT \ln(2)$$

$\rightarrow Q = nRT\ln(2)$

isothermal $\rightarrow$ $\boxed{\Delta S = \frac{Q}{T} = nR\ln(2)}$

$= 1\,\text{mol} \cdot 8.314 \frac{J}{K \cdot mol} \ln(2) = 5.76 \frac{J}{K}$

6

b) isothermal $\rightarrow T_f = T_i$

This is thus not a proper question

Problem 4: (11-04-64)

Carnot engin: $e = 1 - \frac{T_c}{T_h}$

$e_i = 0.60 \rightarrow e_f = 0.55$ as $T_c^i \rightarrow T_c^f$

$T_h^f = T_h^i$

a) Find $T_h^i = T_h^f$

$e_i = 1 - \frac{T_c^i}{T_h^i}$

$\rightarrow e_i - 1 = -\frac{T_c^i}{T_h^i}$

$\rightarrow \frac{T_c^i}{T_h^i} = 1 - e_i$

$\rightarrow \boxed{T_h^i = \frac{T_c^i}{1 - e_i}} = \frac{(27 + 273)\,K}{1 - 0.6}$

$= 750\,K = 477\,^\circ C$

7

b) $e_f = 1 - \frac{T_c^f}{T_h^f} = 1 - \frac{T_c^f}{T_h^i}$

$\rightarrow \frac{T_c^f}{T_h^i} = 1 - e_f$

$\rightarrow \boxed{T_c^f = T_h^i(1 - e_f) = \left(\frac{T_c^i}{1 - e_i}\right)(1 - e_f)}$

$= 750 \cdot (1 - 0.55)\,K$

$\simeq 338\,K \simeq 64.5\,^\circ C$

1

Problem 1: (11-05-50)

Na Cl

Singly charged, what force is between the ions

$d = 2.82 \cdot 10^{-10}\,\mathrm{m} = 2.82\,\text{Å}$

$e = 1.602 \cdot 10^{-19}\,\mathrm{C}$

$k_e = 8.99 \cdot 10^{9}\,\frac{\mathrm{N \cdot m^2}}{\mathrm{C^2}}$

$$\boxed{F = |\vec{F}|} = \frac{1}{4\pi\epsilon_0}\frac{e^2}{d^2} = \boxed{k_e \frac{e^2}{d^2}}$$

$$\rightarrow F = 2.901 \cdot 10^{-9}\,\mathrm{N} \approx 2.90\,\mathrm{nN}$$

Problem 2: (11-05-66)

Two forces on a dust particle

$\vec{E} = 100\,\mathrm{N/C}\ (= 100\,\mathrm{V/m})$

w

$F_g = mg\,,\quad F_E = eE$

$m = 2.0 \cdot 10^{-15}\,\mathrm{g} = 2.0 \cdot 10^{-18}\,\mathrm{kg}$

$F_g = 1.96 \cdot 10^{-17}\,\mathrm{N}$

$F_E = 1.602 \cdot 10^{-17}\,\mathrm{N}$

} Comparable

for e

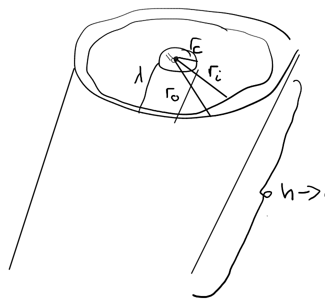
2

If charged -e

$$\rightarrow \boxed{a = -g + \frac{e}{m}E} = -9.81 + \frac{1.602 \cdot 10^{-19}}{2.0 \cdot 10^{-18}}\,100$$

$$= -1.8\,\mathrm{m/s^2}$$

Problem 3: (11-06-52)

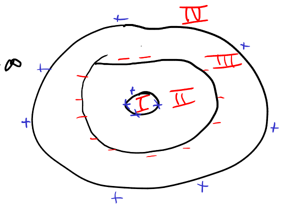


$r_o = 3.0\,\mathrm{cm}$

$r_i = 2.0\,\mathrm{cm}$

$r_c = 1.0\,\mathrm{cm}$

$\lambda = 4\,\mathrm{pC/m}$



Use Gauß

$$\oint_S \vec{E} \cdot \hat{n}\, dA = \frac{q_{enc}}{\epsilon_0}$$

The symmetry of the cylinder makes the electrical field only to have a radial component

$$2\pi r E_r \cdot h = \frac{\lambda h}{\epsilon_0}$$

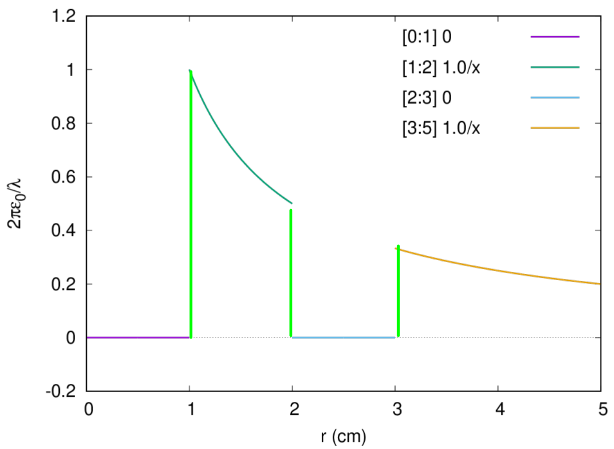
3

$\rightarrow$ I : $\vec{E} = 0$

III : $\vec{E} = 0$

In II and IV we have

$$\vec{E} = \frac{\lambda \hat{r}}{2\pi\epsilon_0 r}$$



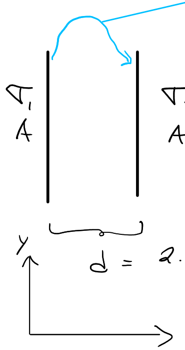
The green vertical lines are only to indicate the dis-continuity of the electrical field at the metal surfaces, that are caused by the surface charge there

4

Problem 4: (11-06-68)

Two parallel plates

move $N_e = 1.0 \cdot 10^{12}$ electrons



A = 400 cm²

Initially $\sigma_1 = \sigma_2 = 0$, but after the move of N_e

a)
$$\boxed{\begin{aligned} \sigma_1 &= +N_e \frac{e}{A} \\ \sigma_2 &= -N_e \frac{e}{A} \end{aligned}}$$

$$\rightarrow \boxed{\vec{E} = \frac{\sigma_1}{\epsilon_0}\hat{x}}$$

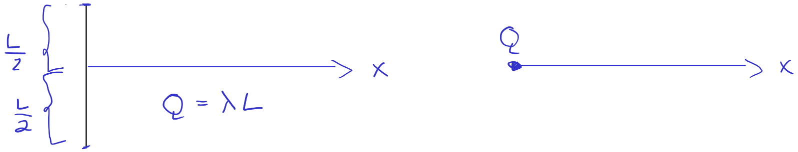
$$= -\frac{1 \cdot 10^{12} \cdot 1.602 \cdot 10^{-19}}{8.85 \cdot 10^{-12} \cdot 400 \cdot 10^{-4}}\,\hat{x}\ \mathrm{N/C}$$

$$= -4.53 \cdot 10^{5}\,\hat{x}\ \mathrm{N/C} = -4.53 \cdot 10^{5}\,\hat{x}\ \mathrm{V/m}$$

1

Problem 1: (11-07-54)

Compare the electrical potential from a point charge and a short line charge



Point charge:

$$V_p(x) = \frac{1}{4\pi\epsilon_0}\frac{Q}{x}, \quad V_p(\infty) = 0 \leftarrow \text{boundary condition}$$

line charge:

Use Ex. 7.13 $\rightarrow$ $V_L(x) = \frac{1}{4\pi\epsilon_0}\lambda \ln\left[\frac{L+\sqrt{L^2+4x^2}}{-L+\sqrt{L^2+4x^2}}\right]$

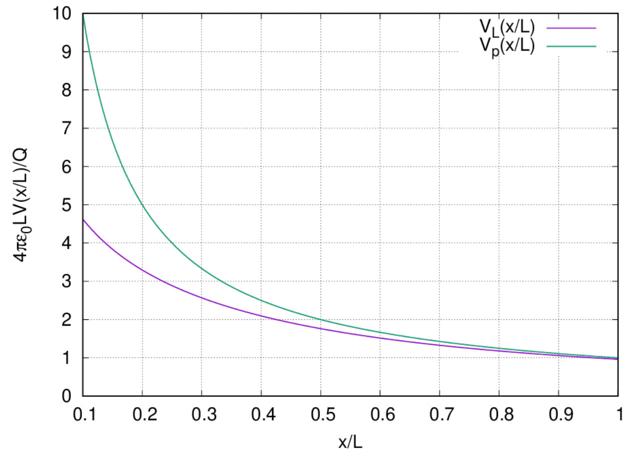
same boundary condition, $\lambda L = Q$

2

rewrite

$$V_L(x) = \frac{Q}{4\pi\epsilon_0 L}\ln\left[\frac{\sqrt{1+4\left(\frac{x}{L}\right)^2}+1}{\sqrt{1+4\left(\frac{x}{L}\right)^2}-1}\right] \rightarrow \boxed{\text{plot } \left(\frac{4\pi\epsilon_0 L V}{Q}\right) \text{ vs. } \frac{x}{L} \quad \text{dimensionless}}$$

$$V_p(x) = \frac{Q}{4\pi\epsilon_0 L}\left(\frac{L}{x}\right)$$

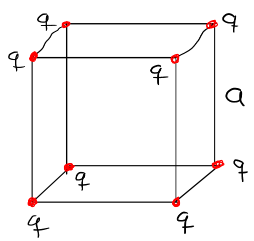


close to the charge the potential of the point charge is stronger. At distance $x/L = 1$ both have the same value, and at infinity they have the same value 0

why? Yes, as far away both tried to look like a point charge. This would never be true for an infinite line charge, which does not have any natural length scale

3

Problem 2: (11-07-74)



$q = +3\mu C$

$a = 2\,cm$

Find the energy of the configuration

$$W = \frac{k}{2}\sum_{\substack{i,j\\ i\neq j}}^{N}\frac{q_i q_j}{r_{ij}}$$

sum over pairs, does not matter where the origin of the coordinate system is,

$7\cdot 8/2$ pairs

$$W = kq^2\left\{\frac{3}{a} + \frac{3}{\sqrt{2}\,a} + \frac{1}{\sqrt{3}\,a}\right]\cdot 28$$

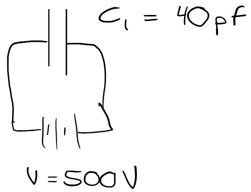
$$= \frac{kq^2}{a}\left\{3 \cdot \frac{3}{\sqrt{2}} + \frac{1}{\sqrt{3}}\right]\cdot 28$$

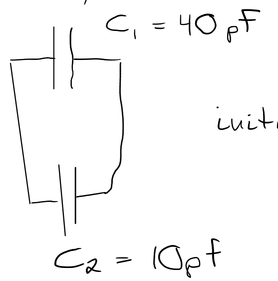
$$\simeq \frac{kq^2}{a}\cdot 160$$

4

Problem 3: (11-08-38)

Initially $\rightarrow$ Finally





initially $Q_2 = 0$

a) Find Q_1^i

$$C_1 = \frac{Q_1^i}{V_1^i} \rightarrow Q_1^i = V_1^i C_1$$

b) Find Q_1^f and Q_2^f

c) $V_1^f = V_2^f$: equilibrium, we us this in b)

$$V_1^f = \frac{Q_1^f}{C_1} \quad \text{and} \quad V_2^f = \frac{Q_2^f}{C_2}$$

$$V_1^f = V_2^f \longrightarrow \boxed{\frac{Q_1^f}{C_1} = \frac{Q_2^f}{C_2}} \quad (1)$$

and Conservation of charge $\longrightarrow$

$$\boxed{Q_1^f + Q_2^f = Q_1^i} \quad (2)$$

Two linear equations for the two unknown quantities Q_1^f and Q_2^f

(1) $\longrightarrow Q_2^f = Q_1^f \frac{C_2}{C_1}$ use in (2)

$$\longrightarrow Q_1^f + Q_1^f \frac{C_2}{C_1} = Q_1^i \longrightarrow Q_1^f \left\{ 1 + \frac{C_2}{C_1} \right\} = Q_1^i$$

$$\longrightarrow \boxed{Q_1^f = \frac{Q_1^i}{1 + \frac{C_2}{C_1}}} \qquad \boxed{Q_2^f = \frac{Q_1^i \frac{C_2}{C_1}}{1 + \frac{C_2}{C_1}}}$$

Problem 4: (11-09-58)

100 W incand. $\sim$ 16 W LED in terms of Light

1 kWhr = \$ 0.10

4 hr per day in one year

$$\longrightarrow P_{LED} \cdot 4 \cdot 365 = 16 \cdot 4 \cdot 365 = \text{Energy}$$
$$= 23\,360 \text{ Whr}$$
$$= 23.360 \text{ kWhr}$$

$\longrightarrow$ Cost $= \$2.34$

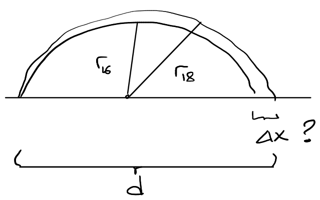
Problem 1: (11-11-56)

$^{16}O \qquad ^{18}O$

$M_{16} = 2.66 \cdot 10^{-26}$ kg $\qquad B = 1.20\,T$

Singly charged

$v = 5.00 \cdot 10^{6}$ m/s $\qquad \dfrac{M_{16}}{M_{18}} = \dfrac{16}{18}$



$$r = \frac{mv}{qB}$$

$$\Delta x = 2r_{18} - 2r_{16} = 2(r_{18} - r_{16})$$

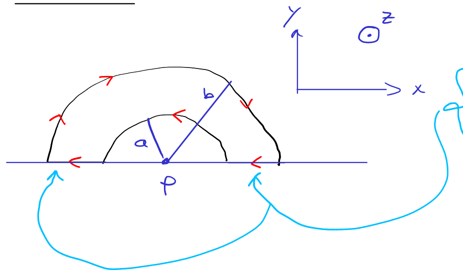
$$= 2\frac{v}{qB}\{M_{18} - M_{16}\}$$

$$\boxed{= \frac{2v}{qB} M_{16}\left\{\frac{M_{18}}{M_{16}} - 1\right\}}$$

$$\Delta x = \frac{2v}{qB} M_{16}\left[\frac{18}{16} - 1\right]$$

$$= \frac{2 \cdot 5 \cdot 10^{6} \cdot 2.66 \cdot 10^{-26}}{1.602 \cdot 10^{-19} \cdot 1.20}\left(\frac{18}{16} - 1\right) \approx \underline{0.173 \text{ m}}$$

Problem 2: (11-12-18)



Use Biot-Savart and Ex. 12.2

B-S gives B(P) = 0 for the straight segments, but not for the arcs

Inner arc:

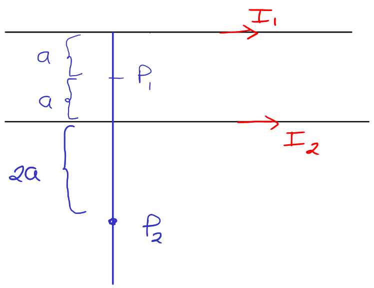
$$\vec{B} = -\frac{\mu_0 I \pi}{4\pi a}\hat{z}$$

outer arc:

$$\vec{B} = +\frac{\mu_0 I \pi}{4\pi b}\hat{z}$$

$$\rightarrow \boxed{\vec{B}_P = -\frac{\mu_0 I}{4}\hat{z}\left(\frac{1}{a} - \frac{1}{b}\right)}$$

Problem 3: (11-12-26)



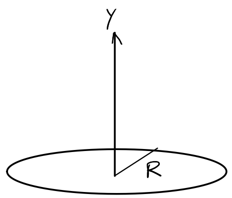
Use $B = \dfrac{\mu_0 I}{2\pi R}$

with right hand rule

P_1 : $\quad \vec{B} = \dfrac{\mu_0 I_2}{2\pi a}\hat{z} - \dfrac{\mu_0 I_1}{2\pi a}\hat{z} = \underline{\dfrac{\mu_0 \hat{z}}{2\pi a}\left[I_2 - I_1\right]}$

P_2 : $\quad \vec{B} = -\dfrac{\mu_0 I_2}{2\pi\, 2a}\hat{z} - \dfrac{\mu_0 I_1}{2\pi\, 4a}\hat{z} = \underline{-\dfrac{\mu_0 \hat{z}}{4\pi a}\left[I_2 + \dfrac{I_1}{2}\right]}$

Problem 4: (11-12-38)



At what distance $B(y) = B(0)/2$

use section 12.4

$$\frac{\mu_0 I \pi R^2}{2\pi (y^2 + R^2)^{3/2}} = \frac{\mu_0 I}{4R}$$

$$\rightarrow \frac{R^2}{2(y^2+R^2)^{3/2}} = \frac{1}{4R} \rightarrow \frac{4R^6}{(y^2+R^2)^3} = 1$$

or $\quad \dfrac{4^{1/3} R^2}{(y^2+R^2)} = 1 \quad \rightarrow \quad 4^{1/3} R^2 = y^2 + R^2$

$$\rightarrow y^2 = (4^{1/3} - 1)R^2 \approx 0.5874\,R^2$$

$$\boxed{y = \sqrt{4^{1/3} - 1} \cdot R} \approx 0.7664\,R$$